Muon g-2
Precision Precession

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OUTLINE

• What is it?
• Why measure it (again)?
• How?
  • Goals and how to achieve them:
    • Brief recap of technique
    • Upgrades!
      • Beam, detectors, field

• Status and Conclusions

† The material for this talk has been shamelessly stolen from many including: B. Lee Roberts, Leah Welty-Reiger, Mark Lancaster, Thomas Teubner, Chris Polly, Andreas Kronfeld, Ruth Van de Water……
Magnetic Moments

• Magnetic moment of elementary particles related to their spin by the “g-factor”

\[ \vec{\mu}_S = g \frac{Qe}{2m} \vec{S} \]

\[ \vec{\tau} = \vec{\mu} \times \vec{B} \]

Larmor frequency

A little history…

1924  Stern-Gerlach
Magnetic moment of silver atom in it’s ground state is 1 Bohr magneton. (10%)

…but not understood as spin 1/2
Spin $\frac{1}{2}$?

1925/26 Uhlenbeck And Goldschmidt proposed electron spin to explain fine structure….

…but prediction off by factor of 2!

Rescued by Thomas precession (1926) - relativistic kinematics effect (successive non-collinear boosts give rotation).
g=2

\[ \left( i \gamma^\mu \left( \partial_\mu + ieA_\mu \right) - m \right) \psi = 0 \]

Non-relativistic reduction ⇒

\[ i\hbar \frac{\partial \psi}{\partial t} = \left\{ \frac{p^2}{2m} - \frac{e}{2m} \left[ 1\vec{L} + 2\vec{S} \right] \cdot \vec{B} \right\} \psi \]

\[ g_L = 1 \]

\[ g_s = 2 \]

The Quantum Theory of the Electron.

By P. A. M. Dirac, St. John’s College, Cambridge.
Greater Experimental Precision...

…1947 (Nafe, Nelson, Rabi) Hyperfine structure of H and D did not fit $g=2$… (It was a 5 sigma effect)

1948 Kusch and Foley : A precision measurement: $g_e=2(1.00119\pm0.00005)$

An anomaly! Define $a = \frac{g - 2}{2}$

It takes QED to begin to explain the anomaly…

$ae = \frac{\alpha}{2\pi} = 0.001161$
More QED…

\[ \frac{g}{2} = 1 + C_1 \left( \frac{\alpha}{\pi} \right) + C_2 \left( \frac{\alpha}{\pi} \right)^2 + C_3 \left( \frac{\alpha}{\pi} \right)^3 + C_4 \left( \frac{\alpha}{\pi} \right)^4 + C_5 \left( \frac{\alpha}{\pi} \right)^5 + \]

Even analytically…

Some very weird diagrams!

4 loops: [Baikov, Broadhurst'95]
5 loops: [Baikov, Chetyrkin, Kühn, Sturm'13; Baikov, Marquard, Maier'13] (see also [Aguilar, Greynat, De Rafael'08])
Together with a succession of experiments, ultra-precise agreement gives the best value of \( \alpha_E \).
Standard Model Physics predicts electron magnetic moment anomaly at ppt level!

But the story is different for the muon… It’s heavier

More sensitive to more contributions…

(Hadronic corrections only enter around 12th decimal place in $a_e$)
• QED well known
• EW contributions also understood (only couple of loop accuracy needed)

• **Hadronic contributions are significant and the biggest source of uncertainty.**

\[
\alpha_{\mu}^{\text{had}} = \alpha_{\mu}^{\text{had},\text{VP LO}} + \alpha_{\mu}^{\text{had},\text{VP NLO}} + \alpha_{\mu}^{\text{had},\text{Light-by-Light}}
\]

LO  
NLO  
L-by-L

Non-perturbative - cannot be calculated. Determined from experiment

Low energy \(e^+e^- \rightarrow \text{hadrons.} \)

