REVIEW OF V_{cb} MEASUREMENTS

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ABSTRACT

A review of measurements of the CKM matrix element, V_{cb}, performed both at LEP and at the $\Upsilon(4S)$ is presented. By averaging results from inclusive semileptonic B decays and from $B \to D^*\ell\nu$ decays (using the zero recoil technique), $|V_{cb}|$ is determined :

$$|V_{cb}| = 0.0395 \pm 0.0020 \; .$$

1. Introduction

In the Wolfenstein approximation[1], the CKM matrix[2] may be represented :

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \tag{1}$$

The element V_{cb} is the third best measured (relative) after V_{ud} and V_{us}, which determine λ. In this representation, V_{cb} therefore determines A.

In this review, experimental determinations of V_{cb} are discussed from inclusive semileptonic B decays, from $B \to D^*\ell\nu$ decays and from $B \to D\ell\nu$ decays.

2. Inclusive semileptonic B decays

The partial width for semileptonic B decays to charmed hadrons, $\bar{B} \to X_c\ell\bar{\nu}$, depends strongly on V_{cb}:

$$\Gamma(\bar{B} \to X_c\ell\bar{\nu}) = \frac{G_F^2 {m_b}^5}{192\pi^3}|V_{cb}|^2 \times f(m_c, m_b, \alpha_s) \tag{2}$$

where X_c represents any state containing a charmed hadron. The function $f(m_c, m_b, \alpha_s)$ represents the correction to the muon decay formalism, depending on the b and c quark masses and the strong coupling constant.

Experimentally, the partial width is determined from the semileptonic branching ratio and the lifetime :

$$\Gamma(\bar{B} \to X_c\ell\bar{\nu}) = \frac{B(\bar{B} \to X_c\ell\bar{\nu})}{\tau_B} \; . \tag{3}$$

There is a slight discrepancy between experiments at the $\Upsilon(4S)$ and those at LEP regarding the value of the semileptonic branching ratio. The $\Upsilon(4S)$ experiments measure $10.43 \pm 0.24\%$[3], while after correcting for the contribution from B_s and b baryon decays, the higher energy experiments yield $11.5 \pm 0.3\%$[3]. Inflating the error to account for this, I assume a value $11 \pm 0.5\%$ which translates to $\mathcal{B}(\bar{B} \to X_c \ell \bar{\nu}) = 10.9 \pm 0.5\%$ after subtracting the small contribution from $b \to u$ decays. The average lifetime for B^0 and B^+ mesons is measured to be 1.59 ± 0.04 ps[3], implying a total experimental uncertainty on $\Gamma(\bar{B} \to X_c \ell \bar{\nu})$ of about 5% (2.5% on $|V_{cb}|$).

There are three major components to the theoretical uncertainty[4, 5]:

1. A 200 MeV uncertainty on the mass of the b quark, m_b, translates into a 10% effect on $\Gamma(\bar{B} \to X_c \ell \bar{\nu})$ if the mass difference $(m_b - m_c)$ is kept fixed.
2. A 60 MeV uncertainty on $(m_b - m_c)$ gives rise to a 5% effect on $\Gamma(\bar{B} \to X_c \ell \bar{\nu})$.
3. The uncertainty from perturbative corrections is estimated to cause a 10% uncertainty on $\Gamma(\bar{B} \to X_c \ell \bar{\nu})$[5] .

Since these uncertainties are uncorrelated (although non-gaussian), I combine them in quadrature to yield a 15% uncertainty on $\Gamma(\bar{B} \to X_c \ell \bar{\nu})$. Using the theoretical prediction[4],

$$|V_{cb}| = (39.9 \pm 1.0 \pm 3.0) \cdot 10^{-3} \tag{4}$$

where the first error is experimental and the second from theory.

