Year 2 Particles and Nuclei - Lectures 4 & 5

We've discussed the idea that there are 3 forces (plus gravity), which are carried by particular particles. Now let's look at the particles they act between.

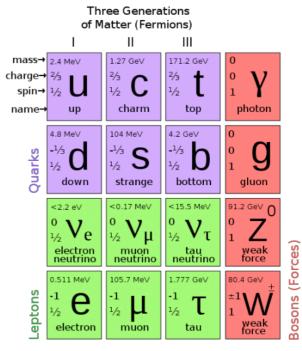
Quarks and Leptons:

We normally divide the particles into 2 groups:

- Quarks: which carry colour charges.
- · Leptons: which do not.

Their properties are summarised in this graphic. Note that some of these values are still evolving (e.g. 173.1 GeV¹ would be a better value for the mass of the top quark).

Before we consider the specific properties of these individual particles though, there are a couple of other terms which appear here which need explaining:



Fermions, Bosons and Spin:

One of the properties listed in the table is the "spin" of each particle. This describes the angular momentum that each particle carries independently of its motion, i.e. even if an electron or a W were at rest it would still have some angular momentum intrinsically associated with it. The name "spin" arose because the nearest classical analogy would be to think of the particle as spinning, like e.g. the Earth spinning about its axis. However, you should not take that picture as literally true (it is extremely easy to show that it is not!) – just think of it as another property of the particle.

In quantum mechanics angular momentum is quantised in units of \hbar . So when we say that a particle, like the W, has spin = 1 we mean that it carries 1 unit of angular momentum. If you measure the projection of the angular momentum onto any axis (i.e. a component of angular momentum) this is quantised, so that for a spin-1 particle the possible values are $+\hbar$,0, $-\hbar$ (i.e. integer multiples of \hbar).

Quarks and leptons carry half a unit of angular momentum. This was rather a surprise when it was discovered, since (a) the idea of spin was itself a surprise, and (b) you can show that the angular momentum due to motion is quantised in integer units, so a half-integer angular momentum was inexplicable! The component still changes in steps of \hbar , so if you measure the projection of angular momentum for a "spin-1/2 particle" there are two possible results: $+\hbar/2$

 $^{^1\} http://pdg8.lbl.gov/rpp2013v2/pdgLive/Particle.action?node=Q007$

("spin up") or $-\hbar/2$ ("spin down"). Half-integer spin was eventually found to be a feature of Dirac's relativistic quantum equation for the electron (which also predicted the existence of antimatter).

Conversely, all of the force-carrying particles are found to be spin-1.

All particles of a particular type are "identical" or "indistinguishable" in that all of their measurable properties are the same. This allows us to talk about e.g. the mass or charge of an electron, because all electrons have the same mass or charge. This turns out to be important when we have a system with more than one particle of the same type in it. In classical mechanics you can always distinguish between two identical particles, because you can follow their trajectories and "keep track" of which is which. In quantum mechanics you cannot do this (observing a particle changes its state), so if the wavefunctions describing the particles overlap then after some time you cannot say with certainty which one is which. So you if you have 2 identical particles, 1 and 2, you cannot distinguish between "particle 1 in state A plus particle 2 in state B" and "particle 1 in state B plus particle 2 in state A", so the wavefunction describing 2 identical particles, one in state A and one in state B, should contain both possibilities, e.g.

$$\psi_a(1)\psi_b(2) + \psi_a(2)\psi_b(1)$$

or

$$\psi_a(1)\psi_b(2) - \psi_a(2)\psi_b(1)$$

There's an important difference between the first example, which is "symmetric" if you swap particles 1 and 2 (unchanged), and the second, which is "anti-symmetric" (changes sign). What happens if both particles are in the same state?

• The symmetric wavefunction just becomes $\psi_a(1)\psi_a(2)$, while the anti-symmetric one disappears!

