Neutrinos and the Universe

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Neutrinos & Baryon Asymmetry in Universe

Neutrinos & the Dark Sector of Universe:

CPT Violation in the Early Universe & Neutrinos

Conclusions - Outlook
Neutrinos & Baryon Asymmetry in Universe

- Neutrino mass types
- Role of (heavy) Majorana
- Right-handed Neutrinos
- In Leptogenesis/Baryogenesis

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Neutrinos & the Dark Sector of Universe:

Sterile Neutrinos as Dark matter
Beyond see-saw for generating
Right-handed Majorana masses –
anomalous generation of neutrino mass
Neutrino Condensates & Dark Energy

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CPT Violation in the Early Universe & Neutrinos

CPT Violation in Early Universe Geometries & particle/antiparticle asymmetries already in thermal equilibrium

Conclusions - Outlook
Generic Concepts

- **Leptogenesis**: physical *out of thermal equilibrium* processes in the (expanding) Early Universe that produce an asymmetry between leptons & antileptons

- **Baryogenesis**: The corresponding processes that produce an asymmetry between baryons and antibaryons

- **Ultimate question**: why is the Universe made only of matter?
• **Leptogenesis**: physical *out of thermal equilibrium* processes in the (expanding) Early Universe that produce an asymmetry between leptons & antileptons

• **Baryogenesis**: The corresponding processes that produce an asymmetry between baryons and antibaryons

• **Ultimate question**: why is the Universe made only of matter?
NEUTRINOS & LEPTOGENESIS

• Matter-Antimatter asymmetry in the Universe ➞ Violation of Baryon # (B), C & CP

• Tiny CP violation (O(10^{-3})) in Labs: e.g. $K^0 \bar{K}^0$

• But Universe consists only of matter

$$\frac{n_B - \bar{n}_B}{n_B + \bar{n}_B} \sim \frac{n_B - \bar{n}_B}{s} = (8.4 - 8.9) \times 10^{-11} \quad T > 1 \text{ GeV}$$

Sakharov : Non-equilibrium physics of early Universe, B, C, CP violation

but not quantitatively in SM, still a mystery
NEUTRINOS & LEPTOGENESIS

• Matter-Antimatter asymmetry in the Universe \[ \rightarrow \] Violation of Baryon # (B), C & CP

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\]

ELECTROWEAK THEORY & FERMION # NON-CONSERVATION

Classical conservations of EW theory: $B$, $L_e$, $L_\mu$, $L_\tau$

Quantum Anomalies in Standard Model (SM):

$$\partial_\mu J^B_\mu = \partial_\mu J^L_\mu = \frac{nf}{32\pi^2} \text{Tr} F_{\mu\nu} \tilde{F}_{\mu\nu} + U(1) \text{ part}$$

Allowed Processes (change of $B$ by multiples of 3)

$$\text{bosons} \leftrightarrow \text{bosons} + 9q + 3l \quad \Rightarrow \quad L_i - B/3 \text{ Conserved}$$

(three quantities)

BUT:

OBSERVED NEUTRINO FLAVOUR OSCILLATIONS

If neutrinos Majorana

L-B conserved (one quantity)

L=total Lepton #

L violated, No conserved numbers
OBSERVED CP VIOLATION UNLIKELY TO EXPLAIN BARYON ASYMMETRY IN THE UNIVERSE

In SM: \[ \text{bosons} \leftrightarrow \text{bosons} + 9q + 3l \]

Rate of B violation in Early Universe

\[ \Gamma \sim \begin{cases} 
(\alpha_W T)^4 \left( \frac{M_{\text{sph}}}{T} \right)^7 \exp \left( -\frac{M_{\text{sph}}}{T} \right), & T \lesssim M_{\text{sph}}, \\
\alpha_W (\alpha_W T)^4 \log(1/\alpha_W), & T \gtrsim M_{\text{sph}}, 
\end{cases} \]

\( \alpha_W = \text{SU}(2) \) fine structure \``constant'' \`

Sphaleron Mass Scale \( (M_W/\alpha_W) = \text{height of energy Barrier separating SU}(2) \text{ vacua with different topologies} \)
OBSERVED CP VIOLATION UNLIKELY TO EXPLAIN BARYON ASYMMETRY IN THE UNIVERSE

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\]

Thermal Equilibrium (i.e. \(\Gamma > H\) (Hubble)) for B non conserv. occurs only for:

\[
T_{\text{sph}}(m_H) < T < (\alpha_W)^5 M_{\text{Pl}} \sim 10^{12} \text{ GeV}
\]

\[
T_{\text{sph}}(m_H) \in [130, 190] \text{GeV}
\]

\[
m_H \in [100, 300] \text{GeV}
\]

Kuzmin, Rubakov, Shaposhnikov
OBSERVED CP VIOLATION UNLIKELY TO EXPLAIN BARYON ASYMMETRY IN THE UNIVERSE

Kuzmin, Rubakov, Shaposhnikov

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BAU could be produced this way only when sphaleron interactions freeze out, i.e.

\[ T \sim T_{\text{sph}} \]
OBSERVED CP VIOLATION UNLIKELY TO EXPLAIN BARYON ASYMMETRY IN THE UNIVERSE

Kuzmin, Rubakov, Shaposhnikov

Rate of B violation in Early Universe

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\end{cases} \]

BAU COULD BE PRODUCED @

\[ T \simeq T_{\text{sph}} \]

\[ T_{\text{sph}}(m_H) \in [130, 190] \text{GeV} \]

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Compute CP Violation Effects
OBSERVED CP VIOLATION UNLIKELY TO EXPLAIN BARYON ASYMMETRY IN THE UNIVERSE

Rate of B violation in Early Universe

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Compute CP Violation Effects

Use CKM Matrix for \(T > T_{\text{sph}}\)
Within the Standard Model, lowest CP Violating structures

\[ d_{CP} = \sin(\theta_{12})\sin(\theta_{23})\sin(\theta_{13})\sin\delta_{CP} \cdot (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)(m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2) \]

Cabbibo-Kobayashi-Maskawa CP Violating phase

Shaposhnikov

\[ D = \text{Im} \ Tr \left[ M_u^2 M_d^2 M_u M_d \right] \]

\[ \delta_{KM} \sim \frac{D}{T_t^{12}} \sim 10^{-20} \]

\[ \langle \langle \frac{n_B - \bar{n}_B}{n_B + \bar{n}_B} \rangle \rangle \sim \frac{n_B - \bar{n}_B}{s} = (8.4 - 8.9) \times 10^{-11} \]

\[ T \sim T_{sph} \]

\[ T_{sph}(m_H) \in [130, 190] \text{GeV} \]
Within the Standard Model, lowest CP Violating structures

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\[ \cdot (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)(m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2) \]

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\[ T_{sph}(m_H) \in [130, 190] \text{GeV} \]

This CP Violation Cannot be the Source of Baryon Asymmetry in The Universe
Role of Neutrinos?

Several Ideas to go beyond the SM (e.g. GUT models, Supersymmetry, extra dimensional models etc.)
Role of Neutrinos?

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• Massive $\nu$ are simplest extension of SM
Role of Neutrinos?

• Several Ideas to go beyond the SM (e.g. GUT models, Supersymmetry, extra dimensional models etc.)

• Massive $\nu$ are simplest extension of SM

• Right-handed massive $\nu$ may provide extensions of SM with:
  - extra CP Violation and thus Origin of Universe’s matter-antimatter asymmetry due to neutrino masses, Dark Matter
THE MOST GENERAL, LORENTZ-INVARIANT NEUTRINO MASS TERM

\[
\mathcal{L}_{D+M}^{D+M} = -\frac{1}{2} \bar{\nu}_L M_L^M (\nu_L)^c - \bar{\nu}_L M^D \nu_R - \frac{1}{2} (\nu_R)^c M_R^M \nu_R + \text{h.c.}
\]

\[
\nu_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} \quad \quad \nu_R = \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix}
\]
BASICS: TYPES OF NEUTRINO MASS TERMS

THE MOST GENERAL, LORENTZ-INVARIANT NEUTRINO MASS TERM

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\[ \nu_{iL}(x) = \sum_{i=1}^{3} U_{li} \nu_{iL}(x) \quad (l = e, \mu, \tau) \]
BASICS: TYPES OF NEUTRINO MASS TERMS

THE MOST GENERAL, LORENTZ-INVARIANT NEUTRINO MASS TERM

\[ \mathcal{L}^{D+M} = -\frac{1}{2} \bar{\nu}_L M^M_L (\nu_L)^c - \bar{\nu}_L M^D \nu_R - \frac{1}{2} (\nu_R)^c M^M_R \nu_R + \text{h.c.} \]

DIRAC

CONSERVE TOTAL LEPTON \((L)\) NUMBER

\[ \nu_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}, \quad \nu_R = \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix} \]

MIXING

\[ \nu_{iL}(x) = \sum_{i=1}^3 U_{li} \nu_{iL}(x) \quad (l = e, \mu, \tau) \]
THE MOST GENERAL, LORENTZ-ININVARIANT NEUTRINO MASS TERM

\[ \mathcal{L}^{D+M} = -\frac{1}{2} \bar{\nu}_L M^M (\nu_L)^c - \bar{\nu}_L M^D \nu_R - \frac{1}{2} (\nu_R)^c M^M_R \nu_R + \text{h.c.} \]

\[ (\nu_{IL})^c = C \bar{\nu}_{IL}^T \]

\[ (\nu_{IR})^c = C \bar{\nu}_{IR}^T \]

C = Charge Conjugation

VIOLATE
LEPTON
(L) NUMBER

MAJORANA
LEFT-HANDED

MAJORANA
RIGHT-HANDED

VIOLATE
LEPTON
(L) NUMBER

\[ \nu_L = \left( \begin{array}{c} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{array} \right) \]

\[ \nu_R = \left( \begin{array}{c} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{array} \right) \]
BASICS: TYPES OF NEUTRINO MASS TERMS

THE MOST GENERAL, LORENTZ-INVARIANT NEUTRINO MASS TERM

\[ \mathcal{L}^{D+M} = -\frac{1}{2} \bar{\nu}_L M_{L}^{M} (\nu_{L})^{c} - \bar{\nu}_L M_{D}^{L} \nu_{R} - \frac{1}{2} (\nu_{R})^{c} M_{R}^{M} \nu_{R} + \text{h.c.} \]

\[ (\nu_{IL})^{c} = C \bar{\nu}_{IL}^{T} \]

\[ (\nu_{IR})^{c} = C \bar{\nu}_{IR}^{T} \]

C = Charge Conjugation

VIOLATE LEPTON (L) NUMBER

MAJORANA LEFT-HANDED

MAJORANA RIGHT-HANDED

VIOLATE LEPTON (L) NUMBER

MAJORANA FIELDS ARE MASS EIGENSTATES

PARTICLE=ANTIPARTICLE

\[ (\nu^{M} (x))^{c} = \nu^{M} (x) \]
THE MOST GENERAL, LORENTZ-ININVARIANT NEUTRINO MASS TERM

\[ \mathcal{L}^{D+M} = -\frac{1}{2} \bar{\nu}_L M^M (\nu_L)^c - \bar{\nu}_L M^D \nu_R - \frac{1}{2} (\nu_R)^c M^M_R \nu_R + \text{h.c.} \]

\[ (\nu_{LR})^c = C \bar{\nu}_{iR}^T \]

\[ C = \text{Charge Conjugation} \]

DIRAC

MAJORANA RIGHT-HANDED

CONSERVE TOTAL LEPTON (L) NUMBER

VIOLATE LEPTON (L) NUMBER

FOR SEESAW: NO LEFT-HANDED MAJORANA
SM Extension with N extra right-handed neutrinos

\[ L = L_{SM} + \bar{N}_I i \partial_\mu \gamma^\mu N_I - F_{\alpha I} \bar{L}_\alpha N_I \tilde{\phi} - \frac{M_I}{2} \bar{N}_I^c N_I + \text{h.c.} \]
SM Extension with N extra right-handed neutrinos

\[ L = L_{SM} + \bar{N}_I i \partial_\mu \gamma^\mu N_I - F_{\alpha I} \bar{L}_\alpha N_I \tilde{\phi} - \frac{M_I}{2} \bar{N}_I^c N_I + \text{h.c.} \]
SM Extension with N extra right-handed neutrinos

\[ L = L_{SM} + \bar{N}_I i \partial_\mu \gamma^\mu N_I - F_{\alpha I} \bar{L}_\alpha N_I \phi - \frac{M_I}{2} \bar{N}_I^c N_I + \text{h.c.} \]

Higgs scalar SU(2)
Dual: \[ \tilde{\phi}_i = \epsilon_{ij} \phi_j^*. \]
SM Extension with N extra right-handed neutrinos

\[ \nu_{MSM} \]

\[ L = L_{SM} + \bar{N}_I i \partial_\mu \gamma^\mu N_I - F_{\alpha I} \bar{L}_\alpha N_I \phi - \frac{M_I}{2} \bar{N}_I^c N_I + \text{h.c.} \]

Yukawa couplings
Matrix (N=2 or 3)

\[ F = \tilde{K}_L f_d \tilde{K}_R^\dagger \]
SM Extension with $N$ extra right-handed neutrinos

$\nu_{\text{MSM}}$

\[
L = L_{\text{SM}} + \bar{N}_I i \partial_\mu \gamma^\mu N_I - F_{\alpha I} \bar{L}_\alpha N_I \bar{\phi} - \frac{M_I}{2} \bar{N}_I^c N_I + \text{h.c.}
\]

Majorana masses to (2 or 3) active neutrinos via seesaw

Yukawa couplings

Matrix ($N=2$ or $3$)

\[
F = \tilde{K}_L f_d \tilde{K}_R^\dagger
\]
SM Extension with N extra right-handed neutrinos

$$\nu_{\text{MSM}}$$

$$L = L_{\text{SM}} + \bar{N}_I i \gamma^\mu \partial_\mu N_I - F_{\alpha I} \bar{L}_\alpha N_I \tilde{\phi} - \frac{M_I}{2} \bar{N}_I^c N_I + \text{h.c.}$$

**Majorana masses to (2 or 3) active neutrinos via seesaw**

**Yukawa couplings Matrix (N=2 or 3 )**

$$F = \tilde{K}_L f_d \tilde{K}_R^\dagger$$

**NB:** Upon Symmetry Breaking

$$\langle \Phi \rangle = \nu \neq 0 \rightarrow \text{Dirac mass term}$$
\[ L = L_{SM} + \bar{N}_I i \partial_\mu \gamma^\mu N_I - F_{\alpha I} \bar{L}_\alpha N_I \phi - \frac{M_I}{2} \bar{N}_I^c N_I + \text{h.c.} \]

Light Neutrino Masses through see saw

\[ m_\nu = -M^D \frac{1}{M_I} [M^D]^T \nu \]

\[ M_D = F_{\alpha I} \nu \]

\[ \nu = \langle \phi \rangle \sim 175 \text{ GeV} \]

\[ M_D \ll M_I \]

Minkowski, Yanagida, Mohapatra, Senjanovic, Sechter, Valle ...
SM Extension with N extra right-handed neutrinos

\[ \nu_{MSM} \]

\[
L = L_{SM} + \bar{N}_I i \partial_\mu \gamma^\mu N_I - F_{\alpha I} \bar{L}_\alpha N_I \phi - \frac{M_I}{2} \bar{N}_I^c N_I + \text{h.c.}
\]

From Constraints (compiled \(\nu\) oscillation data) on (light) sterile neutrinos: Giunti, Hernandez ...

