Black Holes, Extra Dimensions & the LHC

- Black Hole Recap
- The Problematic Standard Model
- Extra Dimensions & the Planck Scale
- Black Hole Production & Decay
- Current Constraints
- Signatures at the LHC
In last ~150 years physics has developed enormously

Three major pillars of modern physics have emerged

- general relativity \( 2 \times 10^{-5} \) Cassini photon freq. shift close to Sun
- thermodynamics \( 1 \times 10^{-7} \) WMAP precision of CMB fluctuations to 1%
- quantum mechanics \( 1 \times 10^{-12} \) Measurement of electron g-2

Tested to unprecedented precision

- Black Hole studies are unique - combines all three areas
- Raises some very interesting questions about the nature of spacetime
- Ideas have very appealing simplicity
- Potential to answer one or several fundamental puzzles
In QM all particles associated with a compton wavelength
\[ \lambda = 1/E \]

In GR any object with energy-momentum \((T_{\mu\nu})\) will cause curvature of space-time \((g_{\mu\nu})\)

\[ R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi \frac{1}{m_p^2} T_{\mu\nu} \]

Riemann tensor \(R_{\mu\nu}\) describes tidal forces: residual acc\(n\) between test masses on initially parallel geodesics

Thus objects warp space-time around themselves and this modifies the objects equations of motion

For fundamental particles expect this influence at Planck Scale - \(M_p\)

\[ M_p = \sqrt{\frac{\hbar c}{G}} \quad \text{where } G = \text{Gravitational constant} \]

\[ M_p \sim 10^{19} \text{ GeV} \quad (\Rightarrow \text{hierarchy problem}) \]
For a spherically symmetric mass distribution the solution is 4d line element given by:

\[ ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -\gamma(r) dt^2 + \gamma(r)^{-1} dr^2 + r^2 d\Omega^2 \]

So, for masses small compared to \( M_p \) then \( \gamma = 1 \)
For large energies metric is distorted by order \( E/M_p^2 \)
At energies close to Planck Mass distortions cannot be neglected

Metric becomes singular at \( r = 2M/M_p^2 = r_s \) the Schwarzschild radius

Schwarzschild radius is solution of GR in case of non-rotating uncharged BHs
First solution to GR discovered 1 month after Einstein's publication
Alternatively, can write \( r_s = \frac{2GM}{c^2} \)

Bring mass \( M \) within a radius \( r_s \) and a singularity will form
Event horizon is all we can observe in our side universe

For Earth \( r_s = 1 \text{cm} \)

A more generic solution was found for charged rotating black holes

Solve classical electro-dynamics in GR field equations yields the Kerr-Newmann metric

Size of event horizon generalises to \( r_h \)

Rotating Kerr solution published 1963
Charged rotating BH Kerr-Newmann solution published 1965
The Problematic Standard Model

Jump to particle physics...

The Standard Model is fantastically successful

... but ...
### The Problematic Standard Model

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61 'fundamental' particles in the SM! (including anti-particles)
The Problematic Standard Model

22 Parameters of the SM to be measured
6 quark masses
3 charged leptons masses
3 coupling constants
4 quark mixing parameters
4 neutrino mixing parameters
1 weak boson mass (other predicted from remaining EW params)
1 Higgs mass

(better than 105 params of generic SUSY)

We have no idea what 96% of the universe is!
unknown form of dark energy
unknown form of dark matter

No treatment of gravity in the Standard Model...
In a symmetric theory gauge bosons are massless
Higgs mechanism explains EW symmetry breaking
→ EW bosons acquire mass

...but there must be a deeper relationship
between Higgs / mass / gravity / dark energy
Dark energy acts to accelerate the expansion of the universe i.e. repulsive gravity

Best guess is:
- constant across cosmos
- property of the vacuum

Summing zero-point vacuum fluctuations of SM fields incl. Higgs yields energy density $10^{120}$ times larger than measured!!!

"the worst theoretical prediction in the history of physics!"*

(not surprising that it's related to what Einstein called "his greatest blunder")

Back to particle physics:
- insufficient CP violation & no Baryon number violation able to account for our matter dominated universe

The Hierarchy Problem

Why is gravity $\sim 10^{33}$ weaker than EW interactions?
Why is Higgs mass ($\sim 100$ GeV) so much smaller than Planck mass ($10^{19}$ GeV)?

