The covalent bond in particle spectrocopy

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See hep.arXiv: 0802.0934

J. Phys. G 35 (2008) 075005

Examples	(MeV)
$f_0(980)$ and $a_0(980)$ -> KK	991
$f_2(1565)$ -> $\omega\omega$	1566
X(3872) -> D(1865)D*(2007)	3872
Y(4660) -> ψ'(3686)$f_0(980)$	4666
$\Lambda_c(2940)$ -> D*(2007)N	2945
$K_0(1430)$ -> Kη' ?	1453
$K_1(1420)$ -> KK*	1388

$$BW = N(s)/D(s) \text{ where } D(s)=M^2 - s - iM\Gamma(s)$$

phase space

$$D(s)= M^2 - s - \Sigma_i \Pi_i(s)$$

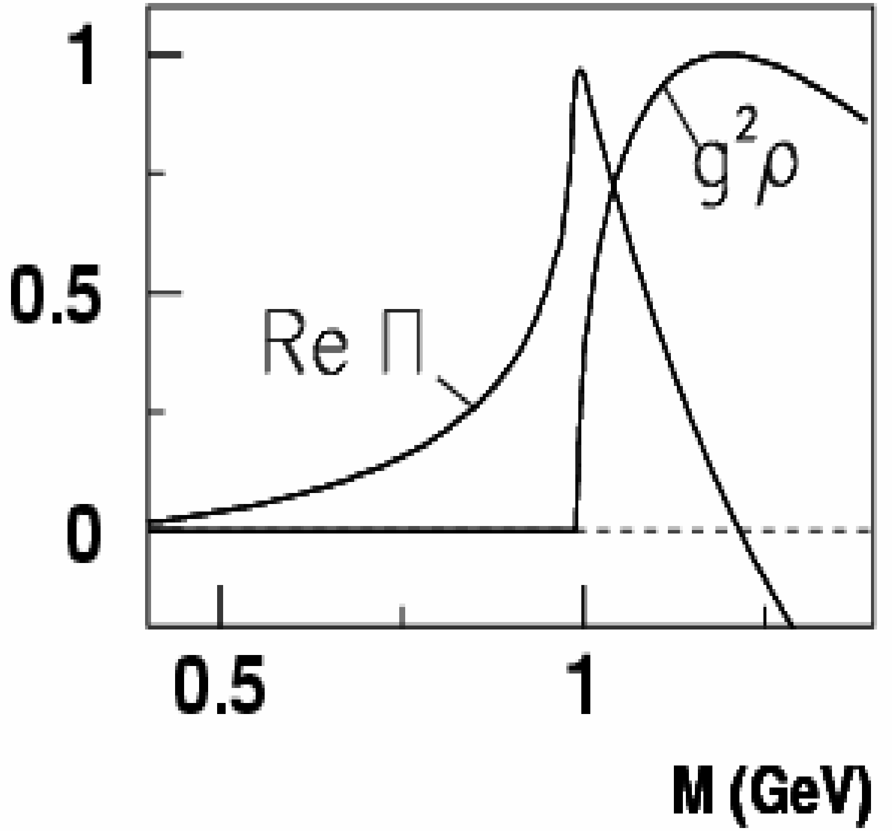
$$\text{Im } \Pi_i = g_i^2 \rho_i(s) FF_i(s)$$

$$\text{Re } \Pi_i = \frac{1}{\pi} P \int_{thr_i} ds' \frac{\text{Im } \Pi_i(s')}{(s' - s)}$$

At threshold, Re Π is positive definite. Form factor is needed to make the integral converge. The full form of the BW is