N=1 excluded by data

Yukawa couplings Matrix (N=2 or 3)

\[ F = \bar{\nu}_L f_d \nu_R^\dagger \]
SM Extension with N extra right-handed neutrinos

$$\nu_{\text{MSM}}$$

$$\begin{align*}
L &= L_{\text{SM}} + \bar{N}_I i \partial_\mu \gamma^\mu N_I - F_{\alpha I} \bar{L}_\alpha N_I \tilde{\phi} - \frac{M_I}{2} \bar{N}_I^c N_I + \text{h.c.}
\end{align*}$$

Majorana masses to (2 or 3) active neutrinos via seesaw

Yukawa couplings
Matrix (N=2 or 3)

Model with 2 or 3 singlet fermions works well in reproducing Baryon Asymmetry and is consistent with Experimental Data on neutrino oscillations

Model with N=3 also works fine, and in fact it allows one of the Majorana fermions to almost decouple from the rest of the SM fields, thus providing candidates for light (kEV region of mass) sterile neutrino Dark Matter.
SM Extension with N extra right-handed neutrinos

$$\nu_{MSM}$$

Boyarski, Ruchayskiy, Shaposhnikov

$$L = L_{SM} + \bar{N}_I i \partial_\mu \gamma^\mu N_I - F_{\alpha I} \bar{L}_\alpha N_I \phi - \frac{M_I}{2} \bar{N}_I^c N_I + \text{h.c.}$$

Yukawa couplings
Matrix (N=3)

$$F = \bar{K}_L f_d \bar{K}_R$$
SM Extension with N extra right-handed neutrinos

\[ \nu_{\text{MSM}} \]

Yukawa couplings

Matrix (N=3)

Mixing

Majorana phases

\[ L = L_{\text{SM}} + \bar{N}_I i \partial_\mu \gamma^\mu N_I - F_{\alpha I} \bar{L}_\alpha N_I \phi - \frac{M_I}{2} \bar{N}_I^c N_I + \text{h.c.} \]

\[ f_d = \text{diag}(f_1, f_2, f_3), \quad \tilde{K}_L = K_L P_\alpha, \quad \tilde{K}_R^\dagger = K_R^\dagger P_\beta \]

\[ P_\alpha = \text{diag}(e^{i\alpha_1}, e^{i\alpha_2}, 1), \quad P_\beta = \text{diag}(e^{i\beta_1}, e^{i\beta_2}, 1) \]

\[ K_L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{L23} & s_{L23} \\ 0 & -s_{L23} & c_{L23} \end{pmatrix} \begin{pmatrix} c_{L13} & 0 & s_{L13} e^{-i\delta_L} \\ 0 & 1 & 0 \\ -s_{L13} e^{i\delta_L} & 0 & c_{L13} \end{pmatrix} \begin{pmatrix} c_{L12} & s_{L12} & 0 \\ -s_{L12} & c_{L12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

\[ c_{Lij} = \cos(\theta_{Lij}) \text{ and } s_{Lij} = \sin(\theta_{Lij}). \]
Thermal Properties

\[ L = L_{SM} + \bar{N}_I i \partial_\mu \gamma^\mu N_I - F_{\alpha I} \bar{L}_\alpha N_I \phi - \frac{M_I}{2} \bar{N}_I^c N_I + h.c. \]

\[ |F|^2 \approx \frac{m_{atm} M_I}{v^2} \sim 2 \times 10^{-15} \frac{M_I}{\text{GeV}} \]

\[ |\Delta m_{atm}^2| \equiv m_{atm}^2 = 2.40^{+0.12}_{-0.11} \times 10^{-3} \text{eV}^2 \]

(Decay) processes in Early Universe

\[ N t \leftrightarrow \nu t, \quad H \leftrightarrow N \nu \text{ or } N \leftrightarrow H \nu \]

Rate: \[ \frac{9 F^2 f_t^2 T}{(64 \pi^3)} \]

\[ f_t = \text{top quark Yukawa coupling} \]

Thermal equilibrium at temperatures

\[ M_0 = M_P/(1.66 \sqrt{g_{eff}}) \]

\[ \text{time} = M_0^2/2T^2 \text{ (radiation era)} \]

\[ T_{eq} \sim \frac{9 f_t^2 m_{atm} M_0}{64 \pi^3 v^2} M_I \sim 5M_I \]

(for \( T_{eq} > 100 \text{ GeV} \))
Thermal Properties

Two distinct physics cases

$(M_W = \text{electroweak scale} = O(100) \text{ GeV})$: 

(i) $M_I > M_W$

(ii) $M_I < M_W$
Thermal Properties

Two distinct physics cases: \( M_I > M_W \) & \( M_I < M_W \)

(i) \( M_I > M_W \) (electroweak scale)

Decay of Right-handed fermions

\[
T_{\text{decay}} \approx \left( \frac{m_{\text{atm}} M_0}{24 \pi v^2} \right)^{\frac{1}{3}} \quad M_I \approx 3 M_I
\]

Out of equilibrium for:

\[ T > T_{\text{eq}} \quad \text{or for} \quad T < T_{\text{decay}} \]

If \( T_{\text{eq}} > T_{\text{sph}} \), Decays of Right-handed Majorana fermions occur for period of active Sphaleron processes

\[
T_{\text{decay}} > T_{\text{sph}}
\]

Thermal Leptogenesis

Fukugita, Yanagida,
Thermal Leptogenesis

Heavy Right-handed Majorana neutrinos enter equilibrium at $T = T_{eq}$.
Thermal Leptogenesis

Heavy Right-handed Majorana neutrinos enter equilibrium at $T = T_{eq}$

Independent of Initial Conditions @ $T >> T_{eq}$
Heavy Right-handed Majorana neutrinos enter equilibrium at $T = T_{eq} > T_{\text{decay}}$.

Enhanced CP Violation

$N_1 \rightarrow H\nu, \bar{H}\bar{\nu}$

Out of Equilibrium Decays

$T \simeq T_{\text{decay}} > T_{sph}$

Produce Lepton asymmetry

Fukugita, Yanagida,

Kuzmin, Rubakov, Shaposhnikov
Independent of Initial Conditions @ T >> T_{eq}

Heavy Right-handed Majorana neutrinos enter equilibrium at T = T_{eq} > T_{decay}

Enhanced CP V. Lepton number Violation

\[ N_I \rightarrow H\nu, \quad \bar{H}\bar{\nu} \]

Out of Equilibrium Decays

\[ T \sim T_{\text{decay}} > T_{\text{sph}} \]

Produce Lepton asymmetry

Equilibrated electroweak B, L violating sphaleron interactions (B-L conserv)

Fukugita, Yanagida,

Kuzmin, Rubakov, Shaposhinkov
**Thermal Leptogenesis**

Heavy Right-handed Majorana neutrinos enter equilibrium at $T = T_{eq} > T_{\text{decay}}$

**Enhanced CP Violation**

Lepton number violation

$N_i \rightarrow H\nu, \bar{H}\bar{\nu}$

**Out of Equilibrium Decays**

$T \simeq T_{\text{decay}} > T_{\text{sph}}$

Produce Lepton asymmetry

Equilibrated electroweak $B, L$ violating sphaleron interactions ($B-L$ conserv)

Observed Baryon Asymmetry in the Universe (BAU)

$$L = \frac{2}{M} l_L l_L \phi\phi + \text{H.c.}$$

where

$$l_L = \begin{pmatrix} \nu_e \\ e_l' \\ \nu_\mu \\ \mu_l' \\ \nu_\tau \\ \tau_l' \end{pmatrix}$$

Independent of Initial Conditions @ $T >> T_{eq}$
**Thermal Leptogenesis**

Heavy Right-handed Majorana neutrinos enter equilibrium at $T = T_{eq} > T_{\text{decay}}$

**enhanced CP V.** Lepton number violation

$N_I \rightarrow H\nu, \bar{H}\bar{\nu}$

Out of Equilibrium Decays

$T \sim T_{\text{decay}} > T_{\text{sph}}$

Produce Lepton asymmetry

Equilibrated electroweak B, L violating sphaleron interactions (B-L conserv)

Independent of Initial Conditions

Observed Baryon Asymmetry In the Universe (BAU)

Fukugita, Yanagida,

Kuzmin, Rubakov, Shaposhinkov
Heavy Right-handed Majorana neutrinos enter equilibrium at \( T = T_{eq} > T_{\text{decay}} \).

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Independent of Initial Conditions

Observed Baryon Asymmetry In the Universe (BAU)

Estimate BAU by solving Boltzmann equations for Heavy Neutrino Abundances

Pilafsis, Riotto…

Buchmuller, di Bari et al.

Fukugita, Yanagida,

Kuzmin, Rubakov, Shaposhinkov
Predicted BAU in such models is found to be of order:

\[ \frac{n_B}{n_\gamma} = \Delta \sim \frac{1}{g_{\text{eff}}} \delta_{CP} \cdot S_{\text{macro}} S_{\text{sph}}, \]

\[ \delta_{CP} = \frac{\Gamma(N \rightarrow H\nu) - \Gamma(N \rightarrow \bar{H}\bar{\nu})}{\Gamma_{\text{tot}}} \]

Majorana fermion
Kinematics

Sphaleron effects $L \rightarrow B$

\[ S_{\text{macro}} S_{\text{sph}} \sim \mathcal{O}\left(\frac{1}{10}\right) \]
Predicted BAU in such models is found to be of order:\n
$$\text{Non mass degenerate Majorana neutrinos}$$

$$|M_I - M_J| \sim M_K$$

$$\frac{n_B}{s} \sim 10^{-3} F^2 \sim 10^{-10}$$

$$F^2 \sim 10^{-7}$$

reproduced observed BAU

$$\frac{n_B}{n_\gamma} = \Delta \sim \frac{1}{g_{\text{eff}}} \delta_{CP} \cdot S_{\text{macro}} S_{\text{sph}},$$

$$\delta_{CP} = \frac{\Gamma(N \rightarrow H\nu) - \Gamma(N \rightarrow \bar{H}\bar{\nu})}{\Gamma_{\text{tot}}}$$

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\[ \frac{n_B}{s} \sim 10^{-3} F^2 \sim 10^{-10} \]

\[ F^2 \sim 10^{-7} \]

Non mass degenerate Majorana neutrinos

reproduced observed BAU

\[ m_\nu = -M_D \frac{1}{M_I} [M_D]^T \]

\[ M_D = F_\alpha I \nu \]

\[ \nu = \langle \phi \rangle = 174 \text{ GeV} \]

\[ M_N \sim 10^{11} \text{ GeV} \]
Predicted BAU in such models is found to be of order:

Non mass degenerate Majorana neutrinos

\[
\frac{n_B}{\rho} \sim 10^{-3} F^2 \sim 10^{-10}
\]

\[F^2 \sim 10^{-7}\]

reproduced observed BAU

\[
m_\nu = -M_D \frac{1}{M_I} [M_D]^T
\]

\[M_D = F_{\alpha I \nu}\]

\[\langle \phi \rangle = 174 \text{ GeV}\]

\[M_N \sim 10^{11} \text{ GeV}\]
Stability of Higgs mass against higher loops in danger!

Non mass degenerate Majorana neutrinos

Predicted BAU in such models is found to be of order:

Pilaftsis, Shaposhnikov...

\[ m_\nu = -M_D \frac{1}{M_I} [M_D]^T \]

\[ M_D = F_{\alpha I} \nu \]

\[ \nu = \langle \phi \rangle = 174 \text{ GeV} \]

\[ M_N \sim 10^{11} \text{ GeV} \]
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Non mass degenerate Majorana neutrinos

Predicted BAU in such models is found to be of order:

$M_N \sim 10^{11} \text{ GeV}$

$m_\nu = -M^D \frac{1}{M_I} [M^D]^T$

$M_D = F_{\alpha I} \nu$

$\nu = \langle \phi \rangle = 174 \text{ GeV}$

Pilaftsis, Shaposhnikov...

Stability of Higgs mass against higher loops in danger!

e.g. one loop

$F^2 M_I^2 / (4\pi) \sim 10^{14} \text{ GeV}^2$

reproduced observed BAU
POSSIBLE RESOLUTION: DEGENERATE RIGHT-HANDED NEUTRINOS

If, say: $N_2$, $N_3$ degenerate in mass

enhanced CP violation contribution from mixing (cf. neutral kaons)

but much smaller Yukawa couplings $F$ allowed

BAU estimated in this case:

$$\frac{n_B}{s} \sim 10^{-3} f^2 \frac{M_2 \Gamma_{tot}}{(M_2 - M_3)^2 + \Gamma_{tot}^2}$$

$$\frac{|M_2 - M_3|}{M_2} \sim f^2 \sim \frac{m_\nu M_W}{v^2} \sim 10^{-13}$$

$$M_I \sim M_W$$
If, say: \( N_2, N_3 \) degenerate in mass

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\]

\[
\frac{|M_2 - M_3|}{M_2} \sim f^2 \sim \frac{m_\nu M_W}{v^2} \sim 10^{-13}
\]

\[ M_I \sim M_W \]

\[ |M_2 - M_3| \sim \Gamma_{\text{tot}} \]

\[ |M_2 - M_3| \sim \Gamma_{\text{tot}} \]

NB: For no Problem for Higgs mass stability
A restricted Case: $N_1$ only out of equilibrium decay
$N_{2,3}$ in thermal equilibrium

One lepton number ($\tau$) resonantly produced by out-of-equilibrium decays

$$-\mathcal{L}_{Y,M} = \frac{1}{2}(\bar{\nu}_{iR})^c (M_S)_{ij} \nu_{jR} + \hat{h}_{ii}^l \bar{L}_i \Phi l_{iR}$$
$$+ h_{ij}^\nu \bar{L}_i \Phi^\dagger \nu_{jR} + \text{H.c.,}$$

$$h^\nu_{iR} = \begin{pmatrix}
\varepsilon_e & ae^{-i\pi/4} & ae^{i\pi/4} \\
\varepsilon_\mu & be^{-i\pi/4} & be^{i\pi/4} \\
\varepsilon_\tau & ce^{-i\pi/4} & ce^{i\pi/4}
\end{pmatrix}.$$
Avoid $L_\tau$ excess $\Rightarrow$ $N_{2,3}$ decay rates suppressed

\[ -\mathcal{L}_{Y,M} = \frac{1}{2} (\bar{\nu}_i R)^c (M_S)_{ij} \nu_j R + h^l_{ii} \bar{L}_i \Phi l_{iR} + h_{ij}^{\nu R} \bar{L}_i \Phi \nu_j R + H.c., \]

\[ h^{\nu R} = \begin{pmatrix} \varepsilon_e & ae^{-i\pi/4} & ae^{i\pi/4} \\ \varepsilon_\mu & be^{-i\pi/4} & be^{i\pi/4} \\ \varepsilon_\tau & ce^{-i\pi/4} & ce^{i\pi/4} \end{pmatrix}. \]
A restricted Case: $N_1$ only out of equilibrium decay
$N_{2,3}$ in thermal equilibrium

Avoid $L_\tau$ excess $\Rightarrow$ $N_{2,3}$ decay rates suppressed

Predicted BAU

\[
\eta_B \sim -10^{-2} \frac{\delta_{N_1}^{\tau}}{K_{N_1}} \frac{\Gamma(N_1 \rightarrow L_\tau \Phi)}{\Gamma(N_{2,3} \rightarrow L_\tau \Phi)} \\
\sim -10^{-2} \frac{\delta_{N_1}^{\tau}}{K_{N_1}} \frac{\varepsilon_\tau^2}{c^2}
\]

\[
\Gamma_{N_1}/H(z = 1) \approx 10 \quad z = m_{N_1}/T
\]
A restricted Case: $N_1$ only out of equilibrium decay
$N_{2,3}$ in thermal equilibrium

Avoid $L_\tau$ excess $\Rightarrow$ $N_{2,3}$ decay rates suppressed

Predicted BAU

\[ \eta_B \sim -10^{-2} \frac{\delta^\tau_{N_1}}{K_{N_1}} \frac{\Gamma(N_1 \to L_\tau \Phi)}{\Gamma(N_{2,3} \to L_\tau \Phi)} \]
\[ \approx -10^{-2} \frac{\delta^\tau_{N_1} \varepsilon^2_{\tau}}{K_{N_1} c^2} \]
\[ \frac{\Gamma_{N_1}}{H(z = 1)} \approx 10 \]

Resonant $\tau$ Leptogenesis

\[ -\mathcal{L}_{Y,M} = \frac{1}{2} (\bar{\nu}_i R)^c (M_S)_{ij} \nu_j R + h_{iiR}^l \bar{L}_i \Phi l_{iR} \]
\[ + h_{ijR}^\nu \bar{L}_i \tilde{\Phi} \nu_{jR} + \text{H.c.,} \]

\[ h_{ijR}^\nu = \begin{pmatrix} \varepsilon_e & ae^{-i\pi/4} & ae^{i\pi/4} \\ \varepsilon_\mu & be^{-i\pi/4} & be^{i\pi/4} \\ \varepsilon_\tau & ce^{-i\pi/4} & ce^{i\pi/4} \end{pmatrix} \]
A restricted Case: $N_1$ only out of equilibrium decay
$N_{2,3}$ in thermal equilibrium