Particles described by the first type of wavefunction are called "bosons", after Satyendra Nath Bose, who first studied the consequences of this type of behaviour, while particles of the second type are called "fermions" after Enrico Fermi, who together with Paul Dirac studied this second case (Dirac always insisted that Fermi was first to look at it).

It turns out that particles with integer spin are bosons, while those with half-integer spin are fermions. As with the existence of spin itself, this is a feature of relativistic quantum mechanics.

This whole issue is very important in statistical physics (thermodynamics). However, for our purposes the most important consequences is contained in the bullet point above: no two identical fermions can exist in the same quantum state. This is called the Pauli Exclusion

Principle. It has profound consequences for the structure of matter: the "shell" structure of electrons, responsible for the variation in chemical properties described in the Periodic Table, is due to the Exclusion Principle. To consider a simple example, consider helium. The lowest-energy electron state in an atom has zero angular momentum (really!). You can have 2 electrons in this state, one spin-up and one spin-down (so their states are not identical). But you can't have a third, since that would have to be in the same state as one of the other two, which is not allowed. Hence in a lithium atom the third electron must be in a higher energy (more loosely-bound) state than the other two, which is why lithium is reactive while helium is inert. This same effect is visible in the structure of nuclei, and was important in understanding the properties of quarks.

Leptons:

The name "lepton" means "light particle", to distinguish them from the baryons ("heavy particles") such as the proton or neutron. As with many historical names it's not wholly valid – the heaviest lepton is almost twice the mass of the lightest baryon – but it has stuck.

The Electron was the first fundamental particle to be discovered. It is the lightest charged particle (mass 511 keV/c^2), and as such should be stable (current limit: mean life > 46×10^{25} years²). We use the magnitude of the electron charge as our basic unit of charge, and so in those units say that the electron has a charge of -1.

<u>The electron neutrino</u> was first proposed in 1930 by Wolfgang Pauli. The motivation for doing so came from the energy spectra of beta radiation.

A "beta ray" is an electron emitted in nuclear decay, e.g.

 $^{3}\text{H} \rightarrow ^{3}\text{He} + \text{e-}$ In these decays no other particles appear to be produced.
If this were truly a 2-body decay, the electron would always be produced with the same energy: the momentum of the electron would equal that of the recoiling He nucleus (as the initial momentum of

F. A. Scott, Phys. Rev. 48, 391 (1935)

V (electron volts)

the ³H atom is negligible), and the kinetic energies of the electron and helium nuclei would equal

 $^{^2\} http://pdg8.lbl.gov/rpp2013v2/pdgLive/Particle.action?node=S003$

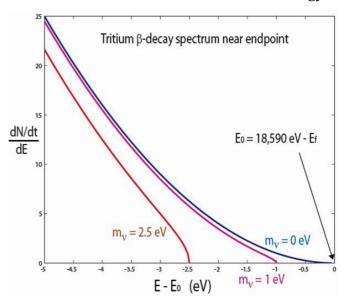
the mass difference between ³H and ³He + e-. Using these two constraints (energy and momentum conservation) and the masses of the 3 particles you calculate an electron energy of 18.59 keV.

In fact what you see is a distribution of electron energies, peaking at some fraction of the maximum, though with a tail all the way up to the value predicted by the mass difference. Pauli proposed that this spectrum could be explained if a third particle was produced in the decay. This particle had to be light (because the maximum beta energy was close to the 2-body value), and neutral (to conserve charge). It also could not be a photon, since it was not observed in any detector.

In 1934 Chadwick discovered the neutron. As this was far too massive to be Pauli's particle, Fermi christened Pauli's proposed particle the "neutrino", or "little neutral one".

The most direct measurements of the neutrino mass come from measurements of the energy

spectrum of tritium beta decay. This plot shows how the spectrum would appear for three different masses of the neutrino. Note the mass scales: we are talking about masses of $O(1eV/c^2)$, whereas even the electron has a mass $\sim 500,000$ times this. Measurements of this sort current set a limit on the neutrino mass of $< 2eV/c^2$. Indirect estimates (from cosmology or other properties of neutrinos) are more stringent.



