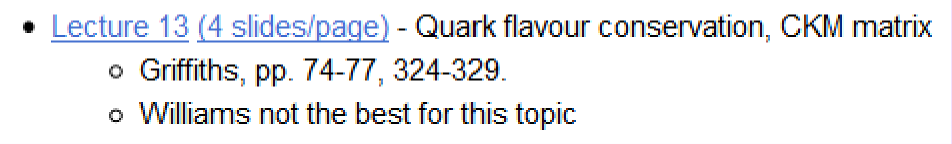
Outline

- Relativistic Kinematics
  - (4-momentum)2 invariance, invariant mass
  - Hypothesis testing, production thresholds
  - Cross-sections, flux and luminosity, accelerators
  - Particle lifetime, decay length, width
- Classification of particles
  - Fermions and bosons
  - Leptons, hadrons, quarks
  - Mesons, baryons
- Quark Model
  - Meson and baryon multiplets
  - Isospin, strangeness, c, b, t quarks
- Particle Interactions
  - Colour charge, QCD, gluons, fragmentation, running couplings
  - Strong and weak decays, conservation rules
  - Virtual particles and range of forces
  - Parity, charge conjugation, CP
  - Weak decays of quarks
  - Charmonium and upsilon systems
- Electroweak Interactions
  - Charged and neutral currents
  - W, Z, LEP experiments
  - Higgs and the future
- LHC Experiments
- Future – introduction to accelerator physics

Today

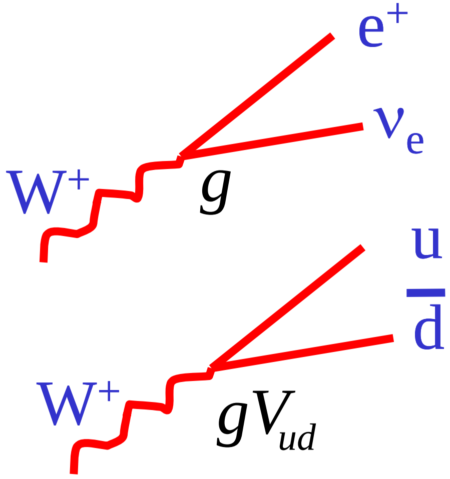
Previous lecture



- Lecture 13 (4 slides/page) - Quark flavour conservation, CKM matrix
  - Griffiths, pp. 74-77, 324-329.
  - Williams not the best for this topic

W coupling to quarks

- Cabibbo theory
  - W boson couples to lepton/neutrinos with universal coupling, g
  - W boson couples to **u** quark and a **d** quark with coupling g.**V**$_{ud}$



- W does not couple directly to physical/mass states of u and d quarks
  - Actually couples to a u-type and a linear combination of d-type quarks in all 3 generations
  - Without this, would be no decay of lightest strange mesons, etc.!

- Effectively, a "rotation" of mass eigenstates to the weak eigenstates

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{vmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{vmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

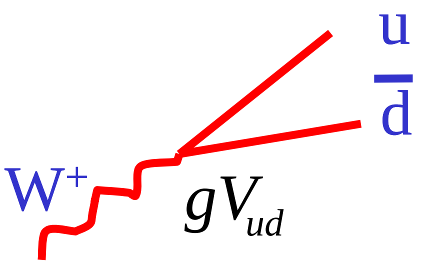
$$i.e. \quad d' = d\cos\theta_C + s\sin\theta_C$$

$$s' = -d\sin\theta_C + s\cos\theta_C$$

$$\theta_C \approx 13.1^0$$

Cabibbo Theory and Golden Rule

- Convention to "rotate" d type quarks, but could equally well rotate u type quarks



- Cabibbo theory is historical name when dealing with 1st and 2nd generations only

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \left| \begin{matrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{matrix} \right| \begin{pmatrix} d \\ s \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

$$= \begin{pmatrix} 0.974 & 0.225 \\ 0.225 & 0.974 \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