Avoid $L_\tau$ excess $\implies$ $N_{2,3}$ decay rates suppressed

Predicted BAU

\[
\eta_B \sim -10^{-2} \frac{\delta_{N_1}^\tau}{K_{N_1}} \frac{\Gamma(N_1 \to L_\tau \Phi)}{\Gamma(N_{2,3} \to L_\tau \Phi)} \\
\sim -10^{-2} \frac{\delta_{N_1}^\tau}{K_{N_1}} \frac{\varepsilon_\tau^2}{c^2} \\
\overset{\text{approx}}{\sim} 10
\]

Approximate $\Gamma_{N_1}/H(z = 1)$ $\approx 10$

$|\delta_{N_1}^\tau| \sim 10^{-5}$ and $\varepsilon_\tau/c \sim 10^{-2}$

Resonant $\tau$ Leptogenesis

Pilaftsis

\[
- \mathcal{L}_Y = \frac{1}{2} (\bar{\nu}_{iR}^c M_S)_{ij} \nu_{jR} + h^l_{ii} \bar{L}_i \Phi l_{iR} \\
+ h^{\nu_R}_{ij} \bar{L}_i \Phi \nu_{jR} + \text{H.c.,}
\]

\[
h^{\nu_R} = \begin{pmatrix}
\varepsilon_e & ae^{-i\pi/4} & ae^{i\pi/4} \\
\varepsilon_\mu & be^{-i\pi/4} & be^{i\pi/4} \\
\varepsilon_\tau & ce^{-i\pi/4} & ce^{i\pi/4}
\end{pmatrix}
\]

Agreement with observed BAU
A restricted Case: $N_1$ only out of equilibrium decay $N_{2,3}$ in thermal equilibrium

Avoid $L_\tau$ excess $\Rightarrow$ $N_{2,3}$ decay rates suppressed

Predicted BAU

$$\eta_B \sim -10^{-2} \frac{\delta_{N_1}^{T\tau}}{K_{N_1}} \frac{\Gamma(N_1 \rightarrow L_\tau \Phi)}{\Gamma(N_{2,3} \rightarrow L_\tau \Phi)}$$

$$\sim -10^{-2} \frac{\delta_{N_1}^{T\tau} \varepsilon_\tau^2}{K_{N_1} c^2}$$

$$|\delta_{N_1}^{T\tau}| \sim 10^{-5} \text{ and } \varepsilon_\tau/c \sim 10^{-2}$$

Estimate agrees with:

**Boltzmann eq calculated neutrino $N_1$ abundance**
**A restricted Case**: $N_1$ only out of equilibrium decay, $N_{2,3}$ in thermal equilibrium

Avoid $L_\tau$ excess  \[ \rightarrow \] N$_{2,3}$ decay rates suppressed

Predicted BAU

\[ \begin{align*}
\eta_B &\sim -10^{-2} \frac{\delta_{N_1}}{K_{N_1}} \frac{\Gamma(N_1 \rightarrow L_\tau \Phi)}{\Gamma(N_{2,3} \rightarrow L_\tau \Phi)} \\
&\sim -10^{-2} \frac{\delta_{N_1}}{K_{N_1}} \frac{\varepsilon_{\tau}^2}{c^2} \\
\Gamma_{N_1}/H(z = 1) &\sim m_{N_1}/T \\
|\delta_{N_1}| &\sim 10^{-5} \text{ and } \varepsilon_{\tau}/c \sim 10^{-2}
\end{align*} \]

Leptogenesis possible for low-mass $N_1$: $M_N = O(M_W - \text{TeV})$

*Resonant $\tau$ Leptogenesis*  \[ \rightarrow \]  

Pilaftsis Underwood
A restricted Case: $N_1$ only out of equilibrium decay
$N_{2,3}$ in thermal equilibrium

Predicted BAU

$M_N = O(M_W - \text{TeV})$

$\mu \rightarrow e\gamma$

$B(\mu \rightarrow e\gamma) \approx 6 \times 10^{-4} (a^2 b^2 v^4) / m_N^4$

$a, b \sim 3 \times 10^{-3}$

$B^{\exp}(\mu \rightarrow e\gamma) \leq 1.2 \times 10^{-11}$

future sensitivity to $10^{-13}$

MEG experiment:

$\mu^+ \rightarrow e^+ + \gamma$

1107.5547 $B(\mu^+ \rightarrow e^+ + \gamma) \leq 2.4 \times 10^{-12}$
\textbf{A restricted Case:} \( N_1 \) only out of equilibrium decay \( N_{2,3} \) in thermal equilibrium

Predicted BAU

\[ M_N = O(M_W - \text{TeV}) \]

\textbf{Resonant} \( \tau \) Leptogenesis

\textbf{Pilaftsis Underwood}

\[ B(\mu \to e\gamma) \approx 6 \times 10^{-4} \left( \frac{a^2 b^2 v^4}{m_N^4} \right) \]

\[ a, b \sim 3 \times 10^{-3} \]

\[ B^{\text{exp}}(\mu \to e\gamma) \lesssim 1.2 \times 10^{-11} \]

future sensitivity to \( 10^{-13} \)

\textbf{Effects at e^+e^- linear collider? Study production of} electroweak scale \( N_{2,3} \) \textbf{via their decays to} e, \( \mu \) (\textbf{not} \( \tau \))
Two distinct physics cases: $M_i > M_w$ & $M_i < M_w$
**Thermal Properties**

Two distinct physics cases: \( M_I > M_W \) & \( M_I < M_W \)

(i) \( M_I > M_W \) (electroweak scale), e.g. \( M_I = O(1) \) GeV

(ii) \( M_I < M_W \) (electroweak scale), e.g. \( M_I = O(1) \) GeV

Keep light neutrino masses in right order, Yukawa couplings must be:

\[
F_{\alpha I} \sim \frac{\sqrt{m_{\text{atm}} M_I}}{v} \sim 4 \times 10^{-8}
\]

**Baryogenesis through coherent oscillations right-handed singlet fermions**

Akhmedov, Rubakov, Smirnov
Heavy Majorana fermions $N_i$ thermalize only for $T < M_w$.

**Out of Equilibrium decays of $N_i$ for $T > M_w$**

BAU depends in this case on initial conditions.

*But* at the end of inflation we may reasonably assume that the $N_i$ populations are washed out, hence set their end-of-inflation concentrations to zero value.

Majorana masses small compared to Sphaleron freeze-out $T$.

*Total Lepton number conserved*

Assume Mass generacy $N_{2,3}$ enhanced CP violation.

*Coherent Oscillations* between these singlet fermions.

*Lepton number of active left-handed $\nu$ transferred to Baryons due to equilibrated sphaleron processes*.

*Total Lepton zero but unevenly distributed between active & sterile $\nu$*.
Assume *Mass degeneracy* $N_{2,3}$, hence enhanced CP violation *Coherent Oscillations* between these singlet fermions.

\[
\omega \sim \frac{|M_2^2 - M_3^2|}{E_I} \sim \frac{M_2 \Delta M(T)}{T}
\]

\[
E_I \sim T
\]

\[
\Delta M(T) \ll M_2 \approx M_3
\]

$N_2 - N_3$ MASS DIFF.
BAU ESTIMATES

Assume *Mass degeneracy* \( N_{2,3} \), hence enhanced CP violation

*Coherent Oscillations* between these singlet fermions

\[
\omega \sim \frac{|M_2^2 - M_3^2|}{E_I} \sim \frac{M_2 \Delta M(T)}{T}
\]

\( E_I \sim T \)

\( \Delta M(T) \ll M_2 \approx M_3 \)

FOR CP VIOLATION TO OCCUR MUST HAVE: Oscillation rate > Hubble rate \( H(T) \)

*Baryogenesis occurs @*: 

\[
T_B \sim \left( M_1 \Delta M(T) M_0 \right)^{1/3}
\]

\( n_B \) / \( s \) \( \approx 1.7 \cdot 10^{-10} \delta_{CP} \left( \frac{10^{-5}}{\Delta M(T)/M_2} \right)^{\frac{2}{3}} \left( \frac{M_2}{10 \text{ GeV}} \right)^{\frac{5}{3}}
\]

\( \delta_{CP} = 4s_{R23}c_{R23} \left[ s_{L12}s_{L13}c_{L13} \left( (c_{L23}^4 + s_{L23}^4)c_{L13}^2 - s_{L13}^2 \right) \cdot \sin(\delta_L + \alpha_2) \right.

\left. + c_{L12}^3 c_{L13}^2 s_{L23} c_{L23} \left( c_{L23}^2 - s_{L23}^2 \right) \cdot \sin \alpha_2 \right] .
\]

eg O(100) GeV
BAU ESTIMATES

Assume **Mass degeneracy** $N_{2,3}$, hence enhanced CP violation. **Coherent Oscillations** between these singlet fermions

$$\omega \sim \frac{|M_2^2 - M_3^2|}{E_I} \sim \frac{M_2 \Delta M(T)}{T}$$

$$E_I \sim T \quad \Delta M(T) \ll M_2 \approx M_3$$

FOR CP VIOLATION TO OCCUR MUST HAVE: Oscillation rate $> Hubble$ rate $H(T)$

Baryogenesis occurs @:

$$T_B \sim \left( M_I \Delta M(T) M_0 \right)^{1/3}$$

eg O(100) GeV

Quite effective Mechanism: Maximal Baryon asymmetry

$$\Delta \equiv \frac{n_B - n_{\bar{B}}}{n_B + n_{\bar{B}}} \sim 1$$

for $T_B = T_{\text{sph}} = T_{\text{eq}}$

**Assumption**: Interactions with plasma of SM particles do not destroy quantum mechanical coherence of oscillations
BAU ESTIMATES

Assume \textit{Mass degeneracy} $N_{2,3}$, hence enhanced CP violation \textit{Coherent Oscillations} between these singlet fermions

$$\omega \sim \frac{|M_2^2 - M_3^2|}{E_I} \sim \frac{M_2 \Delta M(T)}{T}$$

$$E_I \sim T$$

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FOR CP VIOLATION TO OCCUR MUST HAVE: Oscillation rate > Hubble rate $H(T)$

\textit{Baryogenesis occurs @:} $T_B \sim \left(M_I \Delta M(T) M_0\right)^{1/3}$, eg O(100) GeV

$$\frac{n_B}{s} \approx 1.7 \cdot 10^{-10} \delta_{CP} \left(\frac{10^{-5}}{\Delta M(T)/M_2}\right)^{2/3} \left(\frac{M_2}{10 \text{ GeV}}\right)^{5/3}$$

Mass $N_2 (N_3) / (\text{Mass } N_1) = O(10^5)$

\textit{N}_1 \textit{ Lightest Sterile nutrino is a natural DARK MATTER candidate}
PART II
NEUTRINOS
&
THE DARK SECTOR
OF THE UNIVERSE
NEUTRINOS & THE DARK SECTOR OF THE UNIVERSE

Current Energy Budget of the Cosmos

Observations from:

- Supernovae Ia
- CMB
- Baryon Acoustic Oscillations
- Galaxy Surveys
- Structure Formation data
- Strong & Weak lensing

Active $\nu$
TYPES OF DARK MATTER

• HOT DARK MATTER (HDM): form of dark matter which consists of particles that travel with ultrarelativistic velocities: e.g. neutrinos

• COLD DARK MATTER (CDM): form of dark matter consisting of slowly moving particles, hence cold,
  • e.g. WIMPS (stable supersymmetric particles (neutralinos etc.) or MACHOS.

• WARM DARK MATTER (WDM): form of dark matter with properties between those of HDM and CDM, sterile neutrinos, light gravitinos-partner of gravitons in supergravity theories…)


PHYSICS: WMAP and Dark Matter

WMAP results so far:

• Disfavor strongly hot dark matter (neutrinos), $\Omega_\nu h^2 < 0.0076$ ($< m_\nu >_e < 0.23$ eV).

• Warm Dark Matter (gravitino) disfavoured by evidence for re-ionization at redshift $z \sim 20$.

• Cold Dark Matter (CDM) remains: axions, supersymmetric dark matter (lightest SUSY particle (LSP)), superheavy (masses $\sim 10^{14\pm5}$ GeV)

WMAP results: $\Omega_m h^2 = 0.135^{+0.008}_{-0.009}$ (matter), $\Omega_b h^2 = 0.0224 \pm 0.0009$ (baryons), hence, assuming CDM is the difference, $\Omega_{CDM} h^2 = 0.1126^{+0.0161}_{-0.0181}$, (2σ level).
Numerical simulations for structure formation in Cold Dark Matter (CDM) (top) and Warm Dark Matter (WDM) (middle) with mass $m_X = 10$ KeV at $z = 20$. Bottom: Dark halos with mass $> 10^5 M_\odot$ for CDM (left) and for WDM (right).

**IMPORTANT COMMENTS:**

Such structure formation arguments can only place a lower bound on mass of the WDM candidate: $m_X > 10$ KeV.

Above results exclude Light Gravitino Models ($m_X < 0.5 KeV$) of Particle Physics as DM candidates.

NB! WDM with $m_X \geq 100$ KeV becomes indistinguishable from Cold Dark Matter, as far as structure formation is concerned.
Contribution of neutrinos to energy density of Universe: \( \Omega_\nu h^2 = \frac{\sum_i m_i}{94.0 \text{ eV}} \) (sum over light neutrino species (decouple while still relativistic)).

WMAP and other experiments (the Lyman \( \alpha \) data etc) \( \Omega_\nu h^2 < 0.0076 \Rightarrow < m_\nu >_e < 0.23 \text{ eV} \):

Excludes HOT DM.

NB: WMAP still consistent with Majorana neutrinos, and also marginally with \( \beta\beta \)-decay (Heidelberg-Moscow Coll.).
Contribution of neutrinos to energy density of Universe: $\Omega_\nu h^2 = \frac{\sum_i m_i}{94.0 \text{ eV}}$
(sum over light neutrino species (decouple while still relativistic)).

WMAP and other experiments (the Lyman $\alpha$ data etc) $\Omega_\nu h^2 < 0.0076 \Rightarrow \langle m_\nu \rangle < 0.23 \text{ eV}$:
Excludes HOT DM.

NB: WMAP still consistent with Majorana neutrinos, and also marginally with $\beta\beta$-decay (Heidelberg-Moscow Coll.).
### Uncertainty in Cosmological Data Due to Variety of Sources & Measurements

<table>
<thead>
<tr>
<th>Model</th>
<th>Observables</th>
<th>$\sum m_\nu$ (eV) 95% Bound</th>
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<tbody>
<tr>
<td>$\omega CDM + \Delta N_{rel} + m_\nu$</td>
<td>CMB+HO+SN+BAO</td>
<td>$\leq 1.5$</td>
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<td>$\omega CDM + \Delta N_{rel} + m_\nu$</td>
<td>CMB+HO+SN+LSSPS</td>
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<td>$\alpha \omega$ CDM</td>
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</tr>
<tr>
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<td>I [0.048 – 0.21]</td>
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<td>I [0.047 – 0.25]</td>
<td>I [0.014 – 0.25]</td>
</tr>
<tr>
<td>$\Lambda$ CDM + $m_{\nu}$</td>
<td>CMB+LSSPS</td>
<td>N [0.0047 – 0.18]</td>
</tr>
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</tr>
<tr>
<td>$\Lambda$ CDM + $m_{\nu}$</td>
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Contribution of neutrinos to energy density of Universe: $\Omega_\nu h^2 = \frac{\sum_i m_i}{94.0 \text{ eV}}$

(sum over light neutrino species (decouple while still relativistic)).

WMAP and other experiments (the Lyman $\alpha$ data etc) $\Omega_\nu h^2 < 0.0076 \Rightarrow <m_\nu>_e < 0.23 \text{ eV}$:

Excludes HOT DM.