3. $B \to D^* \ell \nu$ decays

The technique for determining V_{cb} from $B \to D^* \ell \nu$ decays relies on HQET. In particular, when the c quark is at rest in the B restframe, the hadronic system is not perturbed by the decay (in the limit of heavy c and b quarks). This allows a calculation of the relevant form factor (the Isgur-Wise function[6] in the limit $m_Q \to \infty$) with only a small uncertainty. To parametrise the problem, we introduce $\omega = v_B \cdot v_{D^*}$, the product of the 4-velocities of the B and D^* or, equivalently, the boost of the D^* in the B restframe. The partial width for the decay $B \to D^* \ell \nu$ may then be written

$$\frac{d\Gamma}{d\omega} = \mathcal{K}(\omega) \cdot \mathcal{F}^2(\omega) \cdot |V_{cb}|^2 \tag{5}$$

where $\mathcal{K}(\omega)$ is a known function, and $\mathcal{F}(\omega)$ is the form factor which is essentially model independent at zero recoil ($\omega = 1$), but model dependent for $\omega > 1$. Neubert[5] writes

$$\begin{aligned} \mathcal{F}(\omega = 1) &= \eta_A (1 + c_2 \frac{\Lambda^2_{QCD}}{m_Q{}^2} + \cdots) & (6) \\ &\equiv \eta_A (1 + \delta_{1/m^2}) & (7) \end{aligned}$$

where η_A is referred to as a short-distance correction and δ_{1/m^2} represents power corrections. The absence of the first-order power correction term is a consequence of Luke's theorem[7]. A recent calculation at two-loop order by Czarnecki[8] yields $\eta_A = 0.960 \pm 0.007$.

A few estimates of the power corrections[9] have been combined by Neubert[5] to give $\delta_{1/\mathrm{m}^2} = -0.055 \pm 0.025$. Combining these results yields

$$\mathcal{F}(1) = 0.91 \pm 0.03. \tag{8}$$

From the experimental point of view, there is no phase space at $\omega = 1$, so one has to extrapolate from data at $\omega > 1$. A Taylor expansion of $\mathcal{F}$ around $\omega = 1$,

$$\mathcal{F}(\omega) = \mathcal{F}(1) \cdot (1 - a^2(\omega - 1) + b(\omega - 1)^2 \cdots) \tag{9}$$

is predicted to work well, since the maximum value of ω is only about 1.5. Experimental results come from the CLEO[10] and ARGUS[11] experiments at the Υ(4S), and from ALEPH[12], DELPHI[13] and OPAL[14] at the Z. The Υ(4S) experiments have the advantage that ω is approximately the boost of the D* in their lab frame, which implies a good resolution for reconstructing ω. Consequently, they suffer from low efficiency near $\omega = 1$ because of a low efficiency to reconstruct the slow pion from the low energy D* decay. For the experiments at the Z, the B mesons are produced with a large and variable boost. This makes the reconstruction of ω tricky, involving some neutrino reconstruction, implying a relatively poor resolution. By contrast, these experiments benefit from an efficiency which varies only mildly with ω, right down to $\omega = 1$.

The decay channels $\bar{\mathrm{B}}^0 \to \mathrm{D}^{*+}\ell\bar{\nu}$ (all experiments) and $\mathrm{B}^- \to \mathrm{D}^{*0}\ell\bar{\nu}$ (CLEO) have both been used, where the D* decays to D$^0\pi$ (this pion is referred to as the slow pion). To identify the D^0, all experiments have used the rather clean $\mathrm{D}^0 \to \mathrm{K}^-\pi^+$ decay mode. An example is given in Figure 1 from OPAL, where the D^{*+} signal, in jets with identified leptons, is clearly seen by plotting the mass difference between D^{*+} and D^0 candidates, $M(\mathrm{K}\pi\pi) - M(\mathrm{K}\pi)$. In this case, the estimated background is determined from the data