Leads to fine tuning problem
self energy corrections to Higgs mass are quadratically divergent upto $10^{19}$ GeV

physical mass = bare mass + “loops” $m_H^2 = m_0^2 + \Delta m_H^2$

since Higgs is scalar field we get:

for top: $\Delta m_H^2 = -\frac{6}{16\pi^2} g_t^2 \Lambda^2$ (g is Yukawa coupling $\propto$ mass)

for EW bosons: $\Delta m_H^2 = +\frac{1}{16\pi^2} g^2 \Lambda^2$

for Higgs: $\Delta m_H^2 = +\frac{1}{16\pi^2} \lambda^2 \Lambda^2$ (λ is Higgs self coupling)

$m_H^2 = m_0^2 + \frac{1}{16\pi^2} (-6g_t^2 + g^2 + \lambda^2) \Lambda^2 - \ldots$ new physics...

For $\Lambda^2 : (10^{19} \text{ GeV})^2$ and $m_H : (100 \text{ GeV})^2$ then

$m_H^2 = m_0^2 + \frac{1}{16\pi^2} (-6g_t^2 + g^2 + \lambda^2) \cdot 10^{38} \approx (100 \text{ GeV})^2$

- if SM is valid to this scale (i.e. no new physics from 1 TeV - $10^{19}$ GeV)
  incredible fine tuning required between bare mass and the corrections to maintain $\sim 100$ GeV Higgs mass
Welcome to the Standard Model
What if there is no new scale in particle physics up to $M_p$?
We will have to live with the fine tuning problem
Use anthropic arguments
(of all possible universes with different physics parameter values
only universes with our parameter settings could lead to humans existing)

Alternative approach

("If the mountain will not come to Mohammed, then Mohammed must go to the mountain.")^{-1}

Perhaps we can bring $M_p$ down to $\sim 1$ TeV

Introduce large extra spatial dimensions (large $\sim 1$ mm)

Standard Model confined to a 3-brane
Embedded in higher dimensional space
Only gravity propagates in extra dimensions
1920s - Kaluza & Klein attempted to unify general relativity & Maxwell's EM incorporated U(1) gauge symmetry into 5d spacetime if extra dimension is compactified then EM & Lorentz symmetries remain photon becomes 4d manifestation of 5d graviton

Theory suffered problems
unable to explain vast difference in strengths of two interactions
unable to combine with quantum mechanics
later discoveries of weak & strong interactions did not fit into the scheme

Supersymmetry & string theory in 1970s / 1980s revived concept of extra dimensions

some of gravity's non-renormalizability could be accommodated in string theory
requires 10 / 11 spatial dimensions
predicted spin 2 massless particle (graviton)
  graviton is expected to be massless (gravity has infinite range)
  graviton is expected to be spin 2
  (since gravity is described by 2\textsuperscript{nd} rank energy-momentum tensor)
Extra Dimensions & The Planck Scale

ADD Model of Large Extra Dimensions

compactified extra dimension of size R

infinite extent usual 3+1 dimensions

flux lines in extra dimensions

test mass

Antoniadis, Arkani-Hamed, Dimopoulos, Dvali: hep-ph/9803315, 9804398, 9807344

• All standard model particles are trapped to surface of this hyper-cylinder
• Particles moving in the bulk have quantised wave functions (like 1d potential well)
• Higher order modes appear as higher energy excitations
• Mass difference between successive states related to size of dimension R
• Can lead to infinite Kaluza-Klein towers of particles
  massless gravitons would appear as a tower of massive states on our brane
  momentum in extra dim appears as additional mass: $M^2 = E^2 - (P_x^2 - P_y^2 - P_z^2) - P_n^2$
Why are the extra dims < 1mm?
gravity has only been tested down to this scale!
current torsion balance experiments set limit on $1/r^2$ dependence to <0.16mm

Where are the extra dimensions?
curled up (compactified) and finite
only visible at small scales / high energies

Relative strength of gravity explained by dilution of gravitons propagating in very large volume of bulk space
Gauss' Law for gravity: surface integral over closed volume containing vector field $g$ gives total enclosed mass $M$

$$\int g \cdot dA = -4\pi M$$

yields Newton's law

$$F = G \frac{m_1 m_2}{r^2}$$

With $n$ extra spatial dimensions each of size $R$

$$F = G_D \frac{m_1 m_2}{r^{2+n}}$$

$$F = \left( \frac{G_D}{R^n} \right) \frac{m_1 m_2}{r^2} \quad \text{i.e.} \quad G = \frac{G_D}{R^n}$$