NB: WMAP still consistent with Majorana neutrinos, and also marginally with $\beta\beta$-decay (Heidelberg-Moscow Coll.).
Caution:
FRW- Comology & local Lorentz invariance assumed.
If Lorentz violated (TeVeS) \( \nu \) of 2 eV mass could have \( \Omega_\nu \approx 0.15 \) to reproduce CMB spectrum (Dodelson-Liguori 2006)

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**WMAP excludes HOT Dark Matter**

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Light Sterile Neutrinos (mass > keV) may provide good dark Matter Candidates

To be DM: lifetime > age of Universe
Light Sterile Neutrinos (mass $> \text{keV}$) may provide good dark Matter Candidates

Boyarski, Ruchayskiy, Shaposhnikov…

$v_{\text{MSM}}$

Lightest of singlet fermions $N_1$ plays that role

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Davidson, Widrow, Shi, Fuller, Dolgov, Hansen, Abazajian, Patel, Tucker…

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Contributions to mass matrix of active neutrinos

$$\delta m_\nu \sim \theta_1^2 M_1$$
Light Sterile Neutrinos (mass > KeV) may provide good dark matter candidates. Boyarski, Ruchayskiy, Shaposhnikov… Davidson, Widrow, Shi, Fuller, Dolgov, Hansen, Abazajian, Patel, Tucker…

Sterile neutrinos and DARK MATTER

$\nu$ MSM

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Boyarски, Ruchayskiy, Shaposhnikov…

\[ \Delta m^2_{\text{sol}} = 7.65^{+0.23}_{-0.20} \times 10^{-5} \text{ eV}^2 \]

\[ \Delta m^2_{\text{atm}} = 2.40^{+0.12}_{-0.11} \times 10^{-3} \text{ eV}^2 \]

\[ M_1 \geq 2 \text{ keV} \Rightarrow \]

mass contribution smaller than solar mass diff experimental error

\[ \delta m_\nu \sim \theta^2_1 M_1 \]

Contributions to mass matrix of active neutrinos
More than one sterile neutrino needed to reproduce Observed oscillations

$\nu_{\text{MSM}}$

Boyarski, Ruchayskiy, Shaposhnikov...

Constraints on two heavy degenerate singlet neutrinos

$N_1$ DM production estimation in Early Universe must take into account its interactions with $N_{2,3}$ heavy neutrinos
More than one sterile neutrino needed to reproduce Observed oscillations

Decaying $N_1$ produces narrow spectral line in spectra of DM dominated astrophysical objects

Constraints on two heavy degenerate singlet neutrinos

$N_1$ DM production estimation in Early Universe must take into account its interactions with $N_{2,3}$ heavy neutrinos
MODEL CONSISTENT WITH BBN, STRUCTURE FORMATION DATA IN THE UNIVERSE & ALL OTHER ASTROPHYSICAL CONSTRAINTS
MASS HIERARCHY ($N_1 \ll N_{2,3}$) AMONG STERILE NEUTRINOS

PHENOMENOLOGICAL…

MICROSCOPIC EXPLANATIONS?
MASS HIERARCHY \((N_1 \ll N_{2,3})\) AMONG STERILE NEUTRINOS

PHENOMENOLOGICAL…

MICROSCOPIC EXPLANATIONS?

(I) **FLAVOUR SYMMETRIES**

\[ M_1 = 0 \] if symmetry unbroken,

Breaking of global Lepton symmetry generate singlet fermion mass hierarchy

\[ M_2 = M_3 \geq \text{GeV} \]

\[ M_2 \geq \text{GeV} \]

\[ L_e - L_\mu - L_\tau \]

\[ L_e \geq L_\mu \geq L_\tau \]

\[ M_1 \sim \text{keV} \]

\[ M_1 = 0 \]

- Shaposhnikov
- Lindner, Merle, Niro

- e.g. GUT models
- Mohapatra, Senjanovic, Ross…
BRANE WORLD RANDALL-SUNDRUM MODELS:  
*Exponential Mass Suppression*

**MASS HIERARCHY** ($N_1 << N_{2,3}$) **AMONG STERILE NEUTRINOS**

PHENOMENOLOGICAL...

**MICROSCOPIC EXPLANATIONS?**

(II) Kushenko, Takahashi, Yanagida

**Shadow world**

Our Brane world

$M_1 \sim \frac{2m_i}{e^2m_i^1 - 1}$

$M_2 \sim 10^{11}\text{GeV}$

$M_3 > M_2$

$M_1 \sim \text{keV}$
MASS HIERARCHY ($N_1 << N_{2,3}$) AMONG STERILE NEUTRINOS

PHENOMENOLOGICAL…

MICROSCOPIC EXPLANATIONS?

(III) FROGATT – NIELSEN MECHANISM:

One fermion acquires mass via Higgs mechanism, others via higher order multiple seesaw

\[
\mathcal{L}_{\text{leptons}} = \sum_{i,j} - Y_{e}^{ij} \bar{e}^i_R H L_j L + \text{h.c.} - Y_{D}^{ij} \bar{N}_i R \tilde{H} L_j L + \text{h.c.} - \frac{1}{2} \bar{N}_i R \tilde{M}_R^{ij} \left( N_{jR} \right)^C \left( \frac{\Theta}{\Lambda} \right)^{g_i + g_j} + \text{h.c.} - \frac{1}{2} Y_{L}^{ij} \bar{L}_{iL} \left( L_{jL} \right)^C \left( i\sigma_2 \Delta \right) L_j L \left( \frac{\Theta}{\Lambda} \right)^{f_i + f_j} + \text{h.c.}
\]
FROGATT – NIELSEN MECHANISM:

One fermion acquires mass via Higgs mechanism, others via higher order multiple see-saw.

\[
\mathcal{L}_{\text{leptons}} = -Y_{e}^{ij} e_{iR} H L_{jL} \left( \frac{\Theta}{\Lambda} \right)^{k_{i}+f_{j}} + \text{h.c.} - Y_{D}^{ij} N_{iR} \tilde{H} L_{jL} \left( \frac{\Theta}{\Lambda} \right)^{g_{i}+f_{j}} + \text{h.c.} \\
- \frac{1}{2} \bar{N}_{iR} \tilde{M}_{R}^{ij} (N_{jR})^{g_{i}+g_{j}} + \text{h.c.} - \frac{1}{2} Y_{L}^{ij} (L_{iL})^{C} (i\sigma_{2}\Delta) L_{jL} \left( \frac{\Theta}{\Lambda} \right)^{f_{i}+f_{j}} + \text{h.c.}
\]

``Flavon'' field

\[\lambda = \frac{\langle \Theta \rangle}{\Lambda} \text{ being a small quantity of the order of the Cabibbo angle: } \lambda \approx 0.22\]
MASS HIERARCHY \((N_1 << N_{2,3})\) AMONG STERILE NEUTRINOS

PHENOMENOLOGICAL…

MICROSCOPIC EXPLANATIONS?

(III) FROGATT – NIELSEN MECHANISM:

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MASS MATRICES

Charged lepton

\[
M_e = v \begin{pmatrix}
  Y_{e11} \lambda|k_1+f_1| & Y_{e12} \lambda|k_1+f_2| & Y_{e13} \lambda|k_1+f_3| \\
  Y_{e21} \lambda|k_2+f_1| & Y_{e22} \lambda|k_2+f_2| & Y_{e23} \lambda|k_2+f_3| \\
  Y_{e31} \lambda|k_3+f_1| & Y_{e32} \lambda|k_3+f_2| & Y_{e33} \lambda|k_3+f_3|
\end{pmatrix}
\]

Dirac neutrino

\[
m_D = v \begin{pmatrix}
  Y_{D11} \lambda|g_1+f_1| & Y_{D12} \lambda|g_1+f_2| & Y_{D13} \lambda|g_1+f_3| \\
  Y_{D21} \lambda|g_2+f_1| & Y_{D22} \lambda|g_2+f_2| & Y_{D23} \lambda|g_2+f_3| \\
  Y_{D31} \lambda|g_3+f_1| & Y_{D32} \lambda|g_3+f_2| & Y_{D33} \lambda|g_3+f_3|
\end{pmatrix}
\]

Merle Niro
Barry, Rodejohann, Zhang
(III) FROGATT – NIELSEN MECHANISM:

One fermion acquires mass via Higgs mechanism, others via higher order multiple see-saw

MASS MATRICES

Right-handed Neutrino sector

then via see-saw
MASS HIERARCHY ($N_1 << N_{2,3}$) AMONG STERILE NEUTRINOS

**Phenomenological...**

**Microscopic Explanations?**

**Froggatt–Nielsen Mechanism:**

One fermion acquires mass via Higgs mechanism, others via higher order multiple see-saw.

\[ \lambda = 0.22 \]

Right-handed Neutrino sector

then via see-saw

\[
M_R = 
\begin{pmatrix}
M_{11}^{12} & M_{12}^{12} & M_{13}^{12} \\
M_{21}^{12} & M_{22}^{12} & M_{23}^{12} \\
M_{31}^{12} & M_{32}^{12} & M_{33}^{12}
\end{pmatrix}
\]

\[
m_\nu = -m_D^T M_R^{-1} m_D = 
\begin{pmatrix}
a_1 \lambda^{2f_1} \\
b_1 \lambda^{f_1+f_2} \\
c_1 \lambda^{f_1+f_3} \\
d_1 \lambda^{2f_2} \\
e_1 \lambda^{f_2+f_3} \\
f_1 \lambda^{2f_3}
\end{pmatrix}
\]
MASS HIERARCHY \( (N_1 << N_{2,3} ) \) AMONG STERILE NEUTRINOS

PHENOMENOLOGICAL…

**MICROSCOPIC EXPLANATIONS?**

(III) FROGATT – NIELSEN MECHANISM:

One fermion acquires mass via Higgs mechanism, others via higher order multiple see-saw

---

**BUT… presently**

Lack of High Energy (UV) COMPLETION
(IV) BEYOND SEE-SAW? UV complete models?

ANOMALOUS GENERATION OF RIGHT-HANDED MAJORANA NEUTRINO MASSES THROUGH TORSIONFUL QUANTUM GRAVITY

Mavromatos, Pilaftsis
arXiv: 1209.6387
(PRD 86, 124038 (2012))
• Field Theories with (Kalb-Ramond) torsion & axion fields: *String inspired models, loop quantum gravity effective field theories... UV complete models*

• Majorana Neutrino Masses from (three-loop) *anomalous* terms with axion-neutrino couplings
Microscopic UV complete underlying theory of quantum gravity: STRINGS

Massless Gravitational multiplet of (closed) strings:
- spin 0 scalar (dilaton)
- spin 2 traceless symmetric rank 2 tensor (graviton)
- spin 1 antisymmetric rank 2 tensor

**KALB-RAMOND FIELD**

\[ B_{\mu \nu} = -B_{\nu \mu} \]

Effective field theories (low energy scale \( E \ll M_s \)) "gauge" invariant

\[ B_{\mu \nu} \rightarrow B_{\mu \nu} + \partial_{[\mu} \theta(x)_{\nu]} \]

Depend only on field strength:

\[ H_{\mu \nu \rho} = \partial_{[\mu} B_{\nu \rho]} \]

**Bianchi identity:**

\[ \partial_{[\sigma} H_{\mu \nu \rho]} = 0 \rightarrow d \star \mathbf{H} = 0 \]
Anomaly (gravitational vs gauge) cancellation in strings require redefinition of $H$ so that Bianchi identity now is extended to:

$$H = d\ B + \frac{\alpha'}{8\kappa} \left( \Omega_L - \Omega_V \right)$$

$$\kappa^2 = 8\pi G_N = \frac{8\pi}{M_P^2}$$

Lorentz (L) & Gauge (V) Chern-Simons three forms

**EXTENDED BIANCHI IDENTITY**

$$d\ H = \frac{\alpha'}{8\kappa} \text{Tr} \left( R \wedge R - F \wedge F \right)$$
ROLE OF H-FIELD AS TORSION

EFFECTIVE GRAVITATIONAL ACTION IN STRING LOW-ENERGY LIMIT CAN BE EXPRESSED IN TERMS OF A GENERALIZED CURVATURE RIEMANN TENSOR WHERE THE CHRISTOFFEL CONNECTION INCLUDES TORSION PROVIDED BY H-FIELD

\[
S^{(4)} = \int d^4 x \sqrt{-g} \left( \frac{1}{2\kappa^2} R - \frac{1}{6} H_{\mu\nu\rho} H^{\mu\nu\rho} \right)
\]

\[
= \int d^4 x \sqrt{-g} \left( \frac{1}{2\kappa^2} \overline{R} - \frac{1}{3} \kappa^2 H_{\mu\nu\rho} H^{\mu\nu\rho} \right)
\]

\[
\overline{\Gamma}_{\nu\rho}^\mu = \Gamma_{\nu\rho}^\mu + \frac{\kappa}{\sqrt{3}} H_{\nu\rho}^\mu \neq \overline{\Gamma}_{\rho\nu}^\mu
\]
IN 4-DIM DEFINE DUAL OF H AS

\[ Y = *H \]

\[ Y_\sigma = -3 \sqrt{2} \partial_\sigma b = \sqrt{-g} \epsilon_{\mu\nu\rho\sigma} H^{\mu\nu\rho} \]

Pseudoscalar
(Kalb-Ramond (KR) axion)
IN 4-DIM DEFINE DUAL OF $H$ AS $Y = *H$

$$Y_\sigma = -3\sqrt{2}\partial_\sigma b = \sqrt{-g} \epsilon_{\mu\nu\rho\sigma} H^{\mu\nu\rho}$$

**Rewrite extended Bianchi identity as**

$$\nabla_\sigma Y^\sigma = \frac{\alpha'}{32\kappa} \sqrt{-g} \epsilon_{\mu\nu\lambda\sigma} \left( R_{\alpha\delta}{}^{\mu\nu} R^{\lambda\sigma\alpha\delta} - F^{\mu\nu} F^{\lambda\sigma} \right)$$

**Hence effective action**

$$S^{(4)} = \int d^4 x \sqrt{-g} \left[ \frac{1}{2\kappa^2} R - \frac{1}{2} \partial_\mu b(x) \partial^\mu b(x) ight. \right.$$

$$\left. + \frac{\alpha'}{192\kappa} \sqrt{2} \epsilon_{\mu\nu\rho\lambda} \left( R_{\alpha\delta}{}^{\mu\nu} R^{\rho\lambda\alpha\delta} - F^{\mu\nu} F^{\rho\lambda} \right) \right]$$
IN 4-DIM DEFINE DUAL OF H AS
\[ Y = \ast H \]
\[ Y_\sigma = -3 \sqrt{2} \partial_\sigma b = \sqrt{-g} \epsilon_{\mu\nu\rho\sigma} H^{\mu\nu\rho} \]

REWRITE EXTENDED BIANCHI IDENTITY AS
\[ \nabla_\sigma Y^\sigma = \frac{\alpha'}{32\kappa} \sqrt{-g} \epsilon_{\mu\nu\lambda\sigma} \left( R_{ad}{}^{\mu\nu} R^{\lambda\sigma ad} - F_{\mu\nu} F_{\lambda\sigma} \right) \]

HENCE EFFECTIVE ACTION
\[ S^{(4)} = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa^2} R - \frac{1}{2} \partial_\mu b(x) \partial^\mu b(x) \right. \]
\[ + \left. \frac{\alpha'}{192\kappa} b(x) \epsilon_{\mu\nu\rho\lambda} \left( R_{ad}{}^{\mu\nu} R^{\rho\lambda ad} - F_{\mu\nu} F_{\rho\lambda} \right) \right] \]

\[ bR\tilde{R} \text{ coupling} \]
In 4-dim define dual of \( H \) as:

\[
Y = *H
\]

\[
Y_\sigma = -3 \sqrt{2} \partial_\sigma b = \sqrt{-g} \epsilon_{\mu\nu\rho\sigma} H^{\mu\nu\rho}
\]

cf. axion-electromagnetic field coupling

\[
a(x) \mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu} =
\]

\[
\mathcal{F}^{\mu\nu} \mathcal{F}_{\mu\nu} - b(x) \epsilon_{\mu\nu\rho\lambda} \left( R^{\mu\nu}_{\text{ad}} R^{\rho\lambda}_{\text{ad}} - F^{\mu\nu} F^{\rho\lambda} \right)
\]

\[bR \tilde{R}\] coupling
**NB:** Torsion Couples to fermions via gravitational covariant derivative \( \rightarrow \) integrating out torsion in path integral results in extra **fermion-fermion-axial current interactions**

\[
\int Db \exp \left[ -i \int \frac{1}{2} db \wedge *db - \frac{1}{f_b} bG(A, \omega) + \frac{1}{2f_b^2} J_5^\mu \wedge J_5^\nu \right]
\]

\[
J_5^\mu = \overline{\psi} \gamma_\mu \gamma_5 \psi
\]

\[
f_b = (3\kappa^2/8)^{-1/2} = \frac{M_P}{\sqrt{3\pi}}
\]

\[
\nabla_\mu J_5^{\mu} = \frac{e^2}{8\pi^2} F^{\mu\nu} \tilde{F}_{\mu\nu} - \frac{1}{192\pi^2} R^{\mu\nu\rho\sigma} \tilde{R}_{\mu\nu\rho\sigma} \\
\equiv G(A, \omega)
\]