+ some lattice QCD for L-by-L contribution

\[
\alpha_{\mu}^{\text{had},\text{LO}} = \frac{m_{\mu}^2}{12\pi^3} \int_{s_{\text{th}}}^{\infty} ds \frac{1}{s} K(s) \sigma_{\text{had}}(s)
\]
$e^+e^- \rightarrow \text{hadrons}$

**New $\pi^+\pi^-$**

**Older $e^+e^-$ data**

**New $K^+K^-$**

$\phi$
How the contributions stack up:

Determination of hadronic contribution to muon $g-2$ has become an industry

<table>
<thead>
<tr>
<th>Contributions</th>
<th>Value ($\times 10^{-11}$) units</th>
</tr>
</thead>
<tbody>
<tr>
<td>QED ($\gamma + \ell$)</td>
<td>$116,584,718.951 \pm 0.009 \pm 0.019 \pm 0.007 \pm 0.077_{\alpha}$</td>
</tr>
<tr>
<td>HVP(lo) [47]</td>
<td>$6.923 \pm 42$</td>
</tr>
<tr>
<td>HVP(lo) [48]</td>
<td>$6.949 \pm 43$</td>
</tr>
<tr>
<td>HVP(ho) [48]</td>
<td>$-98.4 \pm 0.7$</td>
</tr>
<tr>
<td>HLbL [61]</td>
<td>$105 \pm 26$</td>
</tr>
<tr>
<td>EW [54]</td>
<td>$153.6 \pm 1.0$</td>
</tr>
<tr>
<td>Total SM [47]</td>
<td>$116,591,802 \pm 42_{\text{H-LO}} \pm 26_{\text{H-HO}} + 2_{\text{other}} (\pm 49_{\text{tot}})$</td>
</tr>
<tr>
<td>Total SM [48]</td>
<td>$116,591,828 \pm 43_{\text{H-LO}} \pm 26_{\text{H-HO}} + 2_{\text{other}} (\pm 45_{\text{tot}})$</td>
</tr>
</tbody>
</table>

Paralleled by $g-2$ measurements…
Experimental Determination of $\alpha_\mu$.

A succession of improving measurements

...Details to follow!
The current state of the art:

\[ a_{\mu}^{\text{exp}} = 116\,592\,089\,(63) \times 10^{-11} \]
\[ a_{\mu}^{SM} = 116\,591\,802\,(49) \times 10^{-11} \]
\[ a_{\mu}^{\text{exp}} - a_{\mu}^{SM} = 287\,(80) \times 10^{-11} \]

Not same precision as the electron but compensated by higher mass.

Muon anomalous magnetic moment is sensitive to most of the standard model... and to new physics.

A tantalising but inconclusive 3.3-3.6 \( \sigma \) discrepancy
There is no shortage of interest in this intriguing result!

Were it to persist...

• Strong indicator of BSM physics...
  Loop contributions sensitive to new particles running round loop...
  \[
  \mu \text{ better than } e \quad \left( \frac{m_\mu}{m_e} \right)^2 \approx 40,000
  \]
e.g. SUSY

\[ a_\mu(SUSY) \simeq \text{sgn}(\mu) \times 130 \times 10^{-11} \tan \beta \left( \frac{100 \text{ GeV}}{\Lambda} \right)^2 \]

But broad spectrum of sensitivity in TeV mass range...

\[ a_\mu \text{ related to } m_\mu \]

Generically:

\[ a_\mu^{NP} = O(1) \times \left( \frac{m_\mu}{M_{\text{NEW}}} \right)^2 \times \left( \frac{\delta m_\mu^{NP}}{m_\mu} \right)^2 \]

- flavour-conserving, CP-conserving, chirality flipping, loop-induced

Highly model dependent

Extended technicolor (fermion masses)

SUSY (natural, gauge-mediated, compressed), RS ED

Z’, W’, Little Higgs, Universal ED

Value consistent with SM
$a_\mu$ provides discriminating power...