For $r \gg R$ we recover Newtonian gravity

Planck scale: $M_P^2 = \frac{\hbar c}{G}$

In extra dimensions full scale of gravity $M_D$ is given by

$$M_D^{2+n} = \frac{\hbar c}{G_D} = \frac{M_P^2}{R^n}$$

Thus $M_D$ can be $\sim 1$ TeV when $R^n$ is large

For $n=1$ and $M_D=1$ TeV then $R \sim 10^{16}$ m $\Rightarrow$ already excluded!
Randall-Sundrum Model of Warped Extra Dimensions

Spacetime is structured as two separated 3-branes: SM and Planck

Two 3-branes connected with 1 extra dimension

Gravitons propagate in the bulk

Extra dimension highly curved with an exponential warp factor

$\Rightarrow$ introduces scaling between 3-branes

$\text{length} \propto \frac{1}{\mathcal{E}}$


$ds^2 = e^{-k\pi y} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$

$k = \text{warp factor}$

models characterised by scale $k/M_p$
Dark energy is \( \sim 74\% \) of critical density of universe

\[ \Rightarrow \text{density of dark energy } \rho_d \sim 0.0038 \text{ MeV/cm}^3 \]

\[ \Rightarrow \text{distance scale } L_d = \frac{\hbar c}{\sqrt{\rho_d}} \sim 85 \mu \text{m} \]

could be a fundamental distance scale...

Test inverse square law at small distances with torsion balance experiments

Measure torsion forces between test and attractor masses in horizontal plane (actually holes in two rings)

Measure torque vs vertical separation

Sensitive to \( \sim 1 \) nanoradian twists

(angle subtended by 1mm at distance of 1000 km)
Inverse square law holds for $\lambda < 56\mu m$

$\Rightarrow$ extra dims have

$R < 44\mu m$ 95% C.L.
Summary of measurements of \( G \) 1969-1999
Many large discrepancies...


\( \times 10^{-11} \text{m}^2\text{s}^{-2}\text{kg}^{-1} \)

* The error bars represent the quadrated sum of the individually listed Type A and Type B uncertainties.
In collisions Black Hole forms when impact parameter \( < 2r_s \)

\[
M_{BH} = \sqrt{s \cdot x_a \cdot x_b} = \sqrt{\hat{s}}
\]

\( r_s \) increased by factor \( R^n \)

\[
r_s = \frac{2GR^n M_{BH}}{c^2}
\]

Should observe continuous mass spectrum of BHs \( M > M_D \)

In absence of any real theory use classical cross section:

\[
\sigma_{BH}(s) = F\pi r_s^2
\]

parton cross section

\( F = \) production form/fudge factors

\[
\sigma_{BH}(s) = \sum_{a,b} \int \int \ dx_a \ dx_b \cdot f_a(x_a) \cdot f_b(x_b) \cdot \sigma(\hat{s})
\]

convolute PDFs to get total production cross section

Simple but extremely robust prediction!
Micro Black Hole Production

Cross section increases with $s$
For $s > M_D$ BH production will dominate over SM processes
For example very high $E_T$ jets no longer produced $\circledast$ form BH
Energy redistributed as lower momenta thermal emissions

“The end of short distance physics” Giddings, Thomas: hep-ph/0106219v4
Split Fermion Model

BHs do not conserve B, L, or flavour
⇒ Raises problems: proton decay, n-n̄bar oscillations...

Proton kinematically allowed to decay to any lighter fermion
Only protected by B conservation (which must be violated at GUT scale!)
Only option is $e^+ \rightarrow$ thus $p$ decay violates lepton number too

$$p \rightarrow e^+ + \gamma$$
$$p \rightarrow e^+ + \pi^0$$

Many ADD models predict too fast proton decay
(Super Kamiokande limit: $t \sim 10^{33}$y arXiv:0903.0676

Split Fermion Model
In this model spacetime structure is further modified
SM fermions exist on separated 3d branes
SM bosons propagate in the 'mini bulk' between them
Split fermion model may also explain fermion mass hierarchy

Arkani-Hamed, Schmaltz DOI:10.1103/PhysRevD.61.033005
Dai, Starkman, Stojkovic: hep-ph/0605085
Astrophysical black holes characterised by 3 numbers only
- $M$ mass
- $Q$ electric charge
- $J$ angular momentum

Metaphorically: 'bald' BH has only 3 hairs

In context of micro BH - they can also carry colour charge
(astro BHs only absorb colourless hadrons anyway)