- "Fermi Golden Rule" for decay/scattering rates – see QM text
- Rate is product of two terms

$$Rate = \tfrac{2\pi}{\hbar} \left| M_{fi} \right|^2 \rho(E)$$

$$\rho(E)\ is\ phase\ space\ factor, dN/dE$$

- A dynamical term, derived from Feynman diagrams.
    - Enters as the |QM amplitude|^2 , (sometimes called "matrix element"^{2})
    - For our case, amplitude is proportional to the product of couplings present in diagram
- A kinematical term, purely depending on energy/momenta/masses
    - Independent of particle types
    - Variously called "phase space factor" or "density of states"
    - More energy available in final state (apart from masses) increases this statistical counting factor: number of states possible with energy *E* available

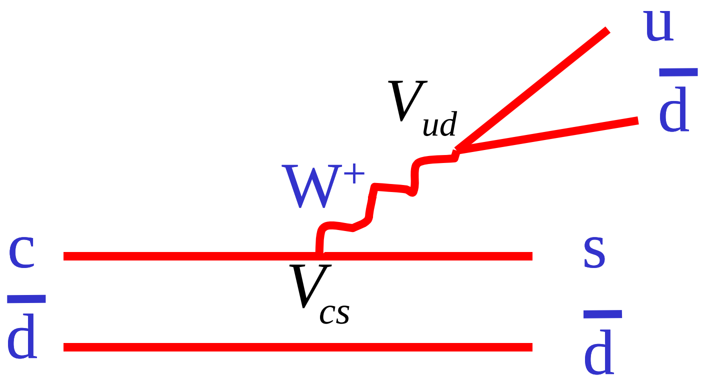
Example: Cabibbo Suppression

$$D^+ \rightarrow \overline{K}^0 \; \pi^+$$

$$(c\overline{d}) \rightarrow (s\overline{d})(u\overline{d})$$

Amplitude

$$\propto V_{ud} V_{cs}$$



Rate $\propto \left|V_{ud} V_{cs}\right|^2 = \left|\cos\theta_C \; \cos\theta_C\right|^2$

“Cabibbo favoured” decay

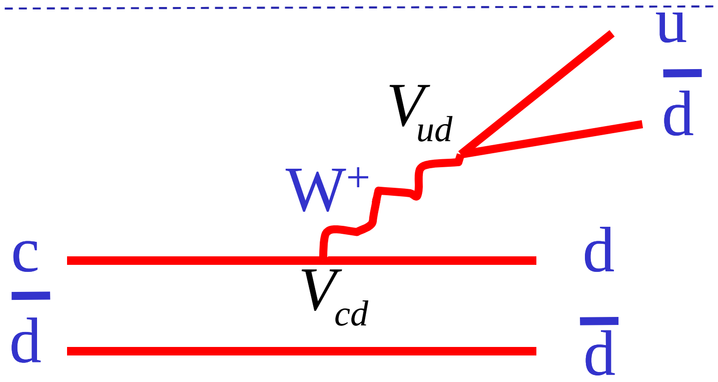
$\propto V_{ud} V_{cd}$

$$D^+ \rightarrow \pi^0 \; \pi^+$$

$$(c\overline{d}) \rightarrow (d\overline{d})(u\overline{d})$$

Amplitude

$$\propto V_{ud} V_{cd}$$



Rate $\propto \left|V_{ud} V_{cd}\right|^2 = \left|\cos\theta_C \; \sin\theta_C\right|^2$

“Cabibbo suppressed” decay

Relative rates $\dfrac{D^+ \rightarrow \overline{K}^0 \; \pi^+}{D^+ \rightarrow \pi^0 \; \pi^+} \propto \left|V_{ud} V_{cs}\right|^2 / \left|V_{ud} V_{cd}\right|^2 = \left|1/\tan\theta_C\right|^2 \sim 1/20$