$$D(s)=M^2 - s - \text{Re } \Pi(s) - i \text{ Im } \Pi(s)$$

f0(980) -> KK as an example



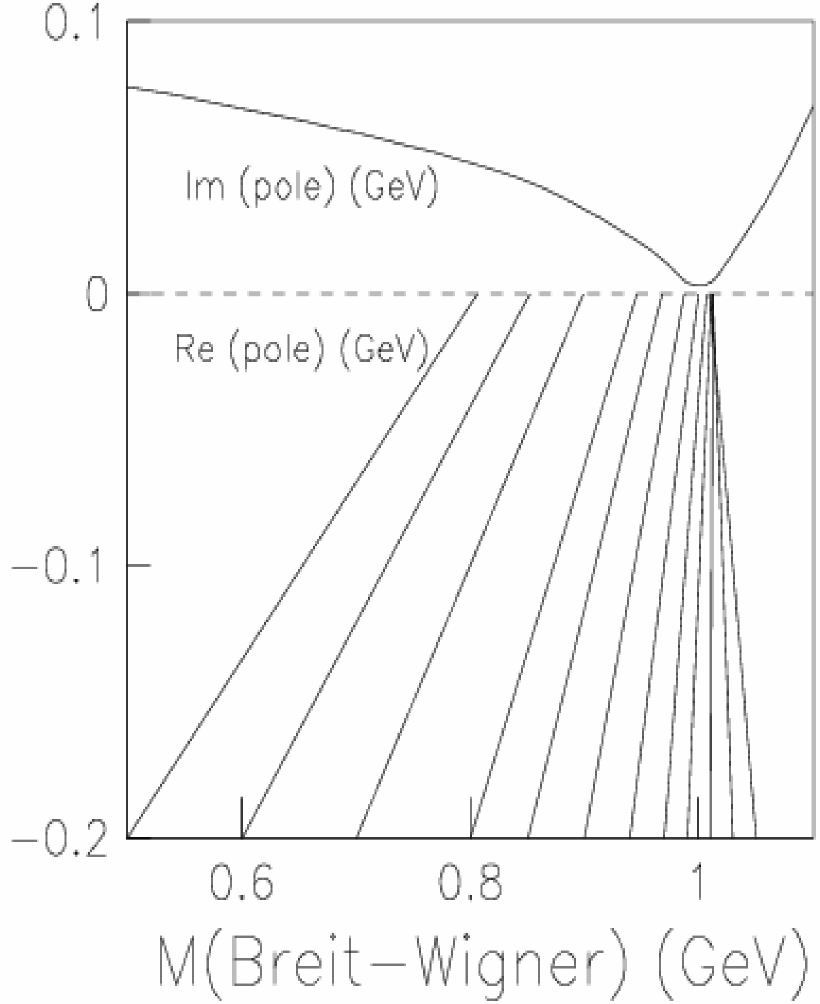
FF = exp(-3k^2)

(R=0.8 fm)

Re Π acts as an effective attraction pulling the resonance to the threshold. (Analogy to S).

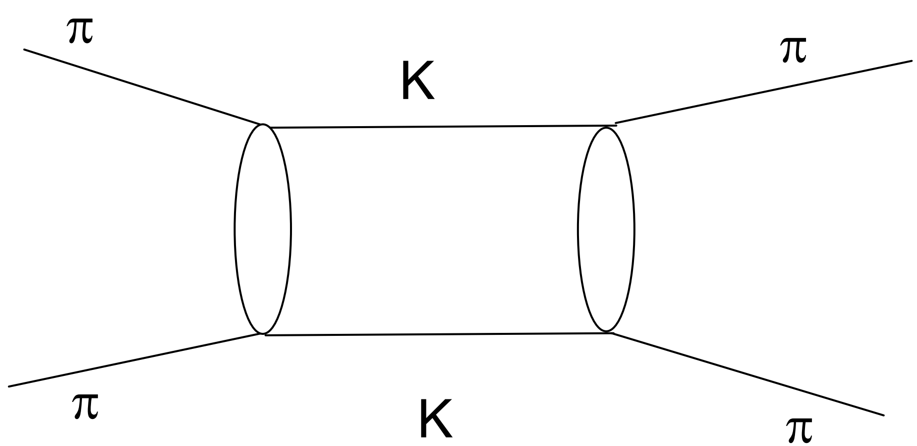
The parameters of $f_0(980)$ are accurately known from BES 2 data on $J/\psi \to \phi\pi\pi$ and ϕKK. One can play the game of varying M of the Breit-Wigner and evaluating the pole position:

M(MeV)	Pole (MeV)
500	806 – i76
700	899 – i59
900	987 – i31
956	1004 – i21
990	1011- i4
1050	1009 – i28
1100	979 – i69



0.1
0
−0.1
−0.2
Im (pole) (GeV)
Re (pole) (GeV)
0.6
0.8
1
M(Breit−Wigner) (GeV)

