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MEASUREMENT OF THE CKM ANGLES α AND γ AT THE *BABAR* EXPERIMENT

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ABSTRACT

The primary objective of the *BABAR* Experiment is to test the Standard Model explanation of CP violation in weak decays by over-constraining the CKM Unitarity Triangle. This includes the measurement of all three angles of the triangle. Although relatively precise measurements of the angle β have been obtained using B decays to charmonium states, the remaining angles, α and γ, pose greater experimental challenges. In this paper, the latest measurements of modes which will constrain α and γ will be presented, including $B^0 \rightarrow \rho^+\rho^-$ for α and a measurement of $\sin(2\beta+\gamma)$ from the $B^0 \rightarrow D^{(*)\pm}\pi^{\mp}$ system.

1 *CP* Violation

In the Standard Model, the imbalance between matter and anti-matter in the universe can be quantifield by measuring the amount of *CP* Violation present

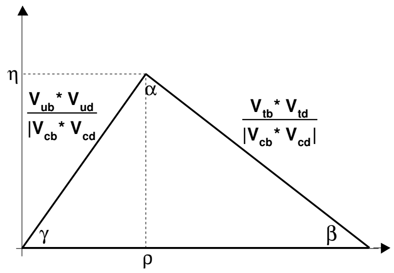

Figure 1: *The triangular representation* $V_{ub}^{*}V_{ud} + V_{cb}^{*}V_{cd} + V_{tb}^{*}V_{td} = 0$*, which describes* CP *violation in the Standard Model for the* B *meson system.*

in weak interactions. *CP* violation is described by a single phase (η) in the quark mixing matrix for three generations, the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The CKM matrix:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{1}$$

is the Standard Model description of *CP* violation, and can be rewritten in the Wolfenstein parameterization [1], as:

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4) \tag{2}$$

The unitarity of the CKM matrix yields several interesting relationships for its components, including $V_{ub}^{*}V_{ud} + V_{cb}^{*}V_{cd} + V_{tb}^{*}V_{td} = 0$ (Figure 1) which describes Standard Model *CP* violation in the B meson system. Measuring the two sides (the base is set to unit value) and all three angles of this triangle in many different processes tests whether this theory of *CP* violation is a full description of the processes which occur in the B meson system. The three angles (α, β and γ) can be written in terms of the couplings between quarks:

$$\alpha \equiv \arg\left[-\frac{V_{td}V_{tb}^{*}}{V_{ud}V_{ub}^{*}}\right] \quad , \quad \beta \equiv \arg\left[-\frac{V_{cd}V_{cb}^{*}}{V_{td}V_{tb}^{*}}\right] \quad , \quad \gamma \equiv \arg\left[-\frac{V_{ud}V_{ub}^{*}}{V_{cd}V_{cb}^{*}}\right] \tag{3}$$

The measurements of these angles can be made in the *CP* asymmetries of decay modes of the B meson. This paper describes recent analyses which aim to measure α and γ from the *BABAR* experiment.

2 The *BABAR* Experiment

The *BABAR* experiment is situated at the PEP-II [2] asymmetric e^+e^- collider at the Stanford Linear Accelerator Center, U.S.A. As the *BABAR* detector is described completely elsewhere [3], only a brief description is included here. A Silicon Vertex Tracker (SVT) surrounds the beam-pipe, providing excellent tracking of charged particles close to interaction point. Surrounding the SVT is a drift chamber (DCH) which provides some particle identification due to its measurements of the energy loss of charged particles and precise measurements of track momenta inside the 1.5T magnetic field applied to the detector using a superconducting coil. The detector of internally reflected Cherenkov radiation (DIRC) provides charged hadron identification, whilst the CsI(Tl) electromagnetic calorimeter (EMC), is used to reconstruct neutral hadrons, detect photons and provide electron identification. Situated next is the magnet, followed by the instrumented flux return (IFR) which is used for the identification of muons and long-lived neutral hadrons.