Quantum anomaly (1-loop) equation
NB: Torsion Couples to fermions via gravitational covariant derivative → integrating out torsion in path integral results in extra fermion-fermion-axial current interactions

\[ \int Db \exp \left[ -i \int \frac{1}{2} db \wedge \ast db - \frac{1}{f_b} bG(A, \omega) + \frac{1}{2f_b^2} \mathbf{J}_5^5 \wedge \mathbf{J}_5^5 \right] \]

\[ J_\mu^5 = \overline{\psi} \gamma_\mu \gamma_5 \psi \]

\[ f_b = (3\kappa^2/8)^{-1/2} = \frac{M_P}{\sqrt{3\pi}} \]

\[ \nabla_\mu J_\mu^5 = \frac{e^2}{8\pi^2} F_{\mu\nu} \tilde{F}_{\mu\nu} - \frac{1}{192\pi^2} R_{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} \]

\[ \equiv G(A, \omega) . \]

Quantum anomaly (1-loop) equation
**NB:** Torsion Couples to fermions via gravitational covariant derivative $\Rightarrow$ integrating out torsion in path integral results in extra **fermion-fermion-axial current interactions**

\[
\int D\theta \exp \left[ -i \int \frac{1}{2} \theta \wedge \ast \theta - \frac{1}{f_b} \theta \theta \mathbb{G}(\mathbb{A}, \omega) + \frac{1}{2 f_b} \mathbb{J}^5 \wedge \mathbb{J}^5 \right]
\]

\[
J^5_\mu = \bar{\psi} \gamma_\mu \psi
\]

\[
\nabla_\mu J^5_\mu = \nabla_\mu \bar{\psi} \gamma_\mu \gamma_5 \psi = \frac{M_P}{\sqrt{3\pi}} \sqrt{\Theta} \theta \mathbb{G}(\mathbb{A}, \omega)
\]

above effective action
generic... in loop quantum gravity etc $\Rightarrow$ b-field
implements constraint
of Bianchi identity (conservation of quantum torsion ``charge'')

(p) equation
ANOMALOUS MAJORANA NEUTRINO MASS TERMS from QUANTUM TORSION

**SHIFT SYMMETRY**  \( b(x) \rightarrow b(x) + c \)

\[ c \, R^{\mu \nu \rho \sigma} \tilde{R}_{\mu \nu \rho \sigma} \text{ and } c \, F^{\mu \nu} \tilde{F}_{\mu \nu}. \] total derivatives

**OUR SCENARIO** Break such *shift symmetry* by coupling first \( b(x) \) to another pseudoscalar field such as QCD axion \( a(x) \) (or e.g. other string axions)

\[
S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} (\partial_\mu b)^2 + \frac{b(x)}{192\pi^2 f_b} R^{\mu \nu \rho \sigma} \tilde{R}_{\mu \nu \rho \sigma} \\
+ \frac{1}{2f_b^2} J^5_\mu J^{5\mu} + \gamma (\partial_\mu b) (\partial^\mu a) + \frac{1}{2} (\partial_\mu a)^2 \\
- y_{ai} a \left( \bar{\psi}_R^C \psi_R - \bar{\psi}_R \psi_R^C \right) \right],
\]
ANOMALOUS MAJORANA NEUTRINO MASS TERMS from QUANTUM TORSION

**SHIFT SYMMETRY** \( b(x) \rightarrow b(x) + c \)

\[ c \, R^{\mu\nu\rho\sigma} \tilde{R}_{\mu\nu\rho\sigma} \text{ and } c \, F^{\mu\nu} \tilde{F}_{\mu\nu}. \]
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+ \frac{1}{2f_b^2} J_\mu^5 J^{\mu 5} + \gamma (\partial_\mu b) (\partial^\mu a) - \frac{1}{2} (\partial_\mu a)^2 \\
y_{aia} \left( \bar{\psi}_R^C \psi_R - \bar{\psi}_R \psi_R^C \right) \right],
\]

Yukawa

neutrino fields
Field redefinition

\[ b(x) \rightarrow b'(x) \equiv b(x) + \gamma a(x) \]

so, effective action becomes

\[
S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} (\partial_\mu b')^2 + \frac{1}{2} \left( 1 - \gamma^2 \right) (\partial_\mu a)^2 + \frac{1}{2 f_b^2} J_\mu^5 J^{5\mu} + \frac{b'(x) - \gamma a(x)}{192\pi^2 f_b} R^{\mu\nu\rho\sigma} \tilde{R}_{\mu\nu\rho\sigma} \right.
\]

\[
- y_a \alpha \left( \bar{\psi}_R^C \psi_R - \bar{\psi}_R \psi_R^C \right) \right] .
\]

must have \(|\gamma| < 1\) otherwise axion field \(a(x)\) appears as a ghost \(\rightarrow\) canonically normalised kinetic terms

\[
S_a = \int d^4x \sqrt{-g} \left[ \frac{1}{2} (\partial_\mu a)^2 - \frac{\gamma a(x)}{192\pi^2 f_b \sqrt{1 - \gamma^2}} R^{\mu\nu\rho\sigma} \tilde{R}_{\mu\nu\rho\sigma} \right.
\]

\[
- \frac{i y_a}{\sqrt{1 - \gamma^2}} a \left( \bar{\psi}_R^C \psi_R - \bar{\psi}_R \psi_R^C \right) + \frac{1}{2 f_b^2} J_\mu^5 J^{5\mu} \right] .
\]
Field redefinition

\[ b(x) \rightarrow b'(x) \equiv b(x) + \gamma a(x) \]

so, effective action becomes

\[
S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} (\partial_{\mu} b')^2 + \frac{1}{2} \left( 1 - \gamma^2 \right) (\partial_{\mu} a)^2 
+ \frac{1}{2 f_b^2} J_5^\mu J_5^{5\mu}
+ \frac{b'(x) - \gamma a(x)}{192 \pi^2 f_b} R_{\mu\nu\rho\sigma} \tilde{R}_{\mu\nu\rho\sigma}
- \gamma a \left( \bar{\psi}_R^C \psi_R - \bar{\psi}_R \psi_R^C \right) \right].
\]

must have \( |\gamma| < 1 \)

otherwise axion field \( a(x) \) appears as a ghost \( \rightarrow \) canonically normalised kinetic terms

\[
S_a = \int d^4x \sqrt{-g} \left[ \frac{1}{2} (\partial_{\mu} a)^2 - \frac{\gamma a(x)}{192 \pi^2 f_b \sqrt{1 - \gamma^2}} R_{\mu\nu\rho\sigma} \tilde{R}_{\mu\nu\rho\sigma}
- \frac{i}{\sqrt{1 - \gamma^2}} \gamma a \left( \bar{\psi}_R^C \psi_R - \bar{\psi}_R \psi_R^C \right) + \frac{1}{2 f_b^2} J_5^\mu J_5^{5\mu} \right].
\]

CHIRALITY CHANGE
$a = \text{axion mixing with KR field } b$

axion field redefinition
THREE-LOOP ANOMALOUS FERMION MASS TERMS

ONE-LOOP

GRAVITON  AXION  GRAVITON

GRAVITON

CHIRALITY CHANGE
THREE-LOOP ANOMALOUS FERMION MASS TERMS

\[ \Lambda = \text{UV cutoff} \]

\[ M_R \sim \frac{1}{(16\pi^2)^2} \frac{y_a \gamma \kappa^4 \Lambda^6}{192\pi^2 f_b (1 - \gamma^2)} = \frac{\sqrt{3} y_a \gamma \kappa^5 \Lambda^6}{49152\sqrt{8} \pi^4 (1 - \gamma^2)} \]
SOME NUMBERS

\[ \Lambda = 10^{17} \text{ GeV} \]
\[ \gamma = 0.1 \]
\[ \Lambda = 10^{16} \text{ GeV} \]

\[ M_R \text{ is at the TeV for } y_a = 10^{-3} \]
\[ M_R \sim 16 \text{ keV, } y_a = \gamma = 10^{-3} \]
SOME NUMBERS

\[ \Lambda = 10^{17} \text{ GeV} \]
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\[ M_R \text{ is} \text{ at the TeV for} \ y_a = 10^{-3} \]
\[ M_R \sim 16 \text{ keV}, \]
\[ y_a = \gamma = 10^{-3} \]

INTERESTING WARM DARK MATTER REGIME

Appropriate Hierarchy for the other two massive Right-handed neutrinos for Leptogenesis-Baryogenesis & Dark matter constraints can be arranged by choosing Yukawa couplings
MODEL CONSISTENT WITH BBN, STRUCTURE FORMATION DATA IN THE UNIVERSE & ALL OTHER ASTROPHYSICAL CONSTRAINTS
More than one sterile neutrino needed to reproduce Observed oscillations

\[ \nu \] 

Boyarski, Ruchayskiy, Shaposhnikov...

Constraints on two heavy degenerate singlet neutrinos

\[ N_1 \text{ DM production estimation in Early Universe must take into account its interactions with } N_{2,3} \text{ heavy neutrinos} \]
Finiteness of the Mass

Multi-Axion Scenarios (e.g. string axiverse)

\[ S_a^{\text{kin}} = \int d^4x \sqrt{-g} \left[ \frac{1}{2} \sum_{i=1}^{n} \left( (\partial_\mu a_i)^2 - M_i^2 \right) + \gamma (\partial_\mu b)(\partial^\mu a_1) \right. \]

\[ -\frac{1}{2} \sum_{i=1}^{n-1} \delta M_{i,i+1}^2 a_i a_{i+1} \quad \right] \]

\[ \delta M_{i,i+1}^2 < M_i M_{i+1} \]

Positive mass spectrum for all axions

Simplifying all mixing equals

\[ M_R \sim \frac{\sqrt{3} y_a \gamma \kappa^5 \Lambda^{6-2n} (\delta M_a^2)^n}{49152 \sqrt{8} \pi^4 (1 - \gamma^2)} \quad n \leq 3 \]

\[ M_R \sim \frac{\sqrt{3} y_a \gamma \kappa^5 (\delta M_a^2)^3}{49152 \sqrt{8} \pi^4 (1 - \gamma^2)} \frac{(\delta M_a^2)^{n-3}}{(M_a^2)^{n-3}} \quad n > 3 \]
FINITENESS OF THE MASS

MULTI-AXION SCENARIOS (e.g. string axiverse)

\[ S_{\text{kin}}^a = \int d^4x \sqrt{-g} \left[ \frac{1}{2} \sum_{i=1}^{n} \left( (\partial_\mu a_i)^2 - M_i^2 \right) + \gamma (\partial_\mu b)(\partial^\mu a_1) \right] \]

\[ -\frac{1}{2} \sum_{i=1}^{n-1} \delta M_{i,i+1}^2 a_i a_{i+1} \]

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\[ M_R \sim \frac{\sqrt{3} y_a \gamma \kappa^5 (\delta M_a^2)^3}{49152\sqrt{8} \pi^4 (1 - \gamma^2)} \frac{(\delta M_a^2)^{n-3}}{(M_a^2)^{n-3}} \quad n > 3 \]

\( M_R \): UV finite for n=3 @ 2-loop independent of axion mass
In this gravitationally-induced right-handed neutrino mass scenario the ordinary (active) neutrinos are supposed to acquire their Majorana masses via standard Yukawa couplings & see-saw type mechanisms.

\[ y_e \bar{\nu}_e R \left( i \tau_2 \phi^* \right)^\dagger \left( \nu_L \right) + h.c. + \text{other flavours} \]

More thoughts and detailed analyses required...
Formation of fermion condensates dynamically in Early Universe as in Nambu-Jona-Lasinio model

Consider Models of (effective) four-fermion interactions of sterile Majorana neutrino in the early Universe

Kapusta, Antusch, Kersten, Lindner, Ratz, Barenboim, Rasero, Bhatt, Desai, Ma, Rajasekaran, U. Sarkar, NEM, ...
Formation of fermion \textbf{condensates} dynamically in Early Universe as in Nambu-Jona-Lasinio model.

Consider Models of (effective) four-fermion interactions of \textit{light sterile} Majorana neutrino in the early Universe.

\[ H_I = -\mathcal{C} \left( \bar{\nu}_M \nu_M \right) \left( \bar{\nu}_M \nu_M \right) \]

\[ \nu_M = \lambda \nu_R + \nu^c_L \]

\[ M_\nu = \begin{pmatrix} m_1 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_3 \\ 0 & 0 & m_3 & M \end{pmatrix} \]

Kapusta, Antusch, Kersten, Lindner, Ratz, Barenboim, Bhatt, Desai, Ma, Rajasekaran, U. Sarkar
NEM,

e.g. through heavy scalar exchange

One light ($O(10^{-3})$ eV) sterile Majorana neutrino forms the condensate.
Formation of fermion condensates dynamically in Early Universe as in Nambu-Jona-Lasinio model

Consider Models of (effective) four-fermion interactions of light sterile Majorana neutrino in the early Universe

\[ H_I = -\mathcal{C} (\bar{\nu}_M \nu_M) (\bar{\nu}_M \nu_M) \]

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\[ M_\nu = \begin{pmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & 0 & m_3 \\ 0 & 0 & m_3 & M \end{pmatrix} \]

Dirac light masses of O(0.1) eV
Formation of fermion **condensates** dynamically in Early Universe as in Nambu-Jona-Lasinio model

Consider Models of (effective) four-fermion interactions of light sterile Majorana neutrino in the early Universe through heavy scalar exchange

\[ H_I = -\mathcal{C} \left( \bar{\nu}_M \nu_M \right) \left( \bar{\nu}_M \nu_M \right) \]

\[ \nu_M = \lambda \nu_R + \nu^c_L \]

\[ M_\nu = \begin{pmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & m_3 & 0 \end{pmatrix} \]

One light \((O(10^{-3}) \text{ eV})\) sterile Majorana neutrino forms the condensate

**Model consistent with solar neutrino data**

Kapusta, Antusch, Kersten, Lindner, Ratz, Barenboim, Bhatt, Desai, Ma, Rajasekaran, U. Sarkar

NEM,
Consider Models of (effective) four-fermion interactions of light sterile Majorana neutrino in the early Universe as in Nambu-Jona-Lasinio model.

\[ H_I = -\mathcal{C} (\bar{\nu}_M \nu_M) (\bar{\nu}_M \nu_M) \]

\[ \nu_M = \lambda \nu_R + \nu^c_L. \]

One light sterile Majorana neutrino forms condensate

\[ \nu_M = \begin{bmatrix} \chi \\ \lambda \chi \end{bmatrix} \]

\[ H_{1}^{MF} = -2 \mathcal{C} \left[ \lambda^* \chi^\dagger_a \chi_b D + \lambda^2 \chi^\dagger_a \chi_b D^* \right] \epsilon_{ab}. \]

\[ \langle \chi_a \chi_b^\dagger \rangle = \epsilon_{ab} D \]

Coherence length, Gap Equation in FRW backgrounds

Dark Energy contribution
Formation of fermion \textit{condensates} dynamically in Early Universe as in Nambu-Jona-Lasinio model

Consider Models of (effective) four-fermion interactions of \textit{light sterile} Majorana neutrino in the early Universe.