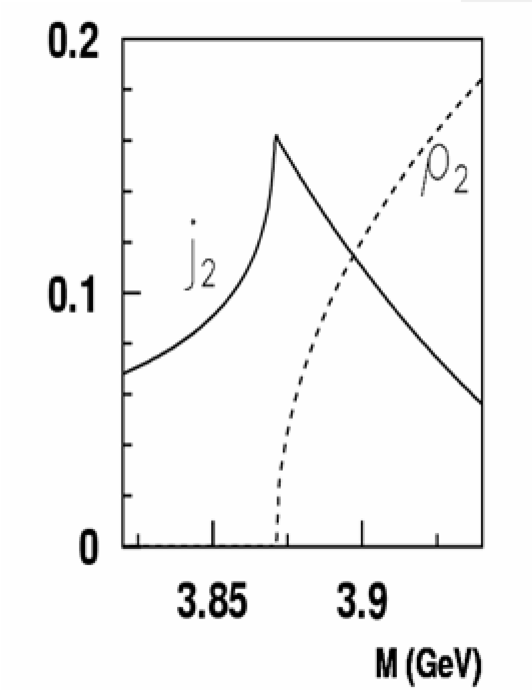
Incidentally, the dispersive term Re Π is equivalent to the loop diagram for producing the open channel:

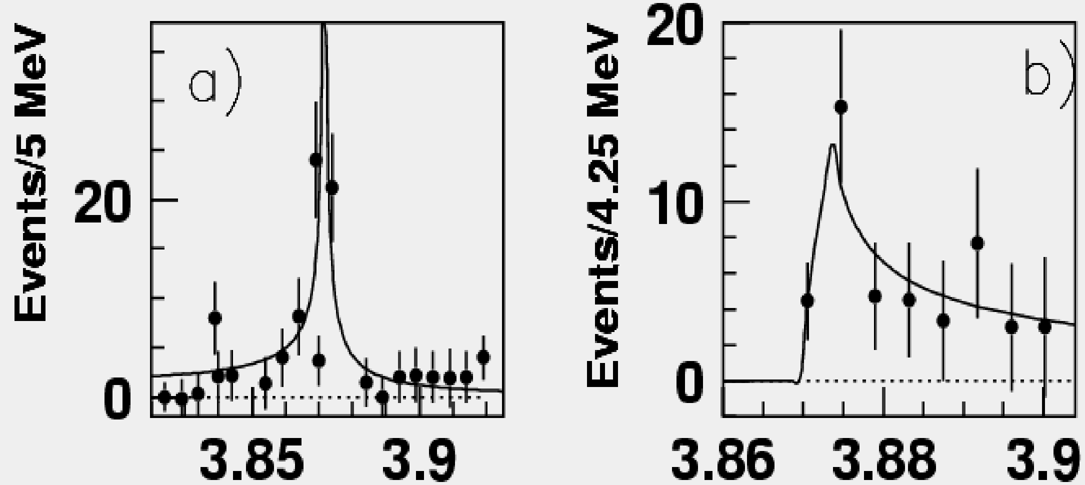


Also, solving the Schrodinger equation (or its relativistic version, the Bethe-Salpter equation, is equivalent to evaluating this loop diagram (and all iterations of it).

I have evaluated Re $\Pi(s)$ for several broad resonances with widths >100 MeV, and refitted data. Unless the data are particularly accurate, or the threshold is strong, the shape of the resonance changes little, and Re $\Pi(s)$ can be absorbed by changes in M and Γ of the BW.

X(3872): found by Babar(2003) in $J/\Psi\pi\pi$, confirmed by CDF, D0 and Belle. Also seen in $J/\Psi\omega$ and DD*. Located at the mass of $D_o(1865)+D_0^*(2007)$.





$J/\psi\ \rho$ D D*

A pure cusp is too broad, so X(3872) must be a resonance or bound state (or virtual state).