3 Measurements of α

Neutral B mesons decay to $\pi^+\pi^-$ and $\rho^+\rho^-$ primarily via a $b \to u\overline{u}d$ tree diagram, with additional contributions from penguin diagrams. The amplitudes of the B^0 (A) and the $\overline{B^0}$ ($\overline{A}$) decay can be represented as a combination of the tree (T) and penguin (P) amplitudes:

$$A = e^{+i\gamma}T + e^{-i\beta}P \quad , \qquad \overline{A} = e^{-i\gamma}T + e^{+i\beta}P \tag{4}$$

whose coefficients give the sensitivity to α. The *CP* asymmetry between the B^0 and the $\overline{B^0}$ decays is given by the equation:

$$\begin{aligned} A_{CP}(t) &= \frac{N(\overline{B^0}(t) \to h^+h^-) - N(B^0(t) \to h^+h^-)}{N(\overline{B^0}(t) \to h^+h^-) + N(B^0(t) \to h^+h^-)} && (5) \\ &= S_{hh}\sin(\Delta m_d \Delta t) - C_{hh}\cos(\Delta m_d \Delta t) && (6) \end{aligned}$$

where the measurable coefficents C_{hh} and S_{hh} are defined as:

$$C_{hh} = \frac{2\mathcal{I}m(\lambda_{hh})}{1+ \mid \lambda_{hh} \mid^2}, \qquad S_{hh} = \frac{1- \mid \lambda_{hh} \mid^2}{1+ \mid \lambda_{hh} \mid^2} \tag{7}$$

and λ_{hh} is given by:

$$\lambda_{hh} \;=\; \frac{q}{p}\frac{\overline{A}}{A} = e^{2i\alpha}\frac{1-\frac{P}{T}e^{-i\alpha}}{1-\frac{P}{T}e^{+i\alpha}} \;=\; \mid \lambda \mid e^{2i\alpha_{\rm eff}} \tag{8}$$

q and p are the B mixing coefficients and h can be a π or a ρ meson. $\alpha_{\rm eff}$ is the experimentally measurable quantity, which is shifted from α by an unknown amount due to penguin pollution.

3.1 $B \to \rho\rho$

Measurements of $B \to \pi\pi$ [4] and $B \to \rho\pi$ [5] have so far failed to yield a tight bound on the value of α, but $B \to \rho\rho$ provides an alternative.

On $113\,{\rm fb}^{-1}$ of data, a measurement of the longitudinal polarisation fraction, $f_L = 1.00 \pm 0.02$, confirmed that this decay is overwhelmingly dominated by the helicity zero state, making an angular analysis unnecessary. A fit to extract the time dependent *CP* parameters S and C for the longitudinal decay yields 314 $\pm$ 34 signal events and:

$$C_{long} = -0.23 \pm 0.24 \pm 0.14 \quad , \quad S_{long} = -0.19 \pm 0.33 \pm 0.11 \tag{9}$$

where the first error is statistical and the second is systematic in both cases, and C_{trans} and S_{trans} were fixed to zero in the fit.

A theoretical bound on the shift between α and $\alpha_{\rm eff}$ is described by the Grossman-Quinn bound [6], which for $B \to \rho\rho$ is written:

$$\mid \alpha - \alpha_{\rm eff} \mid = \frac{\mathcal{B}(B^0 \to \rho^0\rho^0)}{\mathcal{B}(B^0 \to \rho^+\rho^-)} \tag{10}$$

It provides a reasonably tight theoretical constraint on the value of $\mid \alpha - \alpha_{\rm eff} \mid$ of 15.9^o (13^o) at 90% (68.3%) confidence level.

Measurements of C_{long} and S_{long} relate to α up to a four-fold ambiguity [7], and the solution closest to the CKM best fit solution [8] gives $\alpha = (95 \pm 10 \pm 4)^o$, where the first error is statistical and the second is systematic. In addition, the third error is the theoretical error from the Grossman-Quinn bound ($< 13^o$) to account for the shift between α and $\alpha_{\rm eff}$.

