Spacetime curvature and Higgs stability during and after inflation

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Tommi Markkanen\textsuperscript{12} Matti Herranen\textsuperscript{3} Sami Nurmi\textsuperscript{4}
Arttu Rajantie\textsuperscript{2}

\textsuperscript{1}King’s College London
\textsuperscript{2}Imperial College London
\textsuperscript{3}Niels Bohr International Academy, Copenhagen
\textsuperscript{4}University of Jyväskylä

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1. Introduction

2. Higgs stability during inflation (QFT in Minkowski)

3. Higgs stability after inflation

4. Conclusions
Introduction

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4. Conclusions
$V(\phi)$ has a minimum at $\phi = v$

A vacuum at $\phi \neq v$ incompatible with observations

Behaviour very sensitive to $M_h$ and $M_t$

New physics needed to stabilize the vacuum?
Current status

**Figure**: Degrassi et al. (2013)

- **Meta** stable at 99% CL [1]
- Lifetime much longer than $13.8 \cdot 10^9$ years
- Is this also true for the early Universe?

Inflation and the Standard Model

- We assume the SM to be valid at high energies
  - Potential peaks at $\Lambda_{\text{max}}$

- Assuming also an early stage of exponential cosmological expansion (inflation) with a scale $H$
  - Important if $\Lambda_{\text{max}} \lesssim H$
  - State of the art calculations [2]: $\Lambda_{\text{max}} \sim 10^{11}\text{GeV}$

\[ V(\phi) \]

$V_{\text{max}}$

0

$v$

$\Lambda_{\text{max}}$

\[ [2] \text{ Degrazzi et. al.}(2013); \text{ Buttazzo et. al.}(2013) \]
1 Introduction

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4 Conclusions
Inflation induces fluctuations to the Higgs field $\Delta \phi \sim H$

Fluctuations may be treated as stochastic variables [3]

$\Rightarrow$ We can assign a probability density $P(\phi)$ to $\phi$

The essential input for $P(\phi)$ is $\tilde{V}_{\text{eff}}(\phi)$, the effective potential

1-loop Effective potential

- Derivation of $V_{\text{eff}}(\phi)$ is a standard calculation \cite{4}
- A theory with a massive self-interacting scalar field

$$V_{\text{eff}}(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4$$

- Classical

$$+ \frac{M(\phi)^4}{64\pi^2} \left[ \log \left( \frac{M(\phi)^2}{\mu^2} \right) - \frac{3}{2} \right]$$

- Effective mass

- Quantum

- $\mu$ is the renormalization scale
- Similarly one may derive the potential for the SM Higgs

\cite{4} Coleman & Weinberg (1972)
Effective potential for the SM Higgs

\[ V_{\text{eff}}(\phi) = -\frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 + \sum_{i=1}^{5} \frac{n_i}{64\pi^2}M_i^4(\phi) \left[ \log \frac{M_i^2(\phi)}{\mu^2} - c_i \right] \]

\[ M_i^2(\phi) = \kappa_i \phi^2 - \kappa'_i \]

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- Explicit \( \mu \) dependence?
The effective potential is renormalized at a scale $\mu$

$$\lambda_0 \rightarrow \lambda_R + \delta \lambda, \quad \phi \rightarrow (1 + \delta Z)\phi$$

However, the physical result must not depend on $\mu$.

We can impose this by demanding

$$\frac{d}{d\mu} V_{\text{eff}}(\phi) = 0$$

This can be used to improve the perturbative result.

 Leads to *running parameters*, e.g. $\lambda(\mu)$.

Same can be done for the SM.
For large $\phi$, the potential is dominated by the quartic term $\lambda \phi^4$

$$V(\phi) \sim \frac{\lambda(\mu)}{4} \phi^4$$
Scale independence $V_{\text{eff}}$

- One can easily show that for the SM to 1-loop [5]
  \[ \frac{d}{d\mu} V_{\text{eff}} = 0 + O(\bar{h}^2) \]

- We must choose $\mu$ to make the higher order terms as small as possible [6]

The optimal choice

\[ \mu \sim \phi \]

$\Rightarrow$ No large logarithms

- Now we have a well-defined potential with no unknown parameters!

<2->

- It is possible to include (classical) gravity in the quantum calculation, \( R = 12H^2 \)

\[ \Rightarrow \text{The SM includes a non-minimal } \xi \text{-term, } \sim \xi R\phi^2 \]

- **Always** generated by running in curved space
- Virtually unbounded by the LHC, \( \xi_{\text{EW}} < 10^{15} \) [7]

- Curvature induces running of the constants [8]
- Leading potential contributions:

**Flat space, } \phi \gg m \**

\[ V_{\text{eff}}(\phi) \approx \frac{\lambda(\phi)}{4} \phi^4 \]

**Curved space, } H \gg \phi \gg m \**

\[ V_{\text{eff}}(\phi) \approx \frac{\lambda(H)}{4} \phi^4 + \frac{\xi(H)}{2} R\phi^2 \]

1-loop Effective potential in curved space

\[ V_{\text{eff}}(\phi, R) = -\frac{1}{2}m^2(t)\phi(t)^2 + \frac{1}{2}\xi(t)R\phi(t)^2 + \frac{1}{4}\lambda(t)\phi(t)^4 \]

\[ + \sum_{i=1}^{9} \frac{n_i}{64\pi^2}M_i^4(t) \left[ \log \left| \frac{M_i^2(t)}{\mu^2(t)} \right| - c_i \right] ; M_i^2(t) = \kappa_i \phi(t)^2 - \kappa_i' + \theta_i R \]

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For large $H \sim 10^3 \Lambda_{\text{max}}$, the SM is not stable [9].