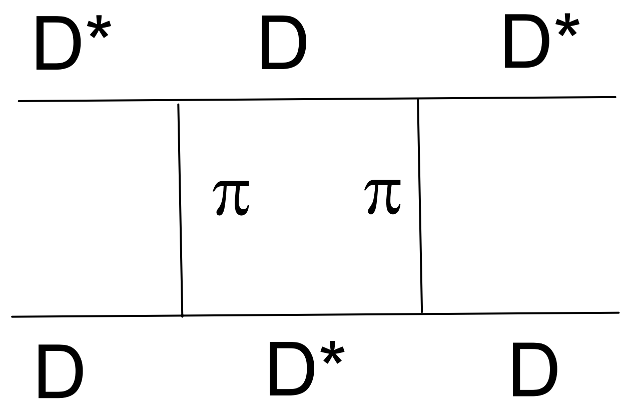
Theorists leapt to the conclusion that this was a $D_0D_0^*$ molecule. However, there are 2 objections to this interpretation: (i) the production cross section in the Fermilab expt is 100 times too large for a molecule with a binding energy of 0.3 MeV. (ii) radiative decays have now been observed to $\gamma J/\Psi$ and $\gamma\Psi(2S)$. Their relative rates are inconsistent with a large molecule,

but ARE consistent with a c-cbar state. It is now clear that X(3872) is largely a c-cbar state which has been attracted to the $D_0D_0^*$ threshold. The decays to $\omega J/\Psi$ and $\rho J/\Psi$ violate isospin conservation because of the 8 MeV splitting between charged and neutral D-D* thresholds.

[Meanwhile several more X,Y,Z states have been claimed from 3915 to 3945 MeV. J^P have been determined only for one of them: Z(3930), but it seems likely they are different decay modes of this one state: the n=2 c-cbar $^3P_2$ state. The X(3872) is the n=2 $^3P_1$ state.]

A full understanding of X(3872) requires an understanding of how meson exchanges contribute to creating it. It couples to D-D*bar.

The D* decays to Dπ (and Dγ). This gives the process:



Including this diagram in the Bethe-Salpeter equation is equivalent to evaluating Re Π(s) from the dispersion relation. This loop diagram therefore contributes to the binding of X(3872); indeed Tornqvist suggested early on that pi exchange created X(3872) as a molecule.

The Tubingen group of Amand Faessler et al. calculated the effect of exchanges of σ, ρ, ω, etc, as well as π exchange. They also included the effects of D^+D^*. They found that these processes accounted for the binding of X(3872), and the observed isospin mixing.

[This is analogous to nuclear binding, eg. in the deuteron]. The attraction of the c-cbar state to the D-D* threshold is due to meson exchanges.

(Lee et al, PRD 80 (2009) 094005)