- \[ H_I = -\mathcal{C} \left( \bar{\nu}_M \nu_M \right) \left( \bar{\nu}_M \nu_M \right) \]

\[ \nu_M = \] Better UV behaviour than flat space e.g. in de Sitter space time absorb UV infinities in Hubble \( H \), and Planck scale \( M_P \)

\[ \langle \chi_a \bar{\chi}_b \rangle = \epsilon_{ab} D \]

- Coherence length, FRW backgrounds

\[ \mathcal{C} = \frac{f^2}{m_s^2} \]

- E.g. through heavy scalar exchange

- Candelas, Reine (1975)

- Kapusta, Antusch, Kersten, Lindner, Ratz, Barenboim, Bhatt, Desai, Ma, Rajasekaran, U. Sarkar NEM,
PART III
CPT VIOLATION IN
THE
EARLY UNIVERSE
&
NEUTRINOS
GENERATE Baryon and/or Lepton ASYMMETRY without Heavy Sterile Neutrinos?

\textit{CPT Invariance Theorem}:
(i) Flat space-times
(ii) Lorentz invariance
(iii) Locality
(iv) Unitarity

(ii)-(iv) Independent reasons for violation

Schwinger, Pauli, Luders, Jost, Bell revisited by: Greenberg, Chaichian, Dolgov, Novikov...
GENERATE Baryon and/or Lepton ASYMMETRY without Heavy Sterile Neutrinos?

**CPT Invariance Theorem:**
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GENERATE Baryon and/or Lepton ASYMMETRY without Heavy Sterile Neutrinos?

Assume CPT Violation. e.g. due to Quantum Gravity fluctuations, strong in the Early Universe
GENERATE Baryon and/or Lepton ASYMMETRY without Heavy Sterile Neutrinos?

Assume CPT Violation.
e.g. due to *Quantum Gravity* fluctuations, *strong* in the Early Universe

**ONE POSSIBILITY:**
particle-antiparticle mass differences

\[ m \neq \bar{m} \]
Equilibrium Distributions different between particle-antiparticles

Can these create the observed matter-antimatter asymmetry?

\[ f(E, \mu) = \frac{1}{\exp[(E - \mu)/T] + 1} \]

\[ \delta n = n - \bar{n} = g_d f \int \frac{d^3p}{(2\pi)^3} \left[ f(E, \mu) - f(\bar{E}, \bar{\mu}) \right] \]

\[ E = \sqrt{p^2 + m^2}, \quad \bar{E} = \sqrt{p^2 + \bar{m}^2} \]

\[ m(\bar{T}) \sim g\bar{T} \]

High-T quark mass >> Lepton mass

\[ \delta m = m - \bar{m} \]

Assume dominant contributions to Baryon asymmetry from quarks-antiquarks

Dolgov, Zeldovich
Dolgov (2009)
Equilibrium Distributions different between particle-antiparticles

Can these create the observed matter-antimatter asymmetry?

\[ f(E, \mu) = \frac{1}{\exp[(E - \mu)/T] + 1} \]

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\[ E = \sqrt{p^2 + m^2}, \quad \bar{E} = \sqrt{p^2 + \bar{m}^2} \]

Assuming dominant contributions to Baryon asymmetry from quarks-antiquarks

\[ \beta_T = \frac{n_B}{n_\gamma} = -8.4 \cdot 10^{-3} \left( 18m_u \delta m_u + 15m_d \delta m_d \right) / T^2 \]

\[ n_\gamma = 0.24T^3 \]

photon equilibrium density at temperature T

Dolgov, Zeldovich

Dolgov (2009)
\[ \beta_T = \frac{n_B}{n_\gamma} = -8.4 \cdot 10^{-3} \left( 18m_u \delta m_u + 15m_d \delta m_d \right) / T^2 \]

\[ n_\gamma = 0.24 T^3 \]

Current bound for proton-anti-proton mass diff.

\[ \delta m_p < 7 \cdot 10^{-10} \text{ GeV} \]

Reasonable to take:

\[ \delta m_q \sim \delta m_p \]

\[ \beta^{(T=0)} = 6 \cdot 10^{-10} \]

Too small

\[ \beta^{T=0} \]

\[ \delta m_q(T = 100 \text{ GeV}) \sim 10^{-5} - 10^{-6} \text{ GeV} \gg \delta m_p \]

NB: To reproduce the observed
\[ \beta_T = \frac{n_B}{n_\gamma} = -8.4 \cdot 10^{-3} \left( 18m_u \delta m_u + 15m_d \delta m_d \right) / T^2 \]

\[ n_\gamma = 0.24T^3 \]

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\[ \beta^{(T=0)} = 6 \cdot 10^{-10} \]

\[ \delta m_q(T = 100 \text{ GeV}) \sim 10^{-5} - 10^{-6} \text{ GeV} \gg \delta m_p \]

**CPT Violating quark-antiquark Mass difference alone CANNOT REPRODUCE observed BAU**

**NB:** To reproduce the observed protons mass difference, need

\[ \beta^{(T=0)} = 6 \cdot 10^{-10} \]
GRAVITATIONALLY-INDUCED CPT VIOLATION
GRAVITATIONAL BACKGROUNDs Generating CPT Violating Effects In the Early Universe: Particle-Antiparticle Differences in Dispersion Relations → Differences in populations → freeze out → Baryogenesis or → Leptogenesis → Baryogenesis
GRAVITATIONAL BACKGROUNDs GENERATING CPT VIOLATING EFFECTS IN THE EARLY UNIVERSE:
PARTICLE-ANTIPARTICLE DIFFERENCES IN DISPERSION RELATIONS
Differences in populations → freeze out → Baryogenesis or
→ Leptogenesis → Baryogenesis
Gravitational Baryogenesis

Quantum Gravity (or something else (e.g. SUGRA)) may lead at low-energies (below Planck scale or a scale $M_\ast$) to a term in the effective Lagrangian (in curved back space-time backgrounds):

$$\frac{1}{M_\ast^2} \int d^4x \sqrt{-g} (\partial_\mu \mathcal{R}) J^\mu$$

Generation (flavour) #

$$\partial_\mu J_\mu^B = \partial_\mu J_\mu^L = \frac{n_f}{32\pi^2} \text{Tr} F_{\mu\nu} \tilde{F}_{\mu\nu} + U(1) \text{ part}$$

SU(2)

Current e.g. baryon-number $J_\mu^B$ current (non-conserved in Standard Model due to anomalies)
Quantum Gravity (or something else (e.g. SUGRA)) may lead at low-energies (below Planck scale or a scale $M_*$) to a term in the effective Lagrangian (in curved back space-time backgrounds):

$$
\frac{1}{M_*^2} \int d^4 x \sqrt{-g} (\partial_\mu R) J^\mu
$$

Term Violates CP but is CPT conserving \textit{in vacuo}.
It \textbf{Violates CPT} in the background space-time of a expanding FRW Universe.

$$
\dot{R} = -(1 - 3w) \frac{\dot{\rho}}{M_P^2} = \sqrt{3} (1 - 3w) (1 + w) \frac{\rho^{3/2}}{M_P^3}
$$

Energy differences between particle vs antiparticle \( \pm \frac{\dot{R}}{M_*^2} \): \textit{Dynamical CPTV}

\textit{LIKE A CHEMICAL POTENTIAL FOR FERMIONS}
Gravitational Baryogenesis

Quantum Gravity (or something else (e.g. SUGRA)) may lead at low-energies (below Planck scale or a scale $M_*$) to a term in the effective Lagrangian (in curved back space-time backgrounds):

$$\frac{1}{M_*^2} \int d^4x \sqrt{-g} (\partial_\mu \mathcal{R}) J^\mu$$

Term Violates CP but is CPT conserving \textit{in vacuo}.

It \textbf{Violates CPT} in the background space-time of an \textit{expanding FRW Universe}:

$$\dot{\mathcal{R}} = -(1 - 3w) \frac{\dot{\rho}}{M_P^2} = \sqrt{3} (1 - 3w)(1 + w) \frac{\rho^{3/2}}{M_P^3}$$

Energy differences between particle vs antiparticle:

$$\pm \frac{\dot{\mathcal{R}}/M_*^2}{M_*^2}$$

\textbf{Baryon Asymmetry} $\frac{n_B}{s} \approx \left. \frac{\dot{\mathcal{R}}}{M_*^2 T} \right|_{T_D}$, calculate for various $w$ in some scenarios.

@ $T < T_D$, $T_D =$ Decoupling $T$
GRAVITATIONAL BACKGROUNDS
GENERATING CPT VIOLATING EFFECTS
IN THE EARLY UNIVERSE:
PARTICLE-ANTIPARTICLE DIFFERENCES
IN DISPERSION RELATIONS →
Differences in populations →
freeze out → Baryogenesis or
→ Leptogenesis → Baryogenesis

REVIEW VARIOUS SCENARIOS
GRAVITATIONAL BACKGROUNDs generating CPT VIOlating EFFECTs in the EArly universe:

PARTICLE-ANTIPARTICLE DIFFERENCES in DISPERSION RELATIONS →

Differences in populations → freeze out → Baryogenesis or → Leptogenesis → Baryogenesis

B-L conserving GUT or Sphaleron

REVIEW VARIOUS SCENARIOS
NB: (}
Heavy Right-handed Majorana neutrinos enter equilibrium at \( T = T_{eq} > T_{\text{decay}} \)

Lepton number violation

\[ N_I \rightarrow H\nu, \ H\bar{\nu} \]

Out of Equilibrium Decays

\[ T \sim T_{\text{decay}} > T_{\text{sph}} \]

Enhanced CP V

Produce Lepton asymmetry

Equilibrated electroweak B+L violating sphaleron interactions

Independent of Initial Conditions

Observed Baryon Asymmetry In the Universe (BAU)

Estimate BAU by solving Boltzmann equations for Heavy Neutrino Abundances

\[ L = \frac{2}{M} l_L l_L \phi \phi + \text{H.c.} \]

where

\[ l_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \]
CPT Violating Thermal Leptogenesis

Early Universe $T > 10^{15}$ GeV

No need for enhanced CPV from Heavy Right-handed Majorana neutrinos?

CPT Violation already in thermal equilibrium

Produce Lepton asymmetry

Equilibrated electroweak $B+L$ violating sphaleron interactions

Independent of Initial Conditions

Observed Baryon Asymmetry In the Universe (BAU)

Estimate BAU by fixing CPTV background parameters In some models this may imply fine tuning ....

$$L = \frac{2}{M} l_L l_L \phi \phi + \text{H.c.}$$

where

$$l_L = \begin{pmatrix} \nu_e \\ e \\ \nu_\mu \\ \mu \\ \nu_\tau \\ \tau \end{pmatrix}_L$$
NB: )
2. CPTV Effects of different Space-Time-Curvature/Spin couplings between neutrinos/antineutrinos

**B. Mukhopadhyay, U. Debnath, N. Dadhich, M. Sinha Lambiase, Mohanty**

**Curvature** Coupling to **fermion spin** may lead to different dispersion relations between neutrinos and antineutrinos (assumed **dominant** in the Early eras) in **non-spherically symmetric** geometries in the Early Universe.
Dirac Lagrangian

\[ \mathcal{L} = \sqrt{-g} \left( i \bar{\psi} \gamma^a D_a \psi - m \bar{\psi} \psi \right) \]

\[ D_a = \left( \partial_a - \frac{i}{4} \omega_{bca} \sigma^{bc} \right), \]

\[ \omega_{bca} = e_{b\lambda} \left( \partial_a e^\lambda_c + \Gamma^\lambda_{\gamma\mu} e_\gamma^c e^\mu_a \right). \]

Gravitational covariant derivative including spin connection

\[ \sigma^{ab} = \frac{i}{2} [\gamma^a, \gamma^b] \]

\[ \mathcal{L} = \mathcal{L}_f + \mathcal{L}_I = \sqrt{-g} \bar{\psi} \left[ (i \gamma^a \partial_a - m) + \gamma^a \gamma^5 B_a \right] \psi, \]

\[ B^d = \epsilon^{abcd} e_{b\lambda} \left( \partial_a e^\lambda_c + \Gamma^\lambda_{\alpha\mu} e^\alpha_c e^\mu_a \right) \]
Dirac Lagrangian (for concreteness, it can be extended to Majorana neutrinos)

\[ \mathcal{L} = \sqrt{-g} \left( i \bar{\psi} \gamma^a D_a \psi - m \bar{\psi} \psi \right) \]

\[ D_a = \left( \partial_a - \frac{i}{4} \omega_{bca} \sigma^{bc} \right), \]

\[ \omega_{bca} = e_b \lambda \left( \partial_a e^\lambda_c + \Gamma^\lambda_{\gamma \mu} e^\gamma_c e^\mu_a \right). \]

Gravitational covariant derivative including spin connection

\[ \sigma^{ab} = \frac{i}{2} \left[ \gamma^a, \gamma^b \right] \]

for the Majorana neutrinos, above \( \mathcal{L}_I \) turns out explicitly as

\[ \mathcal{L}_I = \psi_L^\dagger \gamma^a \psi_L B_a, \quad \mathcal{L}_I = -\bar{\psi}_L^c \gamma^a \psi^c_L B_a \]

\[ B^d = \epsilon^{abcd} e_b \lambda \left( \partial_a e^\lambda_c + \Gamma^\lambda_{\alpha \mu} e^\alpha_c e^\mu_a \right) \]
Dirac Lagrangian (for concreteness, it can be extended to Majorana neutrinos)

\[ \mathcal{L} = \sqrt{-g} \left( i \bar{\psi} \gamma^a D_a \psi - m \bar{\psi} \psi \right) \]

\[ D_a = \left( \partial_a - \frac{i}{4} \omega_{bca} \sigma^{bc} \right), \]

\[ \omega_{bca} = e_b \gamma (\partial_a e_c^\lambda + \Gamma^\lambda_{\gamma \mu} e_\gamma^c e^\mu_a). \]

Gravitational covariant derivative including spin connection

\[ \sigma^{ab} = \frac{i}{2} [\gamma^a, \gamma^b] \]

\[ \mathcal{L} = \mathcal{L}_f + \mathcal{L}_I = \sqrt{-g} \bar{\psi} \left[ (i \gamma^a \partial_a - m) + \gamma^a \gamma^5 B_a \right] \psi, \]

\[ B^d = \epsilon^{abcd} e_b \gamma (\partial_a e_c^\lambda + \Gamma^\lambda_{\alpha \mu} e_\alpha^c e^\mu_a) \]

For homogeneous and isotropic Friedman-Robertson-Walker geometries the resulting \( B^\mu \) vanish
Dirac Lagrangian (for concreteness, it can be extended to Majorana neutrinos)

\[ \mathcal{L} = \sqrt{-g} \left( i \bar{\psi} \gamma^a D_a \psi - m \bar{\psi} \psi \right) \]

\[ D_a = \left( \partial_a - \frac{i}{4} \omega_{bca} \sigma^{bc} \right), \]

\[ \omega_{bca} = e_b \lambda \left( \partial_a e^\lambda_c + \Gamma^\lambda_{\gamma\mu} e^\gamma_c e^\mu_a \right). \]

Gravitational covariant derivative including spin connection

\[ \sigma^{ab} = \frac{i}{2} [\gamma^a, \gamma^b] \]

\[ \mathcal{L} = \mathcal{L}_f + \mathcal{L}_I = \sqrt{-g} \bar{\psi} \left[ (i \gamma^a \partial_a - m) + \gamma^a \gamma^5 B_a \right] \psi, \]

\[ B^d = \epsilon^{abcd} e_b \lambda \left( \partial_a e^\lambda_c + \Gamma^\lambda_{\alpha\mu} e^\alpha_c e^\mu_a \right) \]

Can be constant in a given local frame in Early Universe axisymmetric (Bianchi) cosmologies or near rotating Black holes
DISPERSION RELATIONS OF NEUTRINOS ARE **DIFFERENT** FROM THOSE OF ANTINEUTRINOS IN SUCH GEOMETRIES

\[(p_a \pm B_a)^2 = m^2, \quad \pm \text{refers to chiral fields (here neutrino/antineutrino)}\]

**CPTV Dispersion relations**

\[
E = \sqrt{(\vec{p} - \vec{B})^2 + m^2 + B_0}, \quad \bar{E} = \sqrt{(\vec{p} + \vec{B})^2 + m^2 - B_0}
\]

but (bare) masses are equal between particle/anti-particle sectors

**NOT** the Effective fermion/antifermion masses \((p=0)\)

\[
m_{F \pm}^{\text{eff}} = m \pm B_0
\]