Coupling the Higgs to an inflaton $\sim \Phi^2 \phi^2 \Rightarrow$ stable [10].

How does including curvature change this?

First attempt, set $\xi_{EW} = 0$ and $H \sim 10^3 \Lambda_{\text{max}}$

$$V_{\text{eff}}(\phi) \approx \frac{\lambda(\mu)}{4} \phi^4 + \frac{\xi(\mu)}{2} R \phi^2$$

For large $H$ one has $\lambda(\mu) < 0$, since $\mu^2 = \phi^2 + R$

$\xi$ Can become positive or negative depending on $\xi_{EW}$
For large $H$ one has $\lambda(\mu) < 0$, since $\mu^2 = \phi^2 + R$

$\xi$ Can become positive or negative depending on $\xi_{EW}$
Now choosing $\xi_{\text{EW}} = 0.1$ [11]

$V_{\text{max}}(\text{curved}) \gg V_{\text{max}}(\text{flat})$ (and at a higher scale)

$P \sim \exp\left[-8\pi^2 \left(\frac{V_{\text{max}}}{3H^4}\right)\right] \Rightarrow \text{Stable!}$

The (in)stability of the potential is determined by $\xi_{EW}$.
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inflation probably happened sometime here
Reheating

- Equation of state $w = p/\rho$ changes, $w_{\text{inf}} = -1 \rightarrow w_{\text{reh}}$

- Energy of inflation is transferred to SM degrees of freedom, which (eventually) thermalize $T = 0 \rightarrow T_{\text{reh}}$

- The crucial moment is right after inflation, but before thermalization

- A very complicated and dynamical process [12]
  - Reheating $\Leftrightarrow$ Preheating

  - The Higgs always feels the dynamics of reheating
    (even without a direct coupling to the inflaton)

During reheating the inflaton oscillates ($p = w \rho$)

- The inflaton influences the Higgs via gravity
  
  ⇒ New stability constraints!

Two effects:
- A rapid drop in $w$, on average
- Oscillations in the complete solution
Oscillating mass (example)

For example for a coupling $\mathcal{L}_{\text{int}} \propto g\Phi^2\phi^2$

\[ m_{\text{eff}}^2 \]

Oscillating mass for Higgs

\[ m_{\text{eff}}^2 \sim g\Phi_0^2 \cos^2(t M_{\text{inf}}) \]

\( \square \) Parametric resonance via the Mathieu equation

\[
\frac{d^2 f(z)}{dz^2} + \left[ A_k - 2q \cos(2z) \right] f(z) = 0, \quad z = t M_{\text{inf}}
\]

⇒ Exponential amplification

• May result in a very large fluctuation [13]

Oscillating $R$

- The curvature oscillates during reheating

$$G_{\mu\nu} = \frac{1}{M_{\text{pl}}^2} T_{\mu\nu} \quad \Rightarrow \quad R = \frac{1}{M_{\text{pl}}^2} \left[ 4V_{\text{inf}}(\Phi) - \left( \frac{d\Phi}{dt} \right)^2 \right]$$

- **Tachyonic resonance** [14]
- Oscillations of $R$ via $\xi$ provide efficient reheating
  - *Geometric reheating* [15]

Fluctuations from parametric resonance

- Resonance may give large fluctuations, ⇒ Instabilities ?!
- After one oscillation

\[ n \sim \exp \left\{ \sqrt{\xi} \right\} \]

Superhorizon modes, \( k < aH \)

\[ \Rightarrow \quad \Delta \phi^2 \sim \left( \frac{H}{2\pi} \right)^2 \frac{\exp \left\{ \sqrt{\xi} \right\}}{\sqrt{\xi}} \]

- Potentially a huge effect, \( \Delta \phi \gg \Lambda_I \)

However, the resonance may be shut off by backreaction

- Self-interactions
  \[ \lambda \langle \hat{\phi}^2 \rangle \ll \xi R, \quad \text{if} \quad \lambda > 0 \]

- Gravity
  \[ \rho_{\text{Higgs}} \ll 3M_{\text{pl}}^2 H^2 \]
For $H \gtrsim \Lambda_I \sim 10^{11}$ GeV, $\xi$ is constrained to be $\sim 1/6$. 

$\frac{H}{\Lambda_I} \sim 10^{11}$ GeV, $\xi$ is constrained to be $\sim 1/6$. 

⇒ For $H \gtrsim \Lambda_I \sim 10^{11}$ GeV, $\xi$ is constrained to be $\sim 1/6$. 

Stability results, reheating
Outline

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Conclusions

- For a large $H$, curvature significantly effects the early universe SM instability
  - Running of couplings from $H$
  - A curvature mass $\propto \xi R \phi^2$ is always generated
- Stability during inflation and reheating constrains SM physics, namely for large $H$

$\xi \sim 1/6$

Thank You!