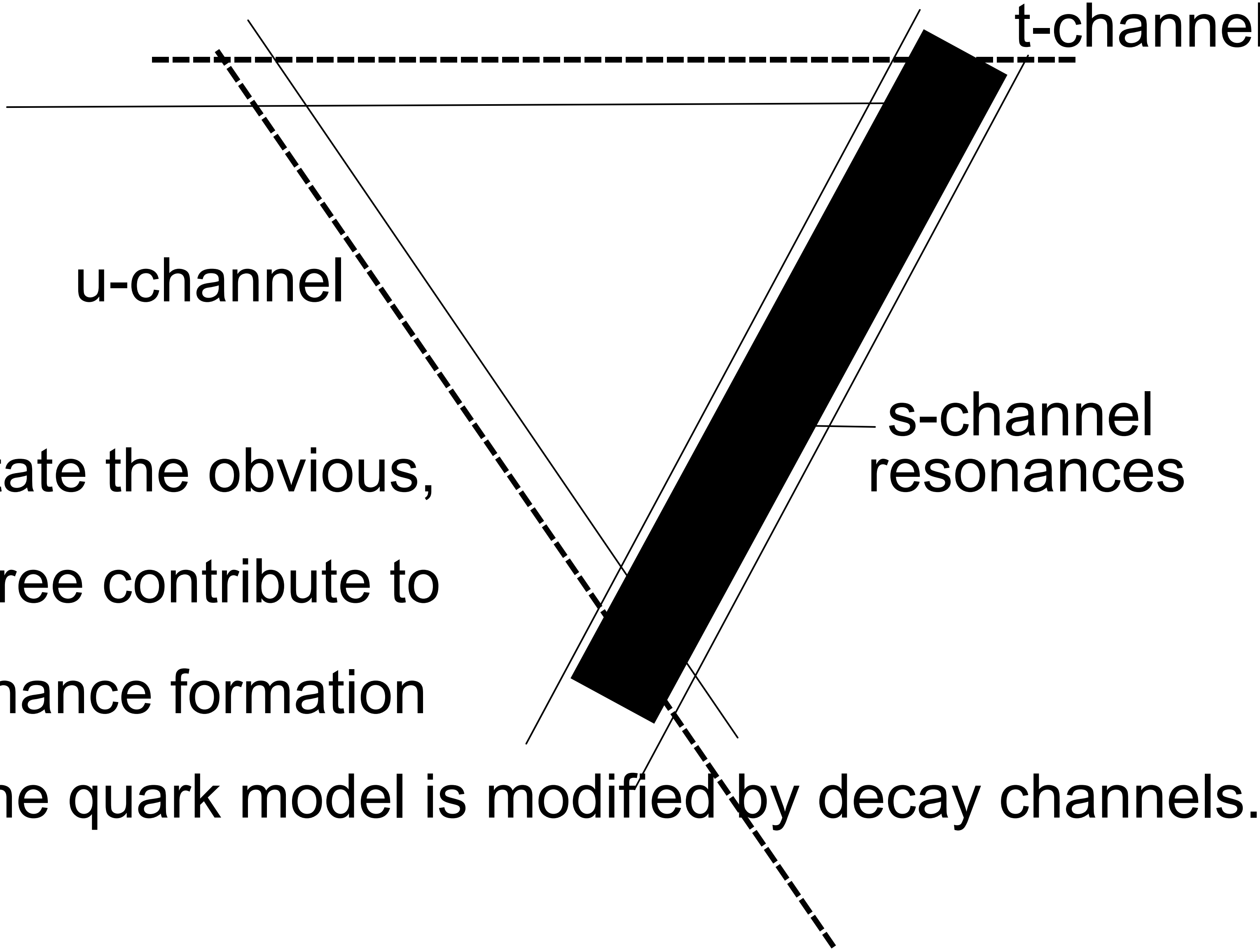
Further examples

1) f2(1560) is seen strongly in $\omega\omega$ and $\rho\rho$. It is lower than its I=1 partner a_0(1680-1720) by a large amount. Its lower mass can be explained by attraction to the $\omega\omega$ and $\rho\rho$ thresholds.

2) There is a 0^+ nonet of f_0(1300-1370), a_0(1450), K_0(1430) and f_0(1710). The a_0(1450) has a dominant decay to $\omega\rho$, which may be pulling its mass up strongly.

4) The PDG lists two closely separated $\eta(1405)$ and $\eta(1475)$. The latter decays only to K*(890)K with L=1. The nominal threshold is at 1394 MeV and the P-wave phase space makes this channel peak at ~1475 MeV. The $\eta(1405)$ decays to $\eta\sigma$, $a_0(980)\pi$ and κK with L=0 and are unaffected by barrier effects. All decays can be fitted well with a single $\eta(1440)$.

In the Mandelstam diagram, there are:



To state the obvious,

all three contribute to

resonance formation

i.e. the quark model is modified by decay channels.

Oset, Oller et al find they can generate MANY states from meson exchanges. This is along the lines of Hamilton and Donnachie, who found in 1965 that meson exchanges have the right signs to generate P_{33}, D_{13}, D_{15} and F_{15} baryons.

Suppose contributions to the Hamiltionian are H_{11} and H_{22}; the eigenvalue equation is

$$\begin{pmatrix} H_{11} & V \\ V & H_{22} \end{pmatrix} \Psi = E\,\Psi$$

H_{11} refers to q-$\bar{q}$; H_{22} to s,t,u exchanges.