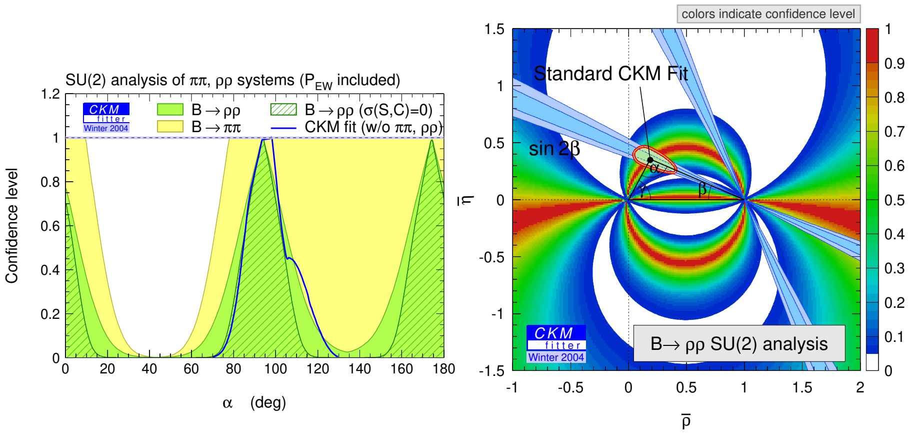

Figure 2: *The $B \to \rho\rho$ analysis constrains the possible values of α. The left-hand plot shows the α plane constrained by the $B \to \pi\pi$ and $B \to \rho\rho$ (with and without experimental errors) analyses. These are overlaid with the global CKM fit without these two analyses included. The right-hand plot shows the constraint on the $\rho - \eta$ plane due to the $B \to \rho\rho$ analysis, which is shown overlaid by the Standard CKM fit* [9].

An isospin analysis provides a complementary measurement of α. Using C_{long} and S_{long}, together with the branching fractions and f_L measurements for $B^+ \to \rho^+\rho^0$, $B^0 \to \rho^0\rho^0$ [10] and $B^0 \to \rho^+\rho^-$ [11] as inputs, and choosing the result nearest the CKM best fit [8], gives $\alpha = (96 \pm 10 \pm 4 \pm 13)^o$ which is consistent with the result from the time dependent fit and is shown in Figure 2.

4 Measurements of γ

γ measurements can be made in modes which have both $b \to c$ and $b \to u$ tree diagrams, which interfere. The magnitude of the interference is determined by the ratio of the two methods of decay.

4.1 $B^0 \to D^{(*)+}\pi^-$

$B^0 \to D^{(*)+}\pi^-$ is sensitive to $\sin(2\beta + \gamma + \delta)$. The 2β term is due to $B^0 - \overline{B^0}$ mixing and the δ represents the strong phase difference between the two decay trees. The time-evolution of the decay is described by:

$$P_{B^0}(D^{\mp}\pi^{\pm}) \propto Ne^{-\Gamma|\Delta t|}(1 \pm C\cos(\Delta m_d \Delta t) + S^{\mp}\sin(\Delta m_d \Delta t)) \quad (11)$$

$$P_{\bar{B}^0}(D^{\mp}\pi^{\pm}) \propto Ne^{-\Gamma|\Delta t|}(1 \mp C\cos(\Delta m_d \Delta t) - S^{\mp}\sin(\Delta m_d \Delta t)) \quad (12)$$

and similar equations for $D^*\pi$, where

$$C = \frac{1-r^2}{1+r^2} \quad \text{and} \quad S^{\mp} = \frac{2r}{1+r^2}\sin(2\beta + \gamma \pm \delta) \quad (13)$$

and the ratio between the suppressed ($b \to u$) and dominant ($b \to c$) amplitudes is described as $r = |V_{ub}^* V_{cd}/V_{cb}V_{ud}^*| \approx 0.02$. As r is small, *CP* asymmetry is also expected to be small in this mode.

BABAR has undertaken two different analysis techniques for this mode, based on partial reconstruction and full reconstruction of the B meson.

The fully reconstructed method has the benefit of having an extremely pure sample, but has a very low efficiency. On $82\,\text{fb}^{-1}$, 5207 ± 87 events are fitted in the $B^0 \to D^+\pi^-$ sample and 4746 ± 78 events in the $B^0 \to D^{*+}\pi^-$ sample. The results of the *CP* measurements were [12]:

$$2r_{D^*\pi}\sin(2\beta+\gamma)\cos(\delta_{D^*\pi}) = -0.068 \pm 0.038 \pm 0.021 \quad (14)$$

$$2r_{D^*\pi}\sin(2\beta+\gamma)\sin(\delta_{D^*\pi}) = 0.031 \pm 0.070 \pm 0.035 \quad (15)$$

$$2r_{D\pi}\sin(2\beta+\gamma)\cos(\delta_{D\pi}) = -0.022 \pm 0.038 \pm 0.021 \quad (16)$$

$$2r_{D\pi}\sin(2\beta+\gamma)\sin(\delta_{D\pi}) = 0.025 \pm 0.068 \pm 0.035 \quad (17)$$