SPS benchmark points

LHC Inverse Problem (300 fb$^{-1}$) can't be distinguished at LHC
[Splitter: Adam, Kneur, Lafayette, Plehn, Rauch, Zerwas '10]

[Hertzog, Miller, de Rafael, Roberts, DS '07]
...also can inform, low mass, below LHC reach...

e.g. Dark photons:

Dark photon contribution of $280 \pm 80 \times 10^{-11}$ to $a_{\mu}$
How do we measure $g-2$?
First make your muons… …from pions.

Fortune of nature number 1

Parity violation delivers conveniently polarised muons:

⇒ beam of polarised muons
inject into a (very) uniform magnetic field…

Muon momentum turns with cyclotron frequency

$$\omega_c = -\frac{QeB}{m\gamma}$$

Spin turns with frequency

$$\omega_s = -g \frac{QeB}{2m} - \left(1 - \gamma\right) \frac{QeB}{\gamma m}$$

Fortune of nature number 2:

$$\omega_a \equiv \omega_s - \omega_c = -\left(\frac{g - 2}{2}\right) \frac{QeB}{m_\mu} = -a_\mu \frac{QeB}{m_\mu}$$

Direct dependence on the anomaly: an immediate 3 orders of magnitude gain over measuring $\mu$ in at-rest experiments!

We need to measure $\omega_a$ and $B$

…and know $m_\mu$ very accurately?
Actually measure:

\[ a_\mu = \frac{\omega_p - \mu_\mu/\mu_p \omega_a}{\omega_p} \]

Normalise magnetic field to Larmor frequency of proton

\[ \lambda \equiv \frac{\mu_\mu}{\mu_p} \]

Measured from hyperfine structure of muonium: currently known to 120ppb†

†JPARC expt. to reduce this to ppb level

“Never measure anything but frequency”

I. Rabi
E821 Experimental Technique

- Muon polarization
- Muon storage ring
- Injection & kicking
- Focus with Electric Quadrupoles
- 24 electron calorimeters

\[ \vec{\omega} = - \frac{e}{m} \alpha_\mu \vec{B} \]

- 25ns bunch of 5 \( \times 10^{12} \) protons from AGS
- \( \pi^- \) Pions
- \( p = 3.1 \text{GeV/c} \)
- Inflector
- \( x_c \approx 77 \text{ mm} \)
- \( \theta \approx 10 \text{ mrad} \)
- \( B \cdot dl \approx 0.1 \text{ Tm} \)
Magic $\gamma$

- Vertical magnetic field – need vertical focussing to stop muons spiralling out of ring
- Achieve using electrostatic dipoles
- The E-field modifies the precession frequency:

$$\vec{\omega}_a = \frac{e}{mc} \left[ a\vec{B} - \left( a - \frac{1}{\gamma^2 - 1} \right) \vec{\beta} \times \vec{E} \right]$$

- Unwelcome source of additional systematics
- Can be made to vanish for ‘magic’ $\gamma$. Extremely lucky that size of $a_\mu$ makes this possible!

$$\gamma_{\text{magic}} = 29.3 \quad \Rightarrow \quad p_\mu = 3.09 \text{ GeV}$$

Method pioneered by 3rd CERN g-2 …but sadly, not every $\mu$ will be magic!
But how to measure $\omega_a$?

Parity violation again!

- Highest energy $e^+$ emitted along direction of $\mu^+$ spin
- Use calorimeters to count $e^+$ above an energy threshold vs. $t$
...an iconic plot

E 821

\[ N(t) = N_0 e^{-t/\gamma\mu} \left[ 1 - A \cos(\omega_a t + \phi) \right] \]

“5-parameter fit”
Measuring $\omega_a$ ...some reality

High frequency modulation because muon bunch initially doesn’t fill ring...decays as bunch spreads. This is good – can get $p$ distribution of muons

Expected for E989: $\omega_c 149$ns

Bunch length 120ns at injection

$$N(t) = N_0 e^{-t/\gamma\tau_\mu} \left[ 1 - A \cos(\omega_a t + \phi) \right]$$

$N, A$ depend on energy
Many sources of systematic error.