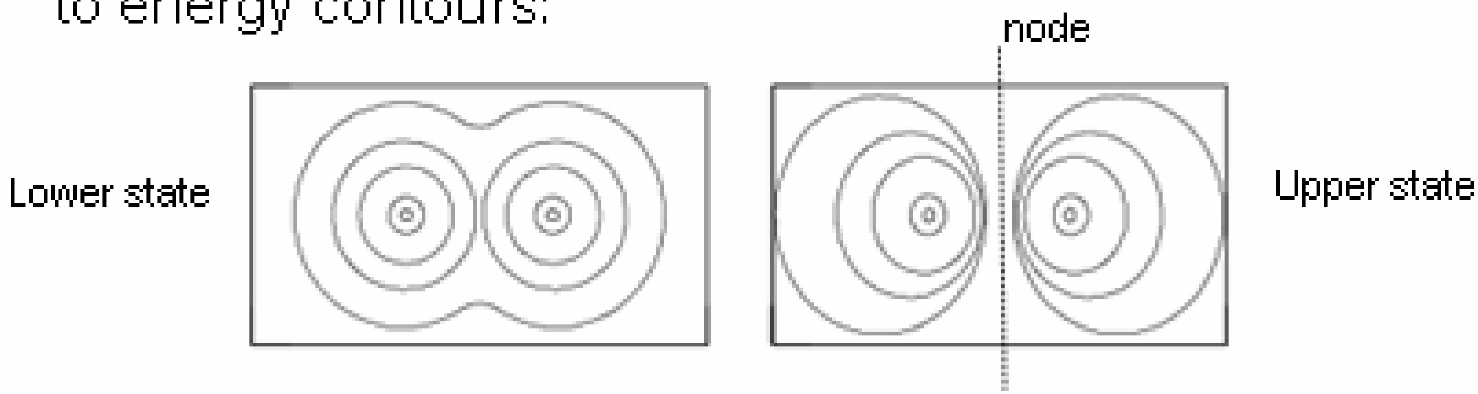
V is the mixing element between them.

Two solutions: $E = (E_1+E_2)/2 \pm [(E_1-E_2)^2 - |V|^2]^{1/2}$

The KEY point is that mixing LOWERS the ground state, hence increasing the binding. (It also pushes the upper state higher).

Analogy with the covalent bond in chemistry

The H_2 molecule may be treated with the molecular orbital approximation or the valence bond approximation; both lead to energy contours:



Binding arises from two effects:

(i) The orbital round one nucleus expands over the interaction region and lowers the momentum, hence kinetic energy:

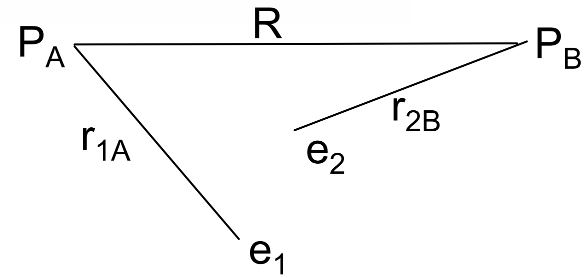
(ii) the lower energy configuration shrinks so that H_2 is more compact than the hydrogen ion H_2^+; this increases the potential energy of the electrons round each proton.

Purely a quantum mechanical effect.

see hep-ph/1001.1712; J.Phys. G37 (2010) 055002

The Hydrogen Molecule: 2 electrons (1 and 2), 2 protons (A and B).

$$H = -\frac{\hbar^2}{2m}(\nabla_1^2 + \nabla_2^2) + e^2\left(\frac{1}{R} + \frac{1}{r_{12}} - \frac{1}{r_{1A}} - \frac{1}{r_{2A}} - \frac{1}{r_{1B}} - \frac{1}{r_{2B}}\right)$$



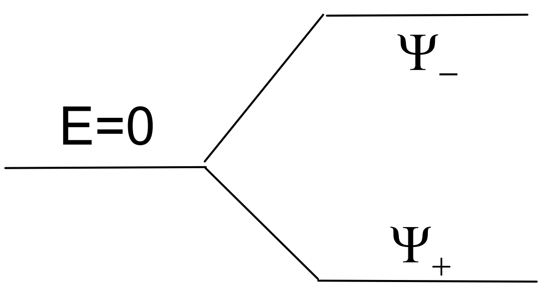
Set $\psi_+ = \psi_a + \psi_b$; $\psi_- = \psi_a - \psi_b$

Try $\psi = \psi_+ + \alpha\psi_-$; then

$$H\begin{pmatrix}\psi_+ \\ \alpha\psi_-\end{pmatrix} = E\begin{pmatrix}\psi_+ \\ \alpha\psi_-\end{pmatrix}$$ **<=**

First approx: neglect e^2/r_{12}, i.e. repulsion between electrons;
solution: $\psi = \psi_+$ (ground state) and ψ_- (excited state).