The partially reconstructed method is used only for the mode $B^0 \to D^{*\pm}\pi^{\mp}$. A useful feature of this decay is the presence of a "fast" π from the B meson decay and a "slow" π from the $D^{*\pm}$ decay. These pions, together with beam constraints, allow the missing mass of the decay to be reconstructed. This mass distribution peaks at the D^0 mass. This method finds 6406 ± 129 events in the lepton tagged sample and 25157 ± 323 in the kaon tagged sample in $82\,\text{fb}^{-1}$ of data. When a time-dependent simultaneous fit is done to the kaon- and lepton-tagged events, the *CP* measurement is [13]:

$$2r\sin(2\beta+\gamma)\cos(\delta) = -0.063 \pm 0.024 \pm 0.014 \quad (18)$$

The combined results for the two methods gives limits of:

$$|\sin(2\beta+\gamma)| > 0.58 \quad \text{(95\% Confidence Level)} \quad (19)$$

$$|\sin(2\beta+\gamma)| > 0.87 \quad \text{(68\% Confidence Level)} \quad (20)$$

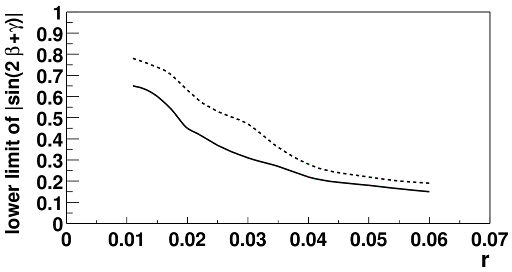

Figure 3: *The comparison between the partially reconstructed limits (solid line) and the combined results of the partially and fully reconstructed fits (dashed line).*

and the difference between the combined limit and the partial measurement can be seen in Figure 3.

4.2 $B^{\pm} \to D^0 K^{\pm}$

One method of extracting γ from the mode $B^{\pm} \to D^0 K^{\pm}$ is by studying the decay of the D^0 to CP even eigenstates, K^+K^- and $\pi^+\pi^-$. These decays are described by $R_{CP\pm}$ and can be compared to the flavor eigenstate decays ($D^0 \to K^-\pi^+, K^-\pi^+\pi^0$ and $K^-\pi^+\pi^-\pi^+$ and the charged conjugate decays) which are described by R:

$$R_{(CP\pm)} = \frac{\Sigma_{B^+,B^-}\Gamma(B \to D^0_{(CP\pm)}K)}{\Sigma_{B^+,B^-}\Gamma(B \to D^0_{(CP\pm)}\pi)} \tag{21}$$

$$\frac{R_{CP\pm}}{R} = 1 + r_{DK}^2 + 2r_{DK}\cos\gamma\cos\delta \tag{22}$$

where r_{DK} is the ratio of the suppressed amplitude to the dominant amplitude, which is expected to be of the order 0.1 - 0.2 for this mode. A charge asymmetry is also expected in this decay, which can be written as:

$$A_{CP\pm} = \frac{\Gamma(B^- \to D^0_{CP\pm}K^-) - \Gamma(B^+ \to D^0_{CP\pm}K^+)}{\Gamma(B^- \to D^0_{CP\pm}K^-) + \Gamma(B^+ \to D^0_{CP\pm}K^+)} \tag{23}$$

$$= \frac{\pm 2r_{DK}\sin\gamma\sin\delta}{R_{CP\pm}} \tag{24}$$

where δ is the relative strong phase between $B^- \to \overline{D^0} K^-$ and $B^- \to D^0 K^-$. Measuring R, $R_{CP\pm}$ and $A_{CP\pm}$ makes it possible to extract r_{DK}, δ and γ.

Using datasets of $56\,\mathrm{fb}^{-1}$ for the measurement of R, and $82\,\mathrm{fb}^{-1}$ for $R_{CP\pm}$ and $A_{CP\pm}$ *BABAR* finds [14]:

$$\begin{aligned} R &= (8.31 \pm 0.35 \pm 0.20)\% && (25)\\ R_{CP\pm} &= (8.8 \pm 0.35 \pm 0.20)\% && (26)\\ A_{CP\pm} &= 0.07 \pm 0.17 \pm 0.06 && (27) \end{aligned}$$

which gives

$$R_{CP\pm}/R = 1.06 \pm 0.19 \pm 0.06. \tag{28}$$

No γ measurement is yet available.