Although Pauli thought that his new particle would be undetectable³, neutrinos can indeed be detected and studied through their interactions via the weak force. While individual neutrinos are practically undetectable, with enough neutrinos and enough matter for them to interact with it is possible to observe and study a small fraction.

The muon was discovered by Anderson and Neddermayer studying cosmic rays in 1936, and confirmed in cloud chamber measurements in 1937. By applying a magnetic field to the detector and observing the curvature of the tracks it was estimated that the muon was intermediate in mass between the electron (511 keV) and the proton (938 MeV). Muons penetrate much more deeply through matter than protons because they do not experience the strong force, while being

³ Famously saying "I have done a terrible thing. I have postulated a particle that cannot be detected"

heavier than electrons they are not so easily scattered and so do not lose energy as rapidly through bremsstrahlung (braking radiation, where an accelerated charged particle emits photons).

Muons are unstable, decaying with a lifetime of $2.2 \times 10^{-6} s$. In almost all cases the only visible decay product is a single electron. However, not only must there be at least one other particle produced (in order to conserve energy and momentum), but the electrons produced have a range of energies, suggesting that there are more than 2 particles produced in the decay. The maximum electron energy is $\sim 0.5 \times 10^{-6} s$. In almost all cases the only visible decay produced in the particle are very light. Hence the decay reaction is:

$$\mu^{-} \rightarrow e^{-} \overline{v}_{e} v_{\mu}$$

$$\mu$$

$$v_{e}$$

$$v_{\mu}$$

Apart from their greater mass, the muon is identical in its properties to the electron. It appears to simply be a heavier version.

<u>Muon neutrino</u>: You'll notice that this reaction includes not just 2 neutrinos, but 2 different types of neutrino. Why have I done that? Well it's not because we enjoy multiplying entities needlessly⁴, but because that's what observational evidence requires. To explain this I have to discuss "conservation laws", and introduce a new one.

Conservation laws

Certain quantities must be conserved, which is to say that in any reaction the total amount of these quantities after the reaction must equal the total amount before. Some we have discussed before are:

- Energy
- Momentum
- Electric charge
- Strong and weak charges

(though generally the last are done automatically if you conserve the others. Indeed I've not actually discussed weak charges quantitatively).

Angular momentum is also a conserved quantity in any reaction.

⁴ which William of Occam warns us against.

All of the above have a strong theoretical base: each can be related to some symmetry in the laws of physics (look up "Noether's Theorem" if you want to know more). But there are others which are purely empirical laws.

"Lepton number" conservation:

There are two types of reaction in which electrons are produced. They are either produced in electron-positron pairs or electron-neutrino pairs. Similarly for muons. So while we see reactions like:

- $\gamma \rightarrow e^+ e^-$; $\gamma \rightarrow \mu^+ \mu^-$
- $n \rightarrow p \ e^- \overline{v}_e$; $\pi^- \rightarrow \mu^- \overline{v}_\mu$

we never see reactions like:

- $\gamma \rightarrow e^+ \mu^-$; $\gamma \rightarrow p e^-$
- $n \rightarrow p \ e^{-\gamma}$; $p + e \rightarrow n + \gamma$

So the total number of leptons seems to be conserved: leptons always seem to be produced in lepton-antilepton pairs, and similarly no reaction or decay seems to change the total number of leptons.

But it seems to be even stronger than that: not only is the total number of leptons conserved, but the numbers of electrons and muons are conserved separately!

- If we assign electrons and electron-neutrinos an "electron number" of 1, and positrons and electron-antineutrinos an "electron number" of -1, then the total electron number before and after any reaction is the same.
- And the same is true for "muon number" (+1 for negative muons and muon-antineutrinos, and -1 for their antiparticles).

But why are we sure that the muon neutrino is different from the electron neutrino?