Second approx: include e^2/r_{12}



The mean energy > E=0, because of repulsion between the two electrons

The eigenvalue equation is written in terms of two basis states ψ_+ and ψ_-; they are mixed by $V = e^2/r_{12}$:

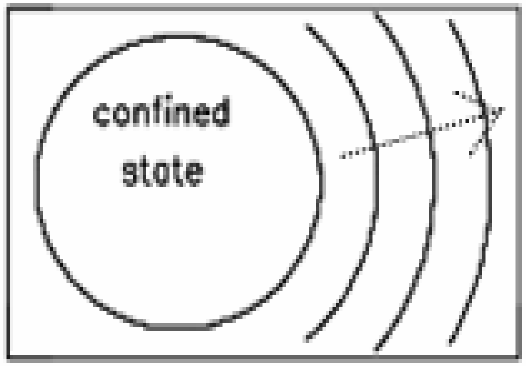
$$\begin{pmatrix} H_{11} & V \\ V & H_{12} \end{pmatrix} \begin{pmatrix} \psi_+ \\ \alpha\psi_- \end{pmatrix} = E \begin{pmatrix} \psi_+ \\ \alpha\psi_- \end{pmatrix}$$

In Particle Physics, the equation is identical but the basis states are $q\bar{q}$ and Meson-Meson. The solution is

$$E = (E_1 + E_2)/2 \quad \pm \quad [(E_1 - E_2)^2 - V^2]^{1/2} \quad \text{(Breit-Rabi)}$$

Key point: The wave function of each electron expands into the overlap region, lowers k, hence zero-point energy.

Analogy with the MIT bag model



In the lower state, the **wave** function is sucked into the overlap region, leading to attraction,

In the upper state, the wave function is repelled from this region.

The Born approximation does not include these effects.

Solving the Schrodinger equation does include them.

A new point

Meson exchange can be attractive or repulsive. For I=0 $\pi\pi$ and I=1/2 $K\pi$, they are attractive; for I=2 $\pi\pi$ and I=3/2 $K\pi$, they are repulsive. Experimentally, the σ pole is observed at 470 MeV, $\Gamma = 500$ MeV; together with $a_0(980)$, $f_0(980)$ and κ they make a nonet. Jaffe (1977) suggested they might be 4-quark states made of diquarks in colour $3 \times \bar{3}$ configurations. He speculated there might also be $6 \times \bar{6}$ configurations making an SU(3) 27 multiplet. These are not observed. I suggest the observed nonet is mostly meson-meson, perhaps with a small $q\bar{q}$ component; the {27} is absent because of repulsive meson exchanges.

Other mesons appear as nonets and baryons as nonets and decuplets. It turns out that in higher representations, meson exchanges are repulsive. I suggest these repulsive effects de-stabilise higher SU(3) representations, so that the lower lying nonets and decuplets are the only stable ones.

A more speculative point is that most of the higher-lying resonances have widths typically 250 MeV, roughly equal to their spacing. This suggests feedback stabilising the width to be equal to the spacing. If a resonance overlaps its radial excitation, there is the usual level-repulsion between them.

Four-quark states distinct from meson-meson seem not to exist so far.

The Higgs – more speculation !

Fermi lab have not seen the Higgs boson on a mass scale of 160 GeV. Perhaps it might appear as a bound state of WW mixed with ZZ, stabilised by decays to WW, WZ and ZZ. Theorists are aware that the dispersive terms associated with decays need to be included in fits to the Higgs.

Alterbatively, a broad Higgs near the unitarity limit would not be surprising; it would be strictly analogous to the broad sigma, which peaks around 1 GeV but has a lower pole at 470 MeV, with large width.

Summary

1) The dispersive term Re $\Pi(s)$ is necessary in the Breit-Wigner denominator of all resonances. It peaks at thresholds and acts as an effective attraction, which explains why several resonances appear at thresholds.

2) There is an exact (and helpful) analogy with the covalent bond in chemistry.

3) Meson and baryon exchanges are repulsive in SU(3) representations other than nonets and decuplets and can explain why we do not see higher representations.

4) There may be a feedback mechanism which limits resonance widths to the spacing between radial excitations.