3 Flavour case: CKM Matrix

- Generalisation to 3 generations: CKM (Cabibbo-Kobayashi-Maskawa) Matrix
- Rotation of mass to weak eigenstates
- 2x2 Cabibbo matrix is merely the upper left part of the full 3x3 CKM matrix
- $\lambda = \sin\theta_C$

Cabibbo

$$\left|\begin{matrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{matrix}\right| = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix}$$

CKM (magnitudes only)

$$\left|\begin{matrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{matrix}\right| \approx \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

$$\left|\begin{matrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{matrix}\right| \approx \begin{pmatrix} 0.974 & 0.225 & \boxed{0.003} \\ 0.225 & 0.973 & \boxed{0.041} \\ \boxed{0.008} & 0.040 & 0.999 \end{pmatrix}$$

- Note: small size of
  - V_{ub}, V_{cb}, V_{td}
  - Observations?? Impact on lifetimes of b, c decays?

Strange Particle Production

- Properties of SI, WI important in production/decay of strange and heavier quarks
- Production requires strong (or EM) to give large cross-section. Requires $s\bar{s}$ production to conserve strangeness
  - Strange hadrons created in pairs - so-called "associated production"
  - Examples of production (on board): p (π^+, π^-, K^-)
- SI decays 13 orders magnitude faster than WI, so WI decays only observed when SI/EM forbidden by (e.g.) flavour conservation rules.
- Heavy strange hadrons (i.e. excited states) decay to lighter strange hadrons by SI
  - Examples of SI decays (on board): p (π^+, π^-, K^-)

  - Conserves flavour and other quantum numbers
  - Typical lifetime of strong decays ~10^{-22}-10^{-23}s, called resonances
- Process continues until arrive at lightest strange hadrons (kaons), which are stable under SI because of strangeness conservation
- WI decays only observed when SI and EM forbidden by, e.g. flavour conservation.

Light Strange Particle Decays

- Lightest strange particles
- Mesons
  - $K^+ = u\ s$, $K^0 = d\ s$, $K^0 = d\ s$, $K^- = u\ s$
    - ⇒ $S = \pm 1$
- Baryons
  - $\Lambda^0 = u\ d\ s$
    - ⇒ $S = -1$
  - $\Sigma^+ = u\ u\ s$, $\Sigma^- = d\ d\ s$,
    - ⇒ $S = -1$
    - ⇒ [not $\Sigma^0 = u\ d\ s$ – not lightest uds: $\boldsymbol{m_{\Sigma^0} - m_\Lambda}$ = (76.959 ± 0.023) MeV
  - $\Xi^0 = u\ s\ s$, $\Xi^- = d\ s\ s$
    - ⇒ $S=-2$
  - $\Omega^- = s\ s\ s$
    - ⇒ $S=-3$
  - Feynman diagrams on board
- These can only decay with change in strangeness, so must be weak decays.
- Typical lifetimes ~ 10^{-8} – 10^{-11} s

Heavy Quark (c, b, t) Decays

- Same pattern as in strange hadrons
- SI, EM and weak neutral current (Z^0) also conserve flavour for C, B, T
  - Each flavour number can change by 1 unit in a single W mediated reaction
- Heavy particles decay to lighter by SI, conserving flavours
  - e.g. $\tau \sim 10^{-23}$s, Feynman diagram on board

$$D^{*+} \to D^0 \pi^0$$
$$(c\bar{d}) \to (c\bar{u})(u\bar{d})$$

- Lightest particle of each flavour decays by WI, changing flavour
  - e.g $D^0 \to K^- \pi^+$ $\tau \sim 4.1\times10^{-13}$s, diagram on board

$$(c\bar{u}) \to (s\bar{u})(u\bar{d})$$

  - e.g. $B^+ \to \overline{D}^0 \pi^+$ $\tau \sim 1.6\times10^{-12}$s, diagram on board

$$(u\bar{b}) \to (u\bar{c})(u\bar{d})$$