Because of an experiment first carried out at the Brookhaven National Laboratory in 1962. It was known that muons were produced, together with neutrinos, in the decay of pions (light quark-antiquark states – will discuss later). In this experiment an intense beam was used to produce large showers of pions, which decayed to muons and neutrinos. A thick iron wall shielded a detector from muons, and the reactions of the neutrinos with the detector were studied. What was observed was that the neutrinos produced together with muons could produce muons when they reacted with the detector, but never produced electrons. This provided evidence that the

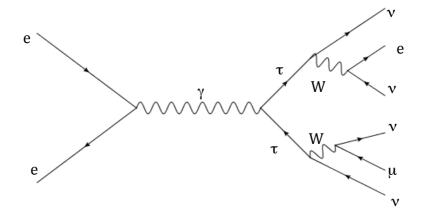
neutrinos produced together with muons are different from those produced together with electrons, and hence that we have two different types of neutrino.

Ironically more refined experiments in recent years have shown that while neutrinos are always produced as one sort or another, they actually do, given long enough, change from one type to another! Thus while electron and muon number are conserved separately in particle reactions and decays, this conservation is not absolute. The more inclusive "lepton number" is however always conserved (so far!).

<u>The tau</u> is an even heavier version of the electron or muon (1.77 GeV/c^2) , or nearly twice the mass of the proton). It was discovered in the early 1970s at the Stanford Linear Accelerator Centre, where experiments studying electron-positron collisions observed reactions of the form:

$$e^+e^- \rightarrow e^\pm \mu^\mp + \text{missing energy}$$

Analysis indicated that there were multiple unobserved particles, presumably neutrinos. These events were interpreted as being due to the production and decay of a new, heavier lepton, dubbed the tau:



Unlike the muon, which being the second-lightest charged particle can only decay to electrons, the tau can decay to hadrons (particles formed from quarks and/or antiquarks) as well as leptons. There is always some missing energy and momentum, indicating that a neutrino is produced in addition.

The tau neutrino is the final lepton. Although it was expected to be distinct from the electron and muon neutrinos, the high mass of the tau made it difficult to produce a sufficiently intense and energetic tau neutrino beam to directly confirm this the way muon neutrinos had been observed. It was finally directly observed by the DoNUT ("Detection of NU Tau") experiment at the Fermi National Accelerator Laboratory in 2000.

Quarks:

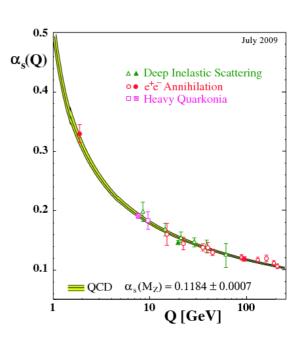
Studying quarks is different from studying leptons because they are never found in isolation. Instead they are always found in one of two types of "bound state": baryons or mesons.

- Baryons are combinations of 3 quarks, one of each colour, forming an overall colourless state.
- Mesons are combinations of quark and antiquark, again giving a colourless state. Collectively these 2 types of particle are called "hadrons".

The fact that both are colourless is important. Because of the strength of the strong force, coloured objects (those with a net strong charge) are never observed as free particles. This was originally an objection to the idea of quarks as physical particles, but is now understood and given a name:

Colour confinement

The key is that the strength of the strong force varies with distance, or equivalently with the exchanged energy/momentum (think of the de Broglie relation, p = h/λ ; large momentum = short wavelength and viceversa). The plot here shows the measured variation of the strong coupling constant (α_s) with the magnitude of the 4-momentum (relativistic energy-momentum vector) transferred in a reaction. So at large distances (low energy-momentum transfer) the force becomes increasingly strong. Hence it becomes impossible to separate a single coloured object from a colour-

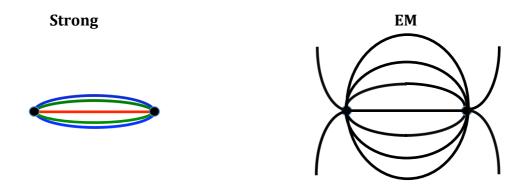


neutral system (e.g. remove a quark from a proton) because of the increasing strength of the interaction when you try to do so.