Particularly insidious are ‘early-to-late’ errors

Example: Effect of pile up.

\[ (\omega_a t + \phi) \]

Time dependence in phase:

\[ \phi(t) = \phi_0 + \alpha t + \beta t^2 + \cdots \equiv \phi_0 + \alpha t \]

\[ \cos(\omega_a t + \phi(t)) \approx \cos((\omega_a + \alpha)t + \phi_0) \]

…but why should \( \phi \) change? Things which change early to late in the fill can lead to a phase change in the accepted events \( \rightarrow \) direct bias to extracted \( \omega_a \).
Higher energy positrons come from further away.

If we get the energy wrong, we get the phase wrong.
If we have pile-up, two low energy positrons fake a high energy positron.
More pile-up early in the fill.

\[
N(t) = \frac{N_0}{\tau} \Lambda(t) V(t) B(t) C(t) [1 - A'(t) \cos(\omega_d t + \phi'(t))], \text{ where}
\]
\[
B(t) = 1 - A_{br} e^{-t/\tau_{br}} \quad \text{with} \quad \tau_{br} = 5 \mu s.
\]
\[
V(t) = (1 - e^{-t/\tau_{yw}} [1 - A_{yw} \cos(\omega_{yw} t + \phi_{yw})]),
\]
\[
A'(t) = A_1 - e^{-t/\tau_{cbo}} \left[ 1 - A_2 \cos(\omega_{cbo} t + \phi_2) \right], \quad \text{and}
\]
\[
\phi'(t) = \phi_1 - e^{-t/\tau_{cbo}} \left[ 1 - A_3 \cos(\omega_{cbo} t + \phi_3) \right].
\]
\[
C(t) = 1 - e^{-t/\tau_{cbo}} \left[ 1 - A_1 \cos(\omega_{cbo} t + \phi_1) \right].
\]
\[
\Lambda(t) = 1 - Ce^{-t_0/\tau} \int_{t_0}^{t} L(t') e^{t'/\tau} dt', \quad \text{Muons lost from ring}
\]

Beam relaxation
Vertical breathing
3 CBO terms

Stephen Maxfield  Seminar  Birmingham Oct2013
…strong incentive to repeat the measurement with increased precision:

**E989 Muon $g-2$ at FNAL** aims to:

Build on BNL result and:

- **increase number of muons by factor ~21**
- **reduce total systematics by factor ~3**

\[
10^{11} \times a^E_{\mu} = 116592089 (54)_{\text{stat}} (33)_{\text{sys}}
\]

\[
(54)_{\text{stat}} \oplus (33)_{\text{sys}} \rightarrow (11)_{\text{stat}} \oplus (11)_{\text{sys}}
\]

0.54ppm $\rightarrow$ 0.14ppm

\[
\delta a_{\mu} \leq \pm 16 \times 10^{-11}
\]

i.e. $\sim 3.5\sigma \rightarrow \geq 5\sigma$
How these goals will be achieved:

Move the entire E821 storage ring to FNAL!

Use the same experimental technique as E821 but:

- exploit the unique FNAL facilities to deliver more muons

Reduce systematics by improving and refining

- the detectors
- the stored beam dynamics
- Uniformity and measurement of magnetic field
Fermilab Muon Campus

Multipurpose Building designed for future experiments as well
At FNAL...