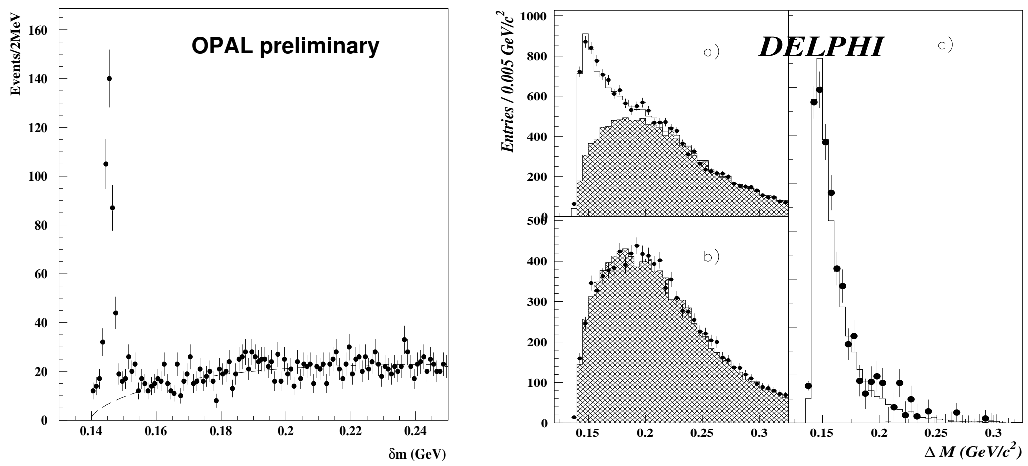

Figure 1: The mass difference $M(\mathrm{K}\pi\pi) - M(\mathrm{K}\pi)$ for the OPAL analysis is shown in the left hand plot. The right hand plot shows the mass difference between D* and D^0 candidates for the inclusive analysis of DELPHI, when the lepton and the slow π have a) opposite charge, b) same charge. The points are data and the histograms simulation, with the shaded part representing combinatorial background. In plot c) the simulated background is subtracted.

using wrong sign combinations and also the reflected pion technique, where the slow pion candidate is taken from the opposite hemisphere. The main background comes from $B \to D^{**}\ell\bar{\nu}$ decays, where the D^{**} decays to $D^{*}X$. ARGUS, ALEPH and DELPHI have included the $D^0 \to K^-3\pi$ decay mode, and ALEPH also studied the $K^0_s\pi^-\pi^+$ decay mode. Both DELPHI and ALEPH have measured the amount of background from D^{**} by studying the impact parameters of extra pions relative to the secondary vertex formed by the D^* and the lepton. This background can be suppressed by considering missing mass (most powerful at the $\Upsilon(4S)$), the transverse momentum of the lepton relative to the jet axis (OPAL) or the lepton momentum at the $\Upsilon(4S)$.

By contrast to these exclusive analyses, DELPHI has performed a semi-inclusive analysis, where the D^0 is reconstructed in an inclusive manner. The lepton and a slow pion candidate were first identified, and the D^0 reconstructed from all the other particles in the jet, suppressing fragmentation particles using impact parameters and rapidity w.r.t. the jet axis. The signal obtained using this technique is shown in Figure 1. The method gains in statistics relative to the exclusive analysis technique, but the signal has a lower purity and the resolution on ω is poorer.

As mentioned above, the reconstruction of ω at the $\Upsilon(4S)$ is relatively simple. At LEP, I use the algorithm of ALEPH as an example. The neutrino energy is determined from the hemisphere missing energy with a resolution of about 2.6 GeV. The B direction is determined from the reconstructed primary and secondary vertices. This constrains the neutrino direction. The remaining neutrino angle is determined from the B^0 mass constraint. The performance of this algorithm is shown in Figure 2; the resolution on ω is about 0.07.