So what if you hit a quark really hard – give it a momentum that's far larger than the mass of the proton, so that the strong force cannot hold the proton together?

What is observed is not a single quark emerging from the proton, but a shower of hadrons. The way this is understood can be pictured using the "flux tube" model:

• When you separate 2 quarks, the strong force (gluons exchanged) between them is concentrated into a narrow tube by the mutual attraction of the gluons to each other. This is unlike the dipole field in the EM force, where the force gets weaker with separation:



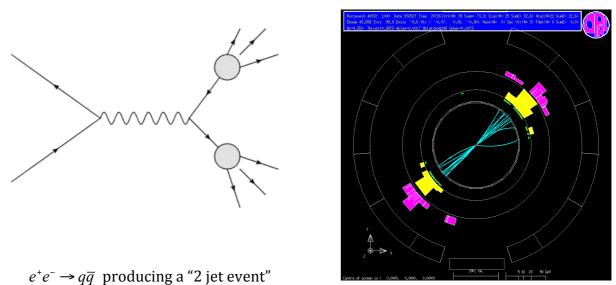
As the separation increases, the stored energy in the "string" gets larger (whereas the EM
potential energy would decrease in magnitude).



• Finally the stored energy becomes so great that the string "breaks". Quark-antiquark pairs are formed, quarks and antiquarks combine into hadrons, and rather than a single isolated quarks we observe a shower of hadrons.

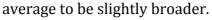


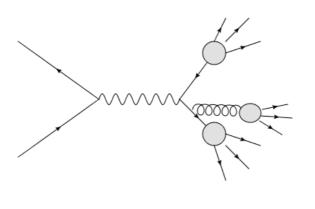
• If the initial quark has a momentum >> hadron masses, the momentum is shared between the produced hadrons and so these form a roughly conical "jet". The energy and direction of the jet correspond more-or-less to that of the original quark.

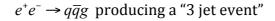


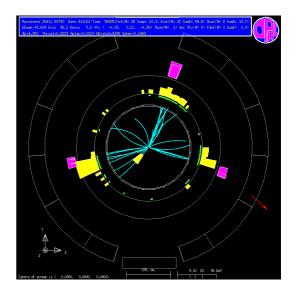
This process of converting energetic quarks to hadron jets is called "fragmentation". The "blobs" in the Feynman diagram indicate that this process is beyond our ability to calculate.

The same is true if you try to isolate an energetic gluon. The gluon will also "fragment" to produce a shower of hadrons. Gluon jets are very similar to quark jets, though tend on







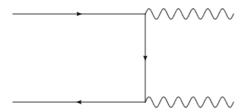


Light mesons and baryons: up and down quarks

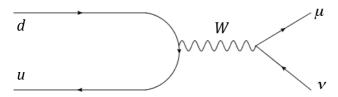
The properties of the quarks were deduced by studying the properties of the mesons and baryons they formed. Confirmation of some of these properties (such as their charges) came from experiments where an electron, photon or Z⁰ interacted directly with the quarks rather than the whole hadron. The first of these were the deep-inelastic electron-proton scattering experiments at SLAC which originally observed the quarks inside the proton. Later scattering experiments, and electron-positron annihilation experiments (such as those illustrated above) provided further confirmation and also allowed study of the weak charges.