Get more and cleaner beam:

~21 times statistics of E821 and

Use beam transfer and delivery as 1900m decay line ⇒ no pion background, **no hadronic flash**

Goal is

1.8 × 10^{11} Detected decays

Systematic errors of better than:

±0.07ppm on \( \omega_a \)

±0.07ppm on \( \omega_p \)
Factor ~3 reduction in systematics built from large number of individual improvements:

<table>
<thead>
<tr>
<th>Category</th>
<th>E821 [ppm]</th>
<th>E989 Improvement Plans</th>
<th>Goal [ppm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain changes</td>
<td>0.12</td>
<td>Better laser calibration</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>low-energy threshold</td>
<td></td>
</tr>
<tr>
<td>Pileup</td>
<td>0.08</td>
<td>Low-energy samples recorded</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td></td>
<td>calorimeter segmentation</td>
<td></td>
</tr>
<tr>
<td>Lost muons</td>
<td>0.09</td>
<td>Better collimation in ring</td>
<td>0.02</td>
</tr>
<tr>
<td>CBO</td>
<td>0.07</td>
<td>Higher $n$ value (frequency)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Better match of beamline to ring</td>
<td>&lt; 0.03</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Improved tracker</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Precise storage ring simulations</td>
<td>0.03</td>
</tr>
<tr>
<td>Total</td>
<td>0.18</td>
<td>Quadrature sum</td>
<td>0.07</td>
</tr>
</tbody>
</table>
Beam injection, beam dynamics

INFLECTOR
Improved acceptance, improved matching between delivery and ring leading to more muons and reduced beam oscillations → possible factor ~4 in storage efficiency.

Possible redesign of inflector

Replace with open ended design and larger aperture. Shielding challenging.
Beam Injection and Ring

Numerous Improvements in collimation, beam tune etc. New inflector, New kickers.

→ Reduction in muon loss
→ Better control of coherent betatron oscillations and their impact on $\omega_a$.

Vertical and horizontal oscillations

Avoid this E821 feature!

Muons not all magic, not all perpendicular to B,E

Pitch corrections needed.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Expression</th>
<th>Frequency</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_a$</td>
<td>$\frac{e}{2\pi mc}a_{\mu}B$</td>
<td>0.23 MHz</td>
<td>4.37 $\mu$s</td>
</tr>
<tr>
<td>$f_c$</td>
<td>$\frac{v}{2\pi R_0}$</td>
<td>6.7 MHz</td>
<td>149 ns</td>
</tr>
<tr>
<td>$f_x$</td>
<td>$\sqrt{1-nf_c}$</td>
<td>6.23 MHz</td>
<td>160 ns</td>
</tr>
<tr>
<td>$f_y$</td>
<td>$\sqrt{n_f_c}$</td>
<td>2.48 MHz</td>
<td>402 ns</td>
</tr>
<tr>
<td>$f_{CBO}$</td>
<td>$f_c - f_x$</td>
<td>0.477 MHz</td>
<td>2.10 $\mu$s</td>
</tr>
<tr>
<td>$f_{VW}$</td>
<td>$f_c - 2f_y$</td>
<td>1.74 MHz</td>
<td>0.574 $\mu$s</td>
</tr>
</tbody>
</table>
Calorimeters

New for E989:

- Segmented: 6x9 PbF$_2$ crystals with SiPM readout. **Attack pileup systematic.**

- Pileup: muon phase correlated with e$^+$ energy
  - overlapping pulses → wrong phase shifts
  - varying fraction of pileup within fill produces early → late shift in average phase → direct impact on $\omega_a$.

- one pulse should not affect gain of subsequent pulse on same channel. Should be able to separate at 5ns level

- Fills ~ 700$\mu$s long
- Gain variations and time shifts over this period feed into systematics. $\Delta G(t) < 0.1\%$

0.05 ppm systematic budget for $\omega_a$
Calorimeters II
Continuous distribution of muons with random decay probability

- Determine arrival time
- Determine energy
- Pileup separation $\Rightarrow$ Wave Form Digitisation
- Laser calibration system

### Improvements
- Gain changes $0.12 \rightarrow 0.02$ppm
- Pileup: $0.08 \rightarrow 0.04$ppm
Tracking detectors

• Measure beam profile at several locations as function of time in fill.