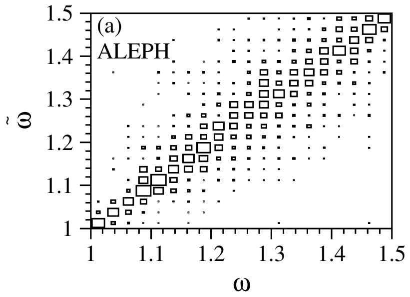

Figure 2: Reconstructed ω vs. true ω for the ALEPH analysis.

The distribution of reconstructed ω in the data is shown in Figure 3 for CLEO and OPAL. In the CLEO distributions, the low efficiency for reconstructing charged slow pions (from B^0) near $\omega = 1$ can be readily seen. Their efficiency for reconstructing slow π^0 (B^-) does not drop to the same extent, but the signal is less pure.

The fit for the corrected distribution $\mathcal{F}(\omega) \cdot |V_{cb}|$ is shown in Figure 4 for CLEO and ALEPH, although the data are binned in reconstructed ω rather than true ω. The fits assume a linear dependence of $\mathcal{F}$ on ω. The results for $\mathcal{F}(1) \cdot |V_{cb}|$ and a^2, the slope, are shown in Figure 5. The averages shown take into account common systematic errors, such as D^* and D^0 branching ratios and the B^0 lifetime for all experiments. Additional sources of error that are common to the LEP experiments include $B(b \to \bar{B}^0)$, the probability of a b quark hadronizing into a $\bar{B}^0$ meson, R_b, the fraction of $Z \to q\bar{q}$ decays that are $Z \to b\bar{b}$, and the D^{**} background. The average for a^2 excludes the OPAL measurement, as no systematic error was quoted. All these results assume the linear form,

$$\mathcal{F}(\omega) = \mathcal{F}(1) \cdot (1 - a^2(\omega - 1)) \ . \tag{10}$$

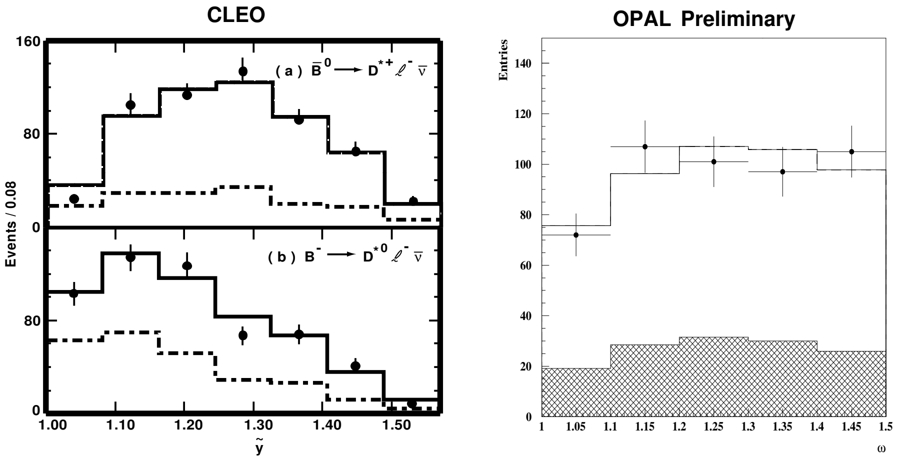

Figure 3: Distributions of ω from the CLEO and OPAL analyses. The lines indicate the expectation after the fit, with the background contributions also indicated.