The first mesons to be discovered were the pions, π^+ , π^- and π^0 . These particles have spin 0 (all mesons have integer spin), which can be simply understood as the quark and antiquark spins being aligned in opposite directions and cancelling each other. π^+ and π^- are each other's antiparticles, with π^+ being formed from u and \overline{d} quarks and π^- from \overline{u} and d. As such they have the same mass (139.6 MeV/c²), since particle and antiparticle have identical masses. π^0 is an mixture of $u\bar{u}$ and $d\bar{d}$ quark wavefunctions (in the same way that a standing wave is a mixture of two travelling waves going in opposite directions), and is slightly lighter (135.0 MeV/ c^2). Both are unstable, but their lifetimes and decays are very different. The π^- has a lifetime of 2.6×10^{-8} s, and decays mainly (99.99% of the time) to $\mu^- \overline{v}_\mu$, and the π^+ has the same lifetime and decays to $\mu^+ v_\mu^-$ (the antiparticles of those produced in the π^- decay). The π^0 on the other hand has a lifetime of 8.5×10^{-17} s and decays mainly (98.8%) to two photons.

The differences in their lifetimes are due to the fact that different forces are responsible for their decays. The π^0 can decay via the electromagnetic interaction, with the quark and antiquark annihilating to produce 2 photons:



The π^- on the other hand has to decay to a charged particle, which leaves the muon or the electron. Lepton number conservation means that there must be a neutrino produced too, which means this has to be a weak decay:



Why the π^- decays to a muon rather than an electron requires a more detailed understanding of the weak force than we have so far introduced.

In addition to the pions there are more massive mesons which can be formed from combinations of u and d quarks, such as the ρ mesons (mass 775 MeV/c²). These are "excited states" of the pions, with the quark's spins aligned rather than opposed. These, and other heavy mesons formed from the light quarks, are heavy enough to decays to two or more pions. These therefore have a very short lifetime, since they can decay via the strong interaction. The lifetimes are of the order of 10^{-22} - 10^{-24} s, but are usually expressed in terms of the uncertainty in the mass of the meson due to the short lifetime. This is another example of the uncertainty principle: if the particle only exists for a short time, it cannot settle to a definite energy (i.e. mass). In the case of the ρ the width of the distribution of its masses is ~145 MeV, corresponding to a lifetime of ~10- 23 - 10^{-24} s.

Similarly there are also heavier baryon partners to the proton and neutron. Of particular interest are the 4 "Delta" baryons, Δ^{++} , Δ^{+} , Δ^{0} and Δ^{-} . In the static quark model these 4 states can be understood as *uuu*, *uud*, *udd* and *ddd*. Their masses are slightly higher than those of the proton or neutron (1230-1234 MeV/c²), and they decay strongly to a nucleon (proton or neutron) plus a pion. Where they differ from the nucleons, which have spin 1/2, is that the Δ s have spin 3/2. Looking at their other properties, and comparing with other baryons, it seems that there is no orbital angular momentum of the quarks and that instead the 3 quarks' spins are parallel

(summing to give S = 3/2). This is important because it would imply that in the Δ^{++} and Δ^{-} we have 3 identical quarks in the same state, which is forbidden by the Pauli Exclusion Principle. This was one of the original motivations for proposing that the quarks have an extra property, with 3 possible values, which became known as "colour", and which we now understand to be the strong force charge.

Strange particles

At the same time, other mesons and baryons were being discovered which didn't fit the expected pattern of behaviour. Mesons called kaons and baryons called lambda were found which were heavy enough to decay strongly to pions (K) or nucleons + pions (Λ). However, while they did decay to these states, their lifetimes were anomalously long (10^{-8} - 10^{-10} s), so long that leptonic decays such as $K^- \to \mu^- \overline{\nu}_\mu$, which can only occur through the weak force, can form a measurable fraction of their decays.

Another anomaly was that these particles were never produced singly, but rather in pairs. These could be pairs of mesons, meson plus baryon or baryon plus antibaryon. So collectively particles which exhibited these properties were referred to as "strange".

This problem was originally solved by proposing that they possessed a new property called "strangeness". Strange particles ($K^+, K^0, \bar{\Lambda}$) would have strangeness +1 and their antiparticles (K^-, \bar{K}^0, Λ) strangeness -1. Strangeness would be conserved by the strong and electromagnetic interactions, which is why these particles would be produced in pairs with opposite strangeness. However, the weak interaction does not conserve strangeness, and so the lightest strange particles could only decay weakly (heavier ones would decay to lighter ones via the weak or EM interactions). Baryons (but no mesons) with strangeness 2 or 3 were also discovered.