**Abundances** of neutrinos in Early Universe, then, **different** from those of antineutrinos if \(B_0\) is **non-trivial**.
**Abundances** of neutrinos in Early Universe *different* from those of antineutrinos if $B_0 \neq 0$

\[
\Delta n = \frac{g}{(2\pi)^3} \int d^3p \left[ \frac{1}{1 + \exp(E_\nu/T)} - \frac{1}{1 + \exp(E_\bar{\nu}/T)} \right].
\]

\[
\Delta n = \frac{g}{(2\pi)^2} T^3 \int_0^\infty \int_0^\pi \left[ \frac{1}{1 + e^{u e B_0/T}} - \frac{1}{1 + e^{u e -B_0/T}} \right] u^2 d\theta du,
\]

\[
u = |\vec{p}|/T
\]

\[
\Delta n_\nu \equiv n_\nu - n_{\bar{\nu}} \sim g^* T^3 \left( \frac{B_0}{T} \right)
\]

with $g^*$ the number of degrees of freedom for the (relativistic) neutrino.
Case I: BARYOGENESIS VIA GUT LEPTOGENESIS

\[ \Delta n_\nu \equiv n_\nu - n_\bar{\nu} \sim g^* T^3 \left( \frac{B_0}{T} \right) \]

@ \( T = T_d \) (decoupling Temp. of Lepton number (L) Violating processes) there is a constant ratio of net neutrino/antineutrino asymmetry (\( \Delta L \)) to entropy density (\( \sim T^3 \))

\[ \Delta L (T < T_d) = \frac{\Delta n_\nu}{s} \sim \frac{B_0}{T_d} \]

for \( T_d \sim 10^{15} \) GeV and \( B_0 \sim 10^5 \) GeV

\( \Delta L \sim 10^{-10} \), in agreement with observations (Leptogenesis)

Communicated to Baryon sector, and thus generates BAU either via a B-L conserving symmetry as in GUT models or via B + L conserving sphaleron processes \( \rightarrow BARYOGENESIS \)
Case II: Black-Hole induced neutrino-antineutrino population difference

\[ \Delta n = \frac{g}{(2\pi)^3} \int_{R_i}^{R_f} dV \int d^3 \vec{p} \left[ \frac{1}{1 + \exp(E_\nu/T)} - \frac{1}{1 + \exp(E_\bar{\nu}/T')} \right] \]

Consider Kerr black holes for which \( \vec{B} \cdot \vec{p} \ll B_0 p^0 \) and show that

\[ \Delta n = \frac{g}{(2\pi)^2} T^3 \int_{R_i}^{R_f} \int_0^\infty \int_0^\pi \left[ \frac{1}{1 + e^{u e^{-B_0/T}}} - \frac{1}{1 + e^{u e^{-B_0/T}}} \right] u^2 d\theta du dV \]

\[ u = |\vec{p}|/T \]

Then, if \( B^0 \ll T \)

\[ \Delta n \sim g T^3 \left( \frac{B_0}{T} \right) \]
Asymmetry depends on the sign of $B^0$

PRIMORDIAL BLACK HOLES WITH MASSES $M_{BH} < 10^{15}$ gm have evaporated today, only BH with masses $M_{BH} > 10^{15}$ gm may survive today

Hawking temperature

$$T = \frac{\hbar}{8\pi k_B M} \sim 10^{-7} K \left( \frac{M_{\odot}}{M} \right)$$

$$T \sim 10^{11} \text{ K} \sim 1.6 \times 10^{-5} \text{ erg}, \overline{B_0} \sim 1.6 \times 10^{-6} \text{ erg}, \text{ then } \Delta n \sim 10^{-16}.$$ 

To reproduce observed Baryon asymmetry $\Delta n = O(10^{-10})$ we need $10^6$ BH with the same sign of $B^0 \Rightarrow \text{fine tuning} \ldots$
3. Fermions in Gravity with TORSION

Dirac Lagrangian (for concreteness, it can be extended to Majorana neutrinos)

\[ \mathcal{L} = \sqrt{-g} \left( i \bar{\psi} \gamma^a D_a \psi - m \bar{\psi} \psi \right) \]

\[ D_a = \left( \partial_a - \frac{i}{4} \omega_{bca} \sigma^{bc} \right), \]

\[ \omega_{bca} = e_b \lambda \left( \partial_a e_c^\lambda + \Gamma_{\gamma \mu}^\lambda e_c^{\gamma} e_\mu^a \right). \]

Gravitational covariant derivative including spin connection

\[ \sigma^{ab} = \frac{i}{2} [\gamma^a, \gamma^b] \]

If torsion then \( \Gamma_{\mu \nu} \neq \Gamma_{\nu \mu} \)

antisymmetric part is the contorsion tensor, contributes to

\[ e_\mu^a e_\nu^b \eta^{a b} = g^{\mu \nu} \]

vielbeins (tetrads) independent from spin connection \( \omega_{\mu}^{ab} \)

now ....
ROLE OF Kalb-Ramond H-FIELD AS TORSION

EFFECTIVE GRAVITATIONAL ACTION IN STRING LOW-ENERGY LIMIT CAN BE EXPRESSED IN TERMS OF A GENERALIZED CURVATURE RIEMANN TENSOR WHERE THE CHRISTOFFEL CONNECTION INCLUDES H-FIELD TORSION

\[ S^{(4)} = \int d^4 x \sqrt{-g} \left( \frac{1}{2\kappa^2} R - \frac{1}{6} H_{\mu\nu\rho} H^{\mu\nu\rho} \right) \]
\[ = \int d^4 x \sqrt{-g} \left( \frac{1}{2\kappa^2} \bar{R} - \frac{1}{3} \kappa^2 H_{\mu\nu\rho} H^{\mu\nu\rho} \right) \]

\[ \Gamma^\mu_{\nu\rho} = \Gamma^\mu_{\nu\rho} + \frac{\kappa}{\sqrt{3}} H^\mu_{\nu\rho} \neq \Gamma^\mu_{\rho\nu} \]

IN 4-DIM DEFINE DUAL OF H AS:

\[ -3 \sqrt{2} \partial_\sigma b = \sqrt{-g} \epsilon_{\mu\nu\rho\sigma} H^{\mu\nu\rho} \]

\[ b(x) = \text{Pseudoscalar (Kalb-Ramond (KR) axion)} \]
$$S_\psi = \frac{i}{2} \int d^4 x \sqrt{-g} \left( \overline{\psi} \gamma^\mu \overline{D}_\mu \psi - (\overline{D}_\mu \overline{\psi}) \gamma^\mu \psi \right)$$

TORSIONFUL CONNECTION, FIRST-ORDER FORMALISM

$$\overline{D}_\mu = \overline{\nabla}_\mu - ieA_\mu$$

$$\overline{\omega}_{ab\mu} = \omega_{ab\mu} + K_{ab\mu}$$

gauge field

contorsion

$$K_{abc} = \frac{1}{2} \left( T_{cab} - T_{abc} - T_{bca} \right)$$

Non-trivial contributions to $B^\mu$

$$B^d = \epsilon^{abcd} e_b^\lambda \left( \partial_\lambda e_c^\alpha e_c^\mu + \Gamma_\alpha^\lambda e_c^\alpha e^\mu_\alpha \right)$$

$$H_{cab}$$

$$\overline{\Gamma}_{\nu\rho}^\mu = \Gamma_{\nu\rho}^\mu + \frac{\kappa}{\sqrt{3}} H_{\nu\rho}^\mu \neq \overline{\Gamma}_{\rho\nu}^\mu$$
FERMIONS COUPLE TO H –TORSION VIA GRAVITATIONAL COVARIANT DERIVATIVE

\[ S_\psi = \frac{i}{2} \int d^4 x \sqrt{-g} \left( \overline{\psi} \gamma^\mu \overline{D}_\mu \psi - (\overline{D}_\mu \overline{\psi}) \gamma^\mu \psi \right) \]

TORSIONFUL CONNECTION, FIRST-ORDER FORMALISM

\[ \overline{D}_\mu = \overline{\nabla}_\mu - i e A_\mu \]

\[ \overline{\omega}_{ab\mu} = \omega_{ab\mu} + K_{ab\mu} \]

\[ K_{abc} = \frac{1}{2} \left( T_{cab} - T_{abc} - T_{bca} \right) \]

Non-trivial contributions to \( B^\mu \)

\[ B^d = \epsilon^{abcd} e_b \lambda \left( \partial_a e_c^\lambda + \Gamma_\alpha^\lambda_{a\mu} e_\alpha^\mu e_\lambda^\mu \right) \]

\[ \Gamma^\mu_{\nu\rho} = \Gamma^\mu_{\nu\rho} + \frac{\kappa}{\sqrt{3}} H^\mu_{\nu\rho} \neq \Gamma^\mu_{\rho\nu} \]

Constant \( H ? \)
**Cosmological Solutions**, non-trivial time-dependent dilatons, axions

In Einstein frame $E$ (Scalar curvature term in gravitational effective action has canonical normalisation):

\[
\begin{align*}
    ds^2 &= g^E_{\mu\nu}(x)dx^\mu dx^\nu = dt^2 - a(t)^2 \delta_{ij} dx^i dx^j \\
    a(t) &= t \\
    \Phi &= -\ln a(t) + \phi_0
\end{align*}
\]

\[
H_{\mu\nu\rho} = e^{2\Phi} \epsilon_{\mu\nu\rho\sigma} \partial^\sigma b(x) \\
b(x) = \sqrt{2}e^{-\phi_0} \sqrt{Q^2} \frac{M_s}{\sqrt{n}} t
\]

Central charge of underlying world-sheet conformal field theory $n \in \mathbb{Z}^+$

\[
c = 4 - 12Q^2 - \frac{6}{n+2} + c_I
\]

``internal'' dims central charge

Kac-Moody algebra level
Exact (conformal Field Theories on World-sheet) Solutions from String theory

**Cosmological Solutions**, non-trivial time-dependent dilatons, axions

In Einstein frame $E$ (Scalar curvature term in gravitational effective action has canonical normalisation):

$$ds^2 = g^{E}_{\mu\nu}(x) dx^\mu dx^\nu = dt^2 - a(t)^2 \delta_{ij} dx^i dx^j$$

$$a(t) = t$$

$$\Phi = -\ln a(t) + \phi_0$$

$$H_{\mu\nu\rho} = e^{2\Phi} \epsilon_{\mu\nu\rho\sigma} \partial^\sigma b(x)$$

$$b(x) = \sqrt{2} e^{-\phi_0} \sqrt{Q^2} \frac{M_s}{\sqrt{n}} t$$

Central charge of underlying world-sheet conformal field theory

$$c = 4 - 12Q^2 - \frac{6}{n+2} + c_I$$

``internal'' dims central charge

Kac-Moody algebra level

$n \in \mathbb{Z}^+$
Covariant Torsion tensor

\[
\bar{\Gamma}^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} + e^{-2\Phi} H^\lambda_{\mu\nu} \equiv \Gamma^\lambda_{\mu\nu} + T^\lambda_{\mu\nu}
\]

Torsion tensor

\[
T_{ijk} \sim \epsilon_{ijk} \dot{b} = \epsilon_{ijk} \sqrt{2Q^2} e^{-\phi_0} \frac{M_s}{\sqrt{n}}
\]

Constant $B^0$

Lepton Asymmetry as in previous cases, e.g. for neutrinos

\[
\Delta L(T < T_d) = \frac{\Delta n_\nu}{s} \sim \frac{B_0}{T_d}
\]

Requires

\[
B^0 \sim 10^5 \text{ GeV} \quad \text{at} \quad T_d
\]
4. STRINGY SPACE-TIME
D(efect)-FOAM & CPTV

BRANE-WORLDS with D-PARTICLE (POINT-LIKE BRANE) DEFECTS

Six tiny dimensions curled up into Calabi-Yau shapes

Open strings are attached to the brane

Closed strings are free to move through bulk

D-particle defect

(Standard Model particles)

(Gravitons)

Recoil of defect

Mass $\sim \frac{M_s}{g_s}$

Our Universe NO LONGER Lorentz Invariant

J ELLIS, NEM, WESTMUCKETT
Colliding Brane world model of Space-Time with point-like space-time defects
String **Splitting** & (``momentary'') **Capture** by the defects

**Matter/D-foam Interactions**

**Intermediate String Creation/Exchange**

**Purely** **Non-local Effect**
CHARGE CONSERVATION MUST BE RESPECTED DURING STRING SPLITTING, INTERMEDIATE CREATION AND STRETCHING:

ONLY ELECTRICALLY NEUTRAL EXCITATIONS (e.g. Photons, Neutrinos) INTERACT VIA CAPTURE DOMINANTLY WITH FOAM

ELLIS, NEM, SAKHAROV, NANOPoulos
CHARGE CONSERVATION MUST BE RESPECTED DURING STRING SPLITTING, INTERMEDIATE CREATION AND STRETCHING:

ONLY ELECTRICALLY NEUTRAL EXCITATIONS (e.g. Photons, Neutrinos) INTERACT VIA CAPTURE DOMINANTLY WITH FOAM DEFECT RECOIL OCCURS

*Time Delays* due to Intermediate String Creation & Oscillations – *Subluminal Vacuum Refractive Index*

J ELLIS, NEM, NANOPOLLOS
Explicit local breaking of SO(3,1) down to SO(2,1) rotation and boosts in transverse directions

Induced metric depends on momenta as well as coordinates (Finsler type): e.g. $u \parallel X_1$

$$h_{01} = g_s \frac{\Delta k_i}{M_s} \equiv u_1$$

``Frame Dragging by recoiling D-particle``
**Space time Foam situations** – Average over both populations of defects & quantum fluctuations

Isotropic & (in)homogeneous foam

for a brane observer:

\[
\langle u_i \rangle \equiv \frac{g_s}{M_s} \langle \Delta k_i \rangle = 0
\]

\[
\frac{g_s^2}{M_s^2} \langle \Delta k_i \Delta k_j \rangle = \sigma^2 \delta_{ij}
\]

Lorentz Invariance on Average

Violated in flcts

\[
g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}
\]

\[
\langle h_{\mu\nu} \rangle = 0
\]

\[
\langle h_{\mu\nu} h_{\rho\sigma} \rangle \neq 0
\]

c.f. Stochastic Foam, through coherent graviton states

leading to light cone fluctuations

Ford (95)
D-foam Induced CPTV for Neutrinos

\[ \langle E_\nu \rangle = \sqrt{p^2 + m_{\nu}^2} \left( 1 + \frac{1}{2} \sigma^2 \right) - \frac{1}{2} \frac{M_s}{g_s} \sigma^2 \]

\[ \langle E_{\bar{\nu}} \rangle = \sqrt{p^2 + m_{\bar{\nu}}^2} \left( 1 + \frac{1}{2} \sigma^2 \right) + \frac{1}{2} \frac{M_s}{g_s} \sigma^2 \]
D-foam Induced CPTV for Neutrinos

\[
\ll E_\nu \gg = \sqrt{p^2 + m_\nu^2} \left( 1 + \frac{1}{2} \sigma^2 \right) - \frac{1}{2} \frac{M_s}{g_s} \sigma^2
\]

\[
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\]

cf. "Frame Dragging" by recoiling D-particle
D-foam Induced CPTV for Neutrinos

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One can thus generate a Lepton asymmetry and, then through B-L conserving processes in the Early Universe a Baryon asymmetry.
D-foam Induced CPTV for Neutrinos

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\[ \langle E_{\bar{\nu}} \rangle = \sqrt{p^2 + m_{\bar{\nu}}^2} \left( 1 + \frac{1}{2} \sigma^2 \right) + \frac{1}{2} \frac{M_s}{g_s} \sigma^2 \pm B_0 \text{ constant } > 0 \text{ if } \sigma^2 \approx \text{const} \text{ in an era} \]

One can thus generate a Lepton asymmetry and, then through B-L conserving processes in the Early Universe a Baryon asymmetry.