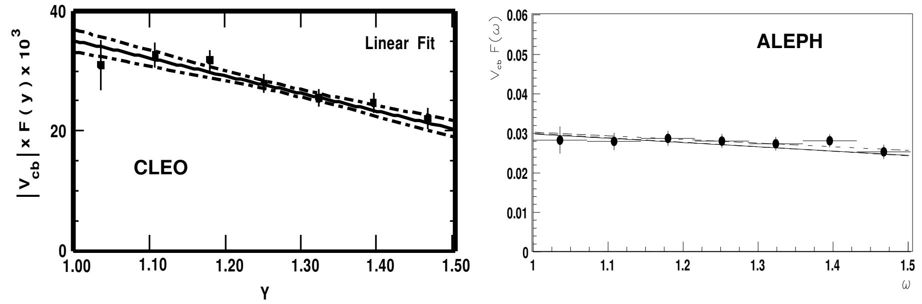

Figure 4: Corrected distribution of $\mathcal{F}(\omega) \cdot |V_{cb}|$ vs. ω for CLEO and ALEPH. The solid lines indicate the results of the fits. For the CLEO plot, the dotted lines indicate the 1σ contour. For the ALEPH plot, the dashed line is the result when the resolution on ω is neglected.

By integrating over ω the experiments can also produce results for $B(\bar{B}^0 \to D^{*+}\ell^-\bar{\nu})$. These results are shown in Figure 6, and their consistency serves as a check on the experimental systematics. Again, the OPAL result is excluded from the average.

The big question mark is whether it is justifiable to assume zero curvature, and whether a positive curvature (which is expected) would have a significant effect on the results. OPAL, ALEPH and CLEO also quote results for a quadratic dependence, $\mathcal{F}(\omega) = \mathcal{F}(1) \cdot (1 - a^2(\omega - 1) + b(\omega - 1)^2)$, where both a and b are allowed to vary. The results for b are shown in Figure 7. Even if systematic errors are ignored, the uncertainty on the average is 1.01, which is not very constraining ! In addition the statistical error on $\mathcal{F}(1) \cdot |V_{cb}|$ is between 1.5 and 2 times larger than in the case of the linear fit, indicating that curvature can have a serious influence on the result. Fortunately, a recent paper by Caprini and Neubert[15] constrains the relationship between a and b:

$$b \approx 0.66a^2 - 0.1 \tag{11}$$

with an uncertainty of roughly ± 0.15[16] on b. There are also similar results from Boyd and Lebed[17].

In this review, I estimate the correction, resulting from this curvature, to a^2 by assuming that the slope of $\mathcal{F}(\omega)$ at the midpoint, $\omega = 1.25$ is unchanged. The change

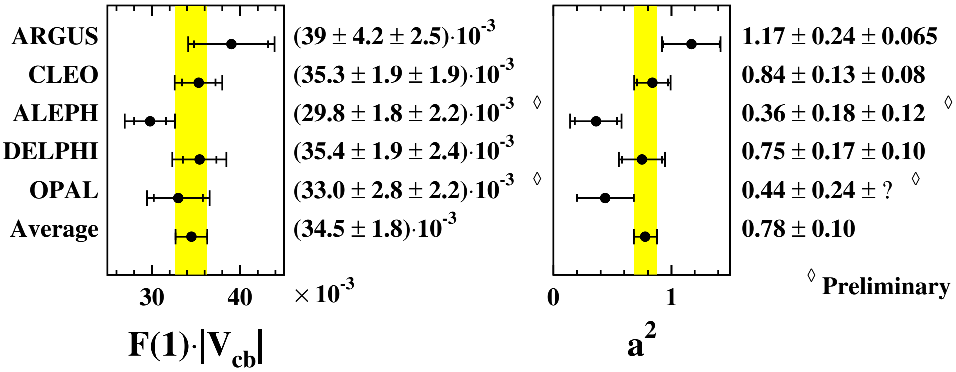

Figure 5: Results for $\mathcal{F}(1) \cdot |\mathrm{V_{cb}}|$ and a^2 from $\mathrm{B} \to \mathrm{D}^*\ell\nu$ decays, assuming a linear extrapolation to $\omega = 1$.