Although you will still read about the strangeness value (or "quantum number"), it is now understood very simply in the quark model. Strangeness merely counts the net number of strange quarks in a hadron: a strange quark has a strangeness -1 and a strange antiquark has strangeness +1 (like electric current, the sign convention is historical and, as it turns out, illogical).

The strong and electromagnetic interactions only produce pairs of quarks and antiquarks of the same type (or "flavour"), i.e. $u\overline{u}$, $d\overline{d}$ or $s\overline{s}$. Only the weak interaction can change quark flavour, in

the same way that it is the only interaction which can change lepton flavour (e.g. muons do not decay to electron + photon). Hence the lightest strange particles are much longer lived than hadrons of similar masses containing only the u and d quarks.

Charm, Bottom and "onia":

The original quark model only had 3 quarks. The remaining 3 were predicted before discovery by Cabibbo, Kobayashi and Maskawa, for reasons we won't go into today.

Charm and bottom quarks (also called "beauty") were first observed in mesons consisting of $c\overline{c}$ (J/ψ) and $b\overline{b}$ pairs (Y) respectively. Later mesons and baryons containing these quarks were observed.

The distinctive thing about these particles is their relatively narrow widths (i.e. long lifetimes) for their high masses. This is due to a couple of factors. We'll use the J/ψ as an example, but the same things apply to the Y.

- The J/ψ mass (3.097 GeV/c²) is less than twice the mass of the lightest charm meson (the D⁰, mass 1.865 GeV/c²). Hence the simplest decay, to a $D^0 \bar{D}^0$ pair, is not allowed.
- So instead the charm quark and antiquark must annihilate each other. However, since the $c\overline{c}$ pair are a "colour singlet" (i.e. uncoloured) they cannot annihilate to a single gluon. In fact the simplest strong decay which is consistent with all of the conservation rules requires 3 gluons to be exchanged (also relatively energetic, since they carry the quark masses, which means the coupling strength is reduced). This makes the decay much less likely:

The decay $J/\psi \rightarrow 3\pi$ via a 3-gluon exchange

In fact the strong decay is suppressed so much that electromagnetic decays to e^+e^- or $\mu^+\mu^-$ are significant as well. It was actually these decays which were first observed.

Light charm or bottom particles, such as the D and B mesons, decay mainly via the weak decay of the heavy quark to a lighter quark. These decays occur via W exchange, so the c quark (charge +2/3) decays to an s or d quark (charge -1/3), and the b quark (charge -1/3) to either c or u. It's also notable that although there is more energy available when decaying to a lighter quark (bigger mass difference = more kinetic energy in the final state, which usually

means the reaction occurs faster) in charm quarks decay to strange \sim 95% of the time, and bottom to charm \sim 99%. Both D and B meson decays have a lifetime of \sim 10⁻¹²s.

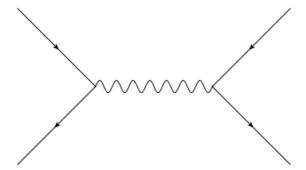
The top quark

The final quark is top, which is considerably heavier than the others. It was finally observed in 1994-5 by the CDF and D0 experiments at Fermilab (near Chicago), using the Tevatron collider to collide beams of protons and antiprotons, each with an energy of 900 GeV.