Correct Sign for Matter dominance over Antimatter due to Energetics no Fine Tuning
D-foam Induced CPTV for Neutrinos

\[
\ll E_\nu \rr = \sqrt{p^2 + m_{\nu}^2} \left( 1 + \frac{1}{2} \sigma^2 \right) - \frac{1}{2} \frac{M_s}{g_s} \sigma^2
\]

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\]

\[\pm B_0 \text{ constant } > 0 \]
if \(\sigma^2 \approx \text{const} \)
in an era

\[
\Delta n = \frac{g}{(2\pi)^3} \int d^3p \left[ \frac{1}{1 + \exp(E_\nu/T)} - \frac{1}{1 + \exp(E_{\bar{\nu}}/T)} \right]
\]

\[
\Delta n_\nu \equiv n_\nu - n_{\bar{\nu}} \sim g^* T^3 \left( \frac{B_0}{T} \right)
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\Delta L(T < T_d) = \frac{\Delta n_\nu}{s} \sim \frac{B_0}{T_d}
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One can thus generate a Lepton asymmetry and through B+L conserving processes in the Early Universe a Baryon asymmetry. The correct value (observed) for BAU is reproduced for, e.g. GUTs

$$\frac{1}{2} \frac{M_s}{g_s} \sigma^2 \sim 10^5 \text{ GeV}$$

for D-foam at $T_d \sim 10^{15} \text{ GeV}$

implying that in these scenarios, for $\sigma^2 < 1$, one must have $M_s/g_s > 200 \text{ TeV}$
D-foam Induced CPTV for Neutrinos

\[ \langle E_\nu \rangle = \sqrt{p^2 + m_{\nu}^2} \left( 1 + \frac{1}{2} \sigma^2 \right) - \frac{1}{2} \frac{M_s}{g_s} \sigma^2 \]

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implying that in these scenarios, for \(\sigma^2 < 1\), one must have \(M_s/g_s > 200 \text{ TeV}\)

PHENOMENOLOGY OF EARLY UNIVERSE NEEDS TO BE CHECKED FOR COMPATIBILITY…. IN PROGRESS
Construct Microscopic Quantum Gravity models with strong CPT Violation in Early Universe, but maybe weak today… Fit with all available data… Estimate in this way matter-antimatter asymmetry in Universe.
CONCLUSIONS-OUTLOOK

- **Neutrinos (Sterile)** may explain matter-antimatter origin in the Universe
- May also provide interesting Dark matter Candidates
- **Neutrino condensates** may contribute to dark energy
• Neutrinos (Sterile) may explain matter-antimatter origin in the Universe
• May also provide interesting Dark matter Candidates
• Neutrino condensates may contribute to dark energy

• Gravitationally-induced anomalous Right-handed Majorana neutrino masses possible, beyond see-saw…
Neutrinos (Sterile) may explain matter-antimatter origin in the Universe

May also provide interesting Dark matter Candidates

Neutrino condensates may contribute to dark energy

Gravitationally-induced anomalous Right-handed Majorana neutrino masses possible, beyond see-saw...

Interesting CPTV Physics for the Early Universe to be investigated
Neutrinos (Sterile) may explain matter-antimatter origin in the Universe

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THANK YOU!
SPARES
Neutrinos (Sterile) may explain matter-antimatter origin in the Universe

May also provide interesting Dark matter Candidates

Neutrino condensates may contribute to dark energy

Mass Varying $\nu$

Gravitationally-induced anomalous Right-handed Majorana neutrino masses possible, beyond see-saw...

Interesting CPTV Physics for the Early Universe to be investigated
OTHER INTERESTING TOPICS

Mass Varying neutrinos & the Dark Sector

Couple Scalar cosmic fields with potential $U(\phi, T)$ and massless fermions $\psi$ through Yukawa couplings

\[
S = S^E_B + S^E_D|_{m=0} + g \int_0^\beta d\tau \int a^3 d^3 x \, \phi \bar{\psi} \psi \\
S^E_B = \int_0^\beta d\tau \int a(t)^3 d^3 x \left[ \frac{1}{2} (\partial_\tau \phi)^2 + \frac{1}{2a^2} (\nabla \phi)^2 + U(\phi) \right]
\]

Fermion mass:

\[
m = g \phi_c
\]
Mass Varying neutrinos & the Dark Sector

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$$S^E_B = \int_0^{\beta} d\tau \int a(t)^3 d^3 x \left[ \frac{1}{2} (\partial_\tau \phi)^2 + \frac{1}{2\alpha^2} (\nabla \phi)^2 + U(\phi) \right]$$

Fermion mass: $m = g \phi_c$ Minimum of action
OTHER INTERESTING TOPICS

Mass *Varying* neutrinos & the Dark Sector

Nelson, Fardon, Weiner Chitov, August, Natarajan, Kahniashvili

Couple Scalar cosmic fields with potential $U(\phi, T)$ and massless fermions $\psi$ through Yukawa couplings

\[
S = S_B^E + S_D^E \bigg|_{m=0} + g \int_0^\beta d\tau \int a^3 d^3 x \ \varphi \tilde{\psi} \psi
\]

\[
S_B^E = \int_0^\beta d\tau \int a(t)^3 d^3 x \left[ \frac{1}{2}(\partial_\tau \varphi)^2 + \frac{1}{2a^2}(\nabla \varphi)^2 + U(\varphi) \right]
\]

Thermodynamic potential density

Fermion mass:

\[
m = g\phi_c
\]

\[
\Omega(\phi_c) = U(\phi_c) + \Omega_D(\phi_c)
\]

\[
Z_D \equiv \text{Tr} e^{-\beta(\hat{H} - \mu \hat{\phi})} = \int D\bar{\psi} D\psi e^{-S_D^E}
\]

\[
\Omega_D \equiv -\frac{1}{\beta a^3 V} \log Z_D
\]
OTHER INTERESTING TOPICS

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\[
\Omega_D \equiv -\frac{1}{\beta a^3 V} \log Z_D
\]

*comoving volume*
OTHER INTERESTING TOPICS

Mass *Varying* neutrinos & the Dark Sector

Couple Scalar cosmic fields with potential $U(\varphi, T)$ and massless fermions $\psi$ through Yukawa couplings

$$S = S^E_B + S^E_D|_{m=0} + g \int_0^\beta d\tau \int a^3 d^3 x \ \varphi \overline{\psi} \psi$$

$$S^E_B = \int_0^\beta d\tau \int a(t)^3 d^3 x \left[ \frac{1}{2} (\partial_\tau \varphi)^2 + \frac{1}{2a^2} (\nabla \varphi)^2 + U(\varphi) \right]$$

Fermion mass:

$$m = g \phi_c$$

$$\Omega(\phi_c) = U(\phi_c) + \Omega_D(\phi_c)$$

$$\frac{\partial \Omega(\varphi)}{\partial \varphi} \bigg|_{\varphi=\phi_c} = 0$$

$$\phi_c = \langle \varphi \rangle$$

$$\frac{\partial^2 \Omega(\varphi)}{\partial \varphi^2} \bigg|_{\varphi=\phi_c} > 0$$

$$T$$ dependent!
OTHER INTERESTING TOPICS

Mass Varying neutrinos & the Dark Sector

Couple Scalar cosmic fields with potential $U(\phi, T)$ and massless fermions $\psi$ through Yukawa couplings

$$S = S_B^E + S_D^E \bigg|_{m=0} + g \int_0^\beta d\tau \int a^3 d^3 x \varphi \bar{\psi}\psi$$

$$S_B^E = \int_0^\beta d\tau \int a(t)^3 d^3 x \left[ \frac{1}{2} (\partial_\tau \varphi)^2 + \frac{1}{2a^2} (\nabla \varphi)^2 + U(\varphi) \right]$$

Fermion mass:

$$m = g\phi_c$$

$$\frac{\partial \Omega(\varphi)}{\partial \varphi} \bigg|_{\varphi=\phi_c} = 0$$

$$\frac{\partial^2 \Omega(\varphi)}{\partial \varphi^2} \bigg|_{\varphi=\phi_c} > 0$$

$$\phi_c = \langle \varphi \rangle$$

Thermodynamic potential density

$$\Omega(\phi_c) = U(\phi_c) + \Omega_D(\phi_c)$$

$$\mathcal{Z}_D \equiv \text{Tr} e^{-\beta (\hat{H} - \mu \hat{Q})} = \int D\bar{\psi} D\psi e^{-S_D^E}$$

$$\Omega_D = -\frac{1}{\beta a^3 V} \log \mathcal{Z}_D$$

T dependent!
MASS-VARYING NEUTRINOS & COSMOLOGY

OTHER INTERESTING TOPICS

Mass \textit{Varying} neutrinos & the Dark Sector

Nelson, Fardon, Weiner Chitov, August, Natarajan, Kahniashvili

Couple Scalar cosmic fields with potential \( U(\varphi, T) \) and massless fermions \( \psi \) through Yukawa couplings

\[
\mathcal{S} = S_B^E + S_D^E \bigg|_{m=0} + g \int_0^\beta d\tau \int a^3 d^3 x \, \varphi \bar{\psi} \psi \\
S_B^E = \int_0^\beta d\tau \int a(t)^3 d^3 x \left[ \frac{1}{2} (\partial_\tau \varphi)^2 + \frac{1}{2a^2} (\nabla \varphi)^2 + U(\varphi) \right]
\]

Fermion mass:

\[
m = g \phi_c
\]

\[
\phi_c = \langle \varphi \rangle
\]

\[
\rho_s \equiv \frac{\langle \hat{N} \rangle}{V} = \frac{\partial \Omega_D}{\partial d} = \rho_0 + \frac{m}{\pi^2} \int_0^\infty \frac{k^2 dk}{\varepsilon(k)} \left[ n_F(\varepsilon_-) + n_F(\varepsilon_+) \right] \hat{N} = \int d^3 x \bar{\psi} \psi
\]
OTHER INTERESTING TOPICS

Mass *Varying* neutrinos & the Dark Sector

\[ U'(\phi_c) + g \rho_s = 0 \]

Fermion mass:
\[ m = g \phi_c \]

High T phase:
\[ \frac{m}{M} \approx \left( \sqrt{6\alpha} \frac{M}{T} \right)^{2/\alpha+2} \propto T^{-\frac{2}{\alpha+2}} \]

Fermionic contribution to thermodynamic potential dominant

Scalar mass
\[ m_\phi \approx \sqrt{\frac{\alpha + 1}{6}} T \]
OTHER INTERESTING TOPICS

Mass Varying neutrinos & the Dark Sector

\[ U'(\phi_c) + g\rho_s = 0 \]

Equation of state:

\[ U(\varphi) = \frac{M^{\alpha+4}}{\varphi^\alpha}, \quad \alpha > 0. \]

Towards \( w = -1 \) for low \( T \) (Cosmo. Const. like)
MASS-VARYING NEUTRINOS & COSMOLOGY

OTHER INTERESTING TOPICS

Mass *Varying* neutrinos & the Dark Sector

Neutrino Dark Energy evolution vs Dark Matter $\Omega_M$

$$M = 2.39 \cdot 10^{-3} \text{ eV (} \alpha = 0.01)$$

\[ \begin{array}{c}
\text{Graph showing evolution of } \Omega_{\phi V}, \Omega_Y, \text{ and } \Omega_M \\
\end{array} \]
OTHER INTERESTING TOPICS

Mass *Varying* neutrinos & the Dark Sector

\[ M = 2.39 \times 10^{-3} \text{ eV} \quad (\alpha = 0.01) \]

Equation of state of entire Universe including radiation contributions:

\[ P_{\text{tot}} = w_{\text{tot}} \rho_{\text{tot}} \]

\[ P_{\text{tot}} = P_\gamma + P_{\varphi \nu} \]

\[ P_\gamma = \frac{1}{3} \rho_\gamma \]

Towards \( w = -1 \) for low \( z \) (Cosmo. Const. like)
OTHER INTERESTING TOPICS

Mass Varying neutrinos & the Dark Sector

Neutrino mass evolution

\[ M = 2.39 \cdot 10^{-3} \text{ eV (} \alpha = 0.01) \]
OTHER INTERESTING TOPICS

Mass *Varying* neutrinos & the Dark Sector

Accetable Cosmology & Neutrino phenomenology

Nelson, Fardon, Weiner Chitov, August, Natarajan, Kahniashvili
Effective (Anti)Neutrino CPTV D-foam Mass

No mixing $\nu \rightarrow \bar{\nu}$

\[
m_{\nu}^{\text{eff}} = m_{\nu}(1 + \frac{1}{2}\sigma^2(T)) - \frac{M_s}{g_s}\sigma^2(T) \simeq m_{\nu} - \frac{M_s}{g_s}\sigma^2(T)
\]

\[
m_{\bar{\nu}}^{\text{eff}} = m_{\nu}(1 + \frac{1}{2}\sigma^2(T)) + \frac{M_s}{g_s}\sigma^2(T) \simeq m_{\nu} + \frac{M_s}{g_s}\sigma^2(T)
\]

\[
\frac{1}{2}\frac{M_s}{g_s}\sigma^2 \sim 10^5 \text{ GeV} \quad @ T_d \sim 10^{15} \text{ GeV}
\]

to generate BAU

Bounds from WMAP Cosmology ($Z < 1000$)

\[
\sigma^2(T) \sim \Delta^2(T)g_s^2\frac{\bar{p}^2}{M_s^2} \sim \frac{g_s^2}{M_s^2}\beta_0(1 + z)^3
\]

\[
\sum_{i=1}^{3} m_{\nu}^{\text{eff}} < 0.69 \text{ eV}
\]

Dust type D-particles

is a safe not strong bound on foam flcts $\sigma^2$ today (within current exp errors)
CPTV \rightarrow \text{neutrino/antineutrino mixing} \& \text{oscillations}

\[ \mathcal{L} = \text{det}(e) \bar{\Psi} \left( \frac{i}{2} \gamma^a \partial_a - m + \gamma^a \gamma^5 B_a \right) \Psi \]

\[ B^d = \epsilon^{abcd} \omega_{bca} \]

\[ E_\nu = \sqrt{(\vec{p} - \vec{B})^2 + m^2 + B_0}, \]

\[ E_{\nu c} = \sqrt{(\vec{p} + \vec{B})^2 + m^2 - B_0} \]

Assume Majorana neutrino in Weyl rep (Lepton number violation unavoidable)

\[ \Psi = \begin{pmatrix} \psi^c_L \\ \psi_L \end{pmatrix} \quad D_\mu \equiv (\partial_0, \partial_i + \gamma^5 B_i). \]

Majorana mass term violates L number

\[ (-g)^{-1/2} \mathcal{L} = (\psi^{\dagger} \psi^{\dagger}) \frac{i}{2} \gamma^0 \gamma^\mu \bar{D}_\mu \left( \begin{array}{c} \psi^c \\ \psi \end{array} \right) - (\psi^{\dagger} \psi^{\dagger}) \begin{pmatrix} -B_0 & -m \\ -m & B_0 \end{pmatrix} \begin{pmatrix} \psi^c \\ \psi \end{pmatrix} \]
\[ \mathcal{L} = \text{det}(e) \bar{\Psi} \left( \frac{i}{2} \gamma^a \hat{\partial}_a - m + \gamma^a \gamma^5 B_a \right) \Psi \]

\[ B^d = \varepsilon^{abcd} \omega_{bca} \]

\[ E_\nu = \sqrt{(\vec{p} - \vec{B})^2 + m^2 + B_0}, \]

\[ E_{\nu^c} = \sqrt{(\vec{p} + \vec{B})^2 + m^2 - B_0} \]

Assume Majorana neutrino in Weyl rep (Lepton number violation unavoidable)

\[ \Psi = \begin{pmatrix} \psi_L^c \\ \psi_L \end{pmatrix} \quad D_\mu \equiv (\partial_0, \partial_i + \gamma^5 B_i). \]

\[ (-g)^{-1/2} \mathcal{L} = (\psi^c \psi^\dagger) \frac{i}{2} \gamma^0 \gamma^\mu \leftrightarrow D_\mu \left( \begin{pmatrix} \psi^c \\ \psi \end{pmatrix} \right) - (\psi^c \psi^\dagger) \begin{pmatrix} -B_0 & -m \\ -m & B_0 \end{pmatrix} \begin{pmatrix} \psi^c \\ \psi \end{pmatrix} \]