Infalling matter has entropy, 2\textsuperscript{nd} law then implies BH have entropy too
BH cannot be a single microstate!
- infalling matter will always increase $r_s$ never decrease
  entropy $\propto$ surface area

Then it follows that an object with entropy has a temperature...

$$r_s = \frac{2GM_{\text{BH}}}{c^2}$$

$$\frac{\partial S}{\partial E} = \frac{1}{T}$$
Hawking Radiation

Near event horizon vacuum fluctuations interact with warped spacetime
Negative energy particle of virtual pair falls into BH, other becomes real
⇒ BH loses mass
radiate a black body spectrum with temp $T_H$

$$T_H = \frac{1}{8\pi} \frac{\hbar c^3}{G k_B M_{BH}}$$

Astro-BHs have temp < CMB
Micro BHs are very hot - radiate intensely
⇒ BH evaporate

Hawking radiation is purely thermal
only depends on M, Q, J, Col
Information Paradox

No hair (bald) theorem of BHs ⇒ violation of baryon nr, lepton nr, flavour

Two BHs of equal M, J, Q, but made of matter and anti-matter are identical

Independent of all other information - i.e. what 'stuff' fell into BH

Information loss paradox - else BH must remember what it swallowed

info remains inside BH? What happens when it decays?

In QM time evolution is unitary transformation:

\[ \langle \psi | \psi \rangle = \langle \psi | U^\dagger U | \psi \rangle = \langle \psi ' | \psi ' \rangle \]

Initial state BH transforms to final state of purely thermal radiation (M,Q,J)
this is a non-unitary transformation forbidden in QM - do not preserve probability!

Hawking now claims non-thermal info-preserving radiation  S. Hawking: hep-th/0507171
Collision produces complex state as horizon forms
Not all energy is trapped behind horizon

Extremely short lifetime $\sim 10^{-25}$ s

Balding
Energy lost as BH settles into 'hairless' state

Evaporation
Thermal Hawking radiation in form of SM particles & gravitons
Greybody factors give emission probs for all quanta

Plank Phase
For $M_{BH} \sim M_D$ unknown
quantum gravity effects dominates. BH left as stable remnant or final burst of particles $???
Limitations of the Models

Clearly much is missing in these models

No knowledge of true quantum gravity

Semi-classical approximation fails for $M_{BH} \sim M_D$

Formation of event horizon $\not\implies$ not all energy trapped inside

Greybody emission factors - QFT in strongly curved spacetime
  they have credence since solutions yield thermal spectra
  i.e. conspiracy of nature to be self-consistent!

Several calculations performed yield agreement at $\sim 1\%$ level

Nevertheless calcs assume fixed metric...

Phenomenological suppression of modes that increase $|Q|$ or Colour

Important to explore full phenomenological space

Include all effects into MC simulations

Gingrich: hep-ph/0609055
MC Generators

Incorporate all effects into MC models
- calculations of energy loss prior to horizon formation
- grey body factors
- rotation of BH (ang.mom)
- recoil of BH
- conservation/violation of B,L,flavour
- number, size & location of extra dimensions

BlackMax  Dai et.al.  arXiv:0711.3012
Charybdis  Frost et.al.  arXiv:0904.0979

Downloads: hepforge.org

BH is formed on quark brane at pp colliders
BH recoils at each emission
Affects emission spectra
Mostly emits quarks/gluons

split fermion model  0.002 fm

lepton brane

extra dim
Current Constraints

Search for deviations from SM cross sections with increasing $m$ $Q^2$ $\sqrt{s}$ ...
Look for $q\bar{q}\rightarrow Gg$ scattering - monojet events (graviton unseen in extra dim)

Graviton scattering derived as low energy effective field theory
Giudice, Rattazzi, Wells: hep-ph/9811291

HERA:
H1: $M_{D^-} > 0.78$ TeV and $M_{D^+} > 0.82$ TeV
ZEUS: $M_{D^-} > 0.9$ TeV and $M_{D^+} > 0.88$ TeV
  coupling $\pm \lambda$ has unknown sign of interference with SM

LEP:
$M_D = 1.5$ TeV for $n = 2$ $R = 0.2 \mu$m
$M_D = 0.75$ TeV for $n = 5$ $R = 400$ fm

CDF:
$M_D = 1.33$ TeV for $n = 2$ $R = 0.27 \mu$m
$M_D = 0.88$ TeV for $n = 6$ $R = 31$ fm

D0 (ll, gg):
$M_D = 1.23$ TeV lower limit

Variety of limits exclude $\sim 1$ TeV
Summary of constraints from astrophysical measurements & colliders