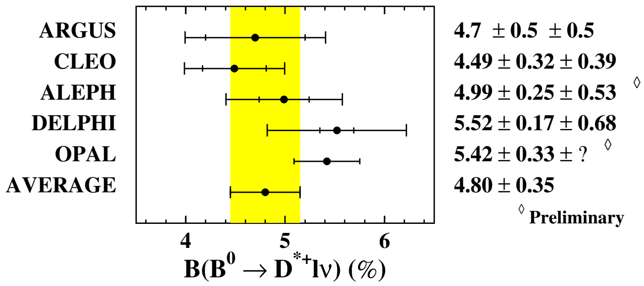

Figure 6: Results for $B(\bar{\mathrm{B}}^0 \to \mathrm{D}^{*+}\ell^-\bar{\nu})$.

in $\mathcal{F}(1) \cdot |\mathrm{V_{cb}}|$ may then be estimated by assuming that $\int \mathcal{F}(\omega)\mathrm{d}\omega$ is unchanged. The integration range of $1.1 < \omega < 1.5$ was chosen, somewhat arbitrarily. The accuracy of this procedure can be controlled by comparing with results obtained by the experiments, with b allowed to float. Good agreement is seen. The corrected average results are

$$\begin{aligned} \mathcal{F}(1) \cdot |\mathrm{V_{cb}}| &= 0.0357 \pm 0.0020 \pm 0.0006 \pm 0.0003 & (12) \\ &= 0.0357 \pm 0.0021 & (13) \end{aligned}$$

and

$$\begin{aligned} a^2 &= 1.07 \pm 0.15 \pm 0.12 \pm 0.04 & (14) \\ &= 1.07 \pm 0.20 \quad . & (15) \end{aligned}$$

In both cases the first error represents the experimental error, the second represents the 0.15 theoretical uncertainty on $b - a^2$ and the third error represents the uncertainty due

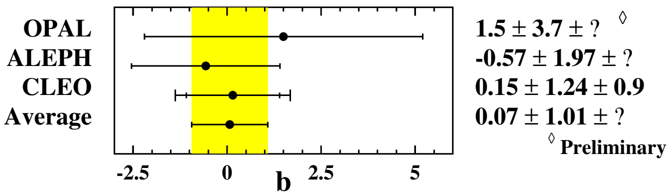

Figure 7: Fitted values for b, the curvature of $\mathcal{F}(\omega)$.

to my correction procedure. Significant differences are seen relative to the results ignoring the curvature, 0.0345 ± 0.0018 and 0.78 ± 0.10. From the Bjorken and Voloshin sum rules[5], a^2 is expected to lie in the range $0.5 < a^2 < 1.1$.

In these decays, the D^* has three possible helicity states in the B restframe. The angular distribution of the D meson in the D^* restframe is sensitive to the relative rates for the transverse and longitudinal states, while the lepton distribution can distinguish between the two longitudinal states. Using this information, CLEO[18] have made preliminary measurements of the ratios of the helicity form factors[5], which are in good agreement with the predictions of HQET. Please consult the references for the details.

4. $\Gamma(B \rightarrow D^* \ell \nu)$

In principle, $|V_{cb}|$ can also be determined from the partial width for $B \rightarrow D^* \ell \nu$ decays. In this case, an absolute theoretical prediction is needed at $\omega > 1$. The results are therefore model dependent. For example, the branching ratio quoted earlier implies

$$\Gamma(\bar{B}^0 \rightarrow D^{*+} \ell^- \bar{\nu}) = (0.0308 \pm 0.0025)\mathrm{ps}^{-1} \tag{16}$$

The predictions from a range of models[19, 20] give

$$\frac{\Gamma(B \rightarrow D^* \ell \nu)}{|V_{cb}|^2} \approx (25 \pm \sim 5)\mathrm{ps}^{-1} \tag{17}$$

where the uncertainty is estimated from the spread of the model results, and the central value is at about the centre of the distribution of model results. Some additional uncertainty may arise from experimental efficiency changes for different models. The result is roughly

$$|V_{cb}| = 0.0351 \pm 0.0014 \pm \sim 0.005 \tag{18}$$

It is clearly undesirable that the dominant uncertainty is estimated in an unsatisfactory way.