5 $B^\mp \to [K^\mp \pi^\pm]_D K^\mp$

When combined with other modes in the Atwood, Dunietz and Soni method [15], it is possible to cleanly extract γ using this mode. *CP* violation could manifest itself as a large difference between the ratios of suppressed ($b \to u$) to dominant tree ($b \to c$) diagrams for B^+ and $B^- \to DK^\mp, D \to K^\mp \pi^\pm$, where D is a D^0 or a $\overline{D^0}$. When D mixing is ignored, the ratio can be expressed as:

$$\begin{aligned} R^\pm_{K\pi} &= \frac{\Gamma([K^\mp\pi^\pm]_D K^\pm)}{\Gamma([K^\pm\pi^\mp]_D K^\mp)} = r_B^2 + r_D^2 + 2 r_D r_B \cos(\pm\gamma + \delta) && (29)\\ r_B &= \left| \frac{A(B^- \to \overline{D^0} K^-)}{A(B^- \to D^0 K^-)} \right| && (30)\\ r_D &= \left| \frac{A(D^0 \to K^+\pi^-)}{A(D^0 \to K^-\pi^+)} \right| = 0.060 \pm 0.003 && (31)\\ \delta &\equiv \delta_B + \delta_D && (32) \end{aligned}$$

where δ is the strong phase difference between the B and D decay amplitudes, r_B is the ratio of the suppressed B decay to the dominant B decay (whose size determines the size of the interference), and r_D is the ratio of the suppressed D decay to the dominant D decay.

However, due to insufficient statistics at this time, the B^+ and B^- samples are combined for this analysis ($109\,\mathrm{fb}^{-1}$), giving:

$$R_{K\pi} = \frac{\Gamma(B^- \to [K^+\pi^-]_D K^-) + \Gamma(B^+ \to [K^-\pi^+]_D K^+)}{\Gamma(B^- \to [K^-\pi^+]_D K^-) + \Gamma(B^+ \to [K^+\pi^-]_D K^+)} \tag{33}$$

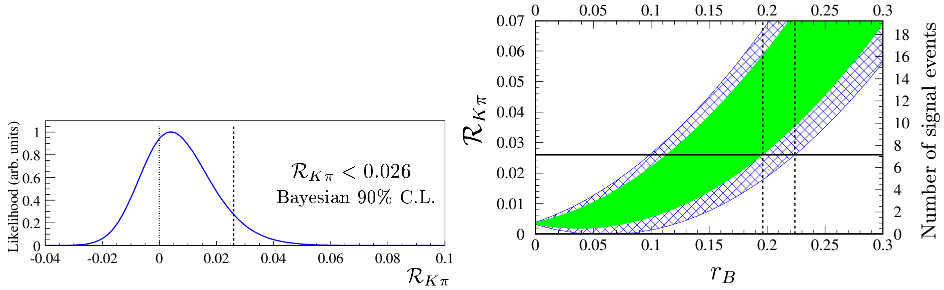

Figure 4: *The left-hand plot shows the Bayesian model of the likelihood used to extract the Upper Limit for $R_{K\pi}$ in $B^{\mp} \to [K^{\mp}\pi^{\pm}]_D K^{\mp}$. The right-hand plot describes the dependence of $R_{K\pi}$ on r_B using $0^o < \gamma,\ \delta < 180^o$ (hashed area) and the range of γ from CKM fits ($48^o < \gamma < 73^o$).*

$$= r_B^2 + r_D^2 + 2 r_D r_B \cos\gamma \cos\delta \tag{34}$$

Using a Bayesian model to determine the Confidence Level, as shown in the left-hand plot of Figure 4, a value of $R_{K\pi} < 0.026$ was found at 90% Confidence Level. Therefore, the $b \to u$ contribution to the amplitude is very small, making it difficult to measure γ in this mode. To calculate r_B, the least restrictive limit is used, computed using maximal destructive interference (right-hand plot of Figure 4). The limit is: $r_B < 0.22$ at a Confidence Level of 90% [16].

6 Conclusion

The *BABAR* Experiment has conducted several analyses with the aim of extracting α and γ. In the $B^0 \to \rho^+\rho^-$ system, $\alpha = (96 \pm 10 \pm 4 \pm 13)^o$ has been measured using an isospin analysis. In $B^0 \to D^{(*)+}\pi^-$, a limit on $\sin(2\beta+\gamma)$ from two different analysis methods was found to be $|\sin(2\beta+\gamma)| > 0.58$ at 95% Confidence Level. Other methods of extracting both angles are under investigation, and tighter constraints on their values will be measured once larger data sets become available.

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