  • Convolute $\mu$ spatial distribution with field to determine effective field seen by $\mu$’s  \[0.03 \rightarrow 0.01\text{ppm}\]

  • Momentum spread and betatron oscillations lead to ppm corrections to $\omega_\alpha$ from non-magic muons - E-field and pitch corrections…
    Beam dynamics corrections  \[0.05 \rightarrow 0.03\text{ppm}\]

• Pileup identification.  \[0.08 \rightarrow 0.04\text{ppm}\]

• Independent momentum measurement. Verify calorimeter gain

• Correct for acceptance changes in calorimeters from betatron oscillations. Validate calorimeter based determinations of pileup corrections, gain, muon loss.  \[0.12 \rightarrow 0.02\text{ppm}\]

• **EDM** measure positron vertical angle – asymmetry.
  Design will allow a factor $\sim 200$/month increase in statistics over Brookhaven measurement $\Rightarrow$ factor of 10 on EDM very quickly and $\sim 100$ eventually.
New Tracking detectors

- Need: long lever arm \( \sim 1\text{mm} \) determination of muon decay up to 10m away
- Continuously distributed decay points, muon momenta \( \Rightarrow \) distributed detector

- Straw chambers
- In vacuum
- 2-3 locations round ring
Straws do the trick:

- Low mass \( \sim 0.1X_0 \) per station, non-magnetic, OK in vacuum
- 5mm diameter, 12cm long straws. Mylar coated with Al+Au.
- 25\( \mu \) gold-plated tungsten wires
- Based on Mu2e straws.
- 80:20 Argon:CO\(_2\) gas
- \( \pm 7.5^\circ \) UV layers to give vertical resolution
What about $\omega_p$?

Goal is to get this to 0.07ppm accuracy…

A lot of shimming

A lot of measuring and monitoring
Field measured with set of NMR probes

- fixed probes
- plunging probes
- trolley probes

Again, requires concerted attack to beat down systematics.

Improvements from reduced position uncertainties, more frequent measurements, better electronics etc.

### Source of errors

<table>
<thead>
<tr>
<th>Source of errors</th>
<th>R01 [ppm]</th>
<th>E989 [ppm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolute calibration of standard probe</td>
<td>0.05</td>
<td>0.035</td>
</tr>
<tr>
<td>Calibration of trolley probes</td>
<td>0.09</td>
<td>0.03</td>
</tr>
<tr>
<td>Trolley measurements of $B_0$</td>
<td>0.05</td>
<td>0.03</td>
</tr>
<tr>
<td>Interpolation with fixed probes</td>
<td>0.07</td>
<td>0.03</td>
</tr>
<tr>
<td>Uncertainty from muon distribution</td>
<td>0.03</td>
<td>0.01</td>
</tr>
<tr>
<td>Inflector fringe field uncertainty</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Time dependent external $B$ fields</td>
<td>–</td>
<td>0.005</td>
</tr>
<tr>
<td>Others †</td>
<td>0.10</td>
<td>0.03</td>
</tr>
<tr>
<td>Total systematic error on $\omega_p$</td>
<td>0.17</td>
<td>0.070</td>
</tr>
</tbody>
</table>

e.g. **Absolute Calibration**

Dedicated 1.45T calibration magnet, more probes.

Possibility of using $^3$He as well as water-based NMR probes.
But first… Vital bits of E821 have to get from BNL to FNAL

The move…
Disassembly September 2012
May 2013 : Yoke pieces arriving at FNAL
Building support structure…
"Yeah, We can move that"
Big barge to limit pitch, roll and heave…
5 nights sheltering from Storm near Norfolk Virginia.


25th June - July 20th
Miss Katie
Lemont, IL. …safely ashore
It fits…
‘Overdaying’ at Costco’s supermarket!
…after 3200 mile journey.
CONCLUSIONS/Timeline

- Important bits of E821 now at Fermilab
  - coils, ~2/3 steel
- Building under construction
  - expect beneficial occupancy February 2014
- Arduous series of CD-1 reviews this year nearly over
  - (then CD-2,… next year!)
- Many upgrades well-underway
- On-schedule for:
  - Magnet powered 2015
  - Beam in 2016 or 2017
Thank You