One distinctive feature of the top quark is that there are no top mesons or baryons. The reason for this is that top decays almost entirely via $t \rightarrow b$ W. Technically that is a weak decay, but the mass difference between the top and bottom quarks, ~ 168 GeV/ c^2 , is large compared even to the mass of the W. Thus this decay is not suppressed by the lack of energy needed to produce a W, the thing that normally makes weak decays weak. Thus the top quark decays very rapidly, with a "width" (uncertainty in its energy) of ~ 2 GeV, corresponding to a lifetime of $\sim 3 \times 10^{-25} \text{s}$. This is too short for the strong force to create quark/antiquark pairs and form hadrons before the top quark decays: to estimate this "hadronisation" time, consider the time that it takes a gluon (travelling at the speed of light) to cross the width of a hadron ($\sim 1 \text{fm}$): time = distance/speed = $\sim 3 \times 10^{-24} \text{s}$. The top quark is therefore the one quark whose properties we can study directly, without having to correct for the other quarks it is bound to in hadrons.

Further evidence for colour: Rhadron

Electron-positron colliders collide beams of electrons and positrons head on. If an electron and a positron annihilate with a centre of mass energy significantly below the mass of the Z, fermion-antifermion pairs are produced through the electromagnetic interaction. The lowest-order process (fewest vertices) is:



If the final state fermion has a charge q, the rate of this reaction is proportional to q^2 .

So if the centre of mass energy of the collision is >> fermion mass, we would expect the ratio

$$\frac{\operatorname{Rate}\left(e^{+}e^{-} \to f \ \overline{f}\right)}{\operatorname{Rate}\left(e^{+}e^{-} \to \mu^{+}\mu^{-}\right)} = q^{2}$$

since the muon has a charge of 1 (in our usual units). We use the $\mu^+\mu^-$ rate as the denominator rather than e^+e^- because the $e^+e^- \rightarrow e^+e^-$ rate includes scattering as well as annihilation, whereas other fermion types can only be produced by annihilation.

So, if we look at the rate of $e^+e^- \rightarrow$ hadrons at an energy of 7-8 GeV, well above the energy needed to produce u, d, s and c quarks, but not enough to produce b quarks, we would expect to see the ratio:

$$R_{hadrons} = \frac{\text{Rate}\left(e^{+}e^{-} \rightarrow hadrons\right)}{\text{Rate}\left(e^{+}e^{-} \rightarrow \mu^{+}\mu^{-}\right)}$$

$$= \sum_{quarks} q^{2}$$

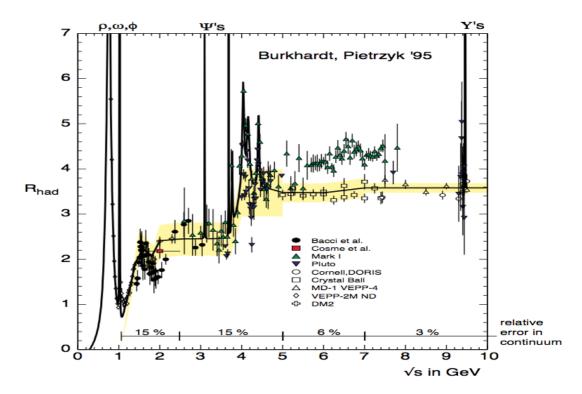
$$= q_{u}^{2} + q_{d}^{2} + q_{s}^{2} + q_{c}^{2}$$

$$= \left(\frac{2}{3}\right)^{2} + \left(\frac{1}{3}\right)^{2} + \left(\frac{1}{3}\right)^{2} + \left(\frac{2}{3}\right)^{2}$$

$$= \frac{10}{9}$$

And at a higher energy (say 20-30 GeV), well above threshold to produce b quarks, this would increase to 11/9.

What is observed is rather different:



The actual observed value of $R_{hadrons}$ in this range is more like 3.5 rather than 1.1.

However, what this simple calculation left out was the fact that each quark comes in 3 colours – there are actually 3 different quarks of each "flavour" (up, down, strange etc). The photon couples equally to quarks of all colours, so the actual rate of quark production should be 3 times the original calculation.

With this colour factor included the prediction becomes much closer to the actual measurement. The remaining discrepancy is resolved when "higher order" processes including additional gluons (such as the "three jet" process illustrated earlier) are included in the rate calculations.