Supernovae & neutron stars probe low n
Colliders probe large n
Lower limits on fraction of trapped energy (indep. of $M_D$)

Form factors

$r_h$ is generalisation of $r_s$ for spinning BHs

$b = \text{impact parameter}

b_{\text{max}} = \text{horizon radius } 2r_h

Large $b \Rightarrow$ large ang mom states

For 'head on' collisions ($b=0$) $\sim 70\%$ of energy is trapped in event horizon

For large impact parameter only 1% - 50% of energy forms BH
Cross Sections for LHC

Cross section lower limits

\[ \sqrt{s} = 14 \text{ TeV} \quad M_D = 1 \text{ TeV} \]

- parton cross section
- single top: 250 pb

\[ n=1 \quad n=7 \]

- pp cross section incl. trapped energy

Potentially very large cross sections predicted
Horizon radius increases with \( n \) \( \Rightarrow \) cross sections increase with \( n \)
BlackMax prediction for non-rotating BHs

Semi-classical approach fails when $M_{BH} \sim M_{D}$

Don’t expect BH to form - but gravitational scattering...?

Close to $M_{D}$ observe jump in $2 \rightarrow 2$ scattering?

May be dominant effect

Meade, Randall: arXiv 0808.3017
BlackMax prediction for non-rotating BHs

Cross sections vary by ~ factor 10 for $n=1 \leq 7$
Factor ~30 suppression for $M_D = 1 \leq 3$ TeV
LHC Signatures

Emission spectra change depending on the models chosen

Typical ratio $\sim$ 8:1 hadrons:leptons

Leptons heavily suppressed in split fermion model

Graviton modes suppressed at low $n$

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<th>scenario</th>
<th>q+g</th>
<th>leptons</th>
<th>neutrinos</th>
<th>W/Z</th>
<th>G</th>
<th>H</th>
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<td>79.0%</td>
<td>9.5%</td>
<td>3.9%</td>
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<td>1.8%</td>
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<td>6.5%</td>
<td>2.5%</td>
<td>9.6%</td>
<td>??</td>
<td>0.7%</td>
<td>2.6%</td>
</tr>
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Uncalculated graviton greybody factors for $J>0$
Expected to be large - super irradiance
Gravitons are spin-2 tensors
LHC Signatures

High multiplicity events: 10-40 particles from heavy state
Hard $P_T$ spectrum of decay particles

\[ \langle N \rangle \text{ falls as } n \text{ increases} \]  
(BH temp increases)

Multiplicity compared to SM
LHC Signatures

Multiplicity of particles by type in different models

Higher multiplicity for larger mass

Quasi-democratic decays - fewer tops due to energy-momentum constraints

More particles than anti-particles due to pp initial state
$\mathcal{L} = 1 \text{ fb}^{-1} \quad M_{BH} > 5 \text{ TeV} \quad M_D = 1 \text{ TeV} \quad n=2$

$\Sigma |P_T| > 2.5 \text{ TeV}$

Requirement of additional high $P_T$ lepton reduces QCD b/g dramatically

If Atlas / CMS cannot trigger these events we should give up now!

highest threshold jet trigger (400 GeV $P_T$) unprescaled, $\epsilon = 100\%$
LHC Signatures

**Missing $E_T$ spectrum**

 Alternative selection: $E_T > 500$ GeV

![ATLAS](image)

**Largely from graviton emission in balding and Hawking phases**

**Compare:**
- SUSY models at 3 different scales
- Soft SM expectation

**But:**
- Difficult to calibrate
- Limits $M_{BH}$ measurement
String theory may be candidate theory for quantum gravity
Requires 6-7 extra spatial dimensions
String balls: high entropy low mass string states - BH progenitors

True theory is missing

3d brane

Gravity

closed strings free to propagate

SM particles are open strings confined to brane

extra dim
• TeV scale gravity can potentially address many shortcomings of SM
• No fundamental theory yet - but very rich phenomenology!
• Large parameter space to be explored
• Some models do appear contrived...
  ... but nature is weird (who could have predicted quantum mechanics?)
• Nevertheless, we should look because we can!
• The 'holy grail' of quantum gravity may be experimentally within reach

“The landscape is magic, the trip is far from being over”

Carlo Rovelli
Quantum Gravity
STRING THEORY SUMMARIZED:

I just had an awesome idea.
Suppose all matter and energy
is made of tiny, vibrating "strings."

Okay. What would
that imply?

I dunno.