5. $B \rightarrow D \ell \nu$ decays

Both ALEPH[12] and CLEO[21] have repeated the zero recoil technique using the decay $\bar{B}^0 \rightarrow D^+ \ell \bar{\nu}$, with the D^+ decaying to $K^- \pi^+ \pi^+$. Unfortunately these decays are suppressed near $\omega = 1$ relative to the D^* decays, since the D^+ carries no spin, and the lepton and anti-neutrino have opposite helicity. The distribution of $\mathcal{F}(\omega) \cdot |V_{cb}|$ vs. ω is shown for CLEO in Figure 8, which also summarises results for $\mathcal{F}(1) \cdot |V_{cb}|$. These results assume a linear extrapolation, but still are not competitive with the D^* results. We can, however, take the ratio to derive

$$\frac{\mathcal{F}_{D^+}(1)}{\mathcal{F}_{D^*}(1)} = 1.03 \pm 0.18 \tag{19}$$

using linear extrapolation for both numerator and denominator. This can be compared to a prediction of 1.08 ± 0.06[20], and thus represents a test of HQET. Unfortunately, the experimental errors should be expected to increase somewhat when the effect of curvature is included, even when constrained as in the Caprini and Neubert paper.

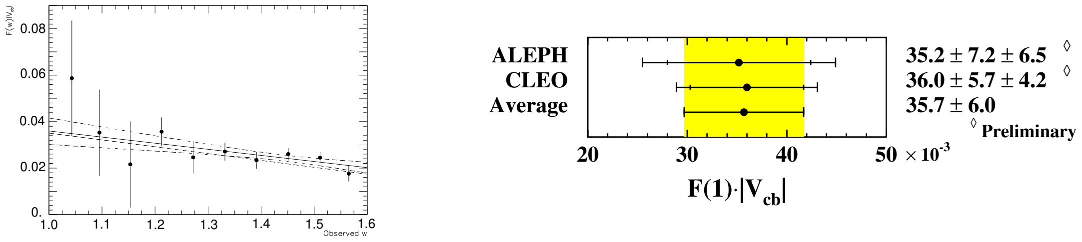

Figure 8: The left hand plot shows the observed distribution of $\mathcal{F}(\omega) \cdot |V_{cb}|$ vs. ω from CLEO, for $B \to D\ell\nu$ decays. The solid line and two dashed lines indicate the fit and its uncertainty. A third dashed line indicates the fitted result for the D^* analysis. The right hand plot summarises results for $\mathcal{F}(1) \cdot |V_{cb}|$ in $B \to D\ell\nu$ decays.

6. Conclusion

The different results for $|V_{cb}|$ are summarised in Figure 9. The result from $\Gamma(B \to D^*\ell\nu)$

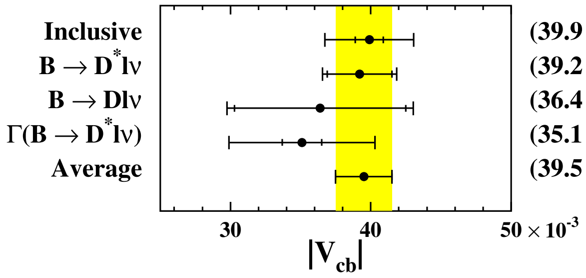

Figure 9: Summary of $|V_{cb}|$ results.

is excluded from the average, because it is very model dependent. The $B \to D\ell\nu$ result is also excluded, because of the linear extrapolation. The remaining results, from $B \to D^*\ell\nu$ decays at zero recoil and from inclusive semileptonic B decays, have a similar precision and their errors are largely uncorrelated. For the zero recoil technique, it has been shown that the curvature of $\mathcal{F}(\omega)$ should not be ignored. The D^* results can be expected to improve when the experiments analyse more decay channels and, of course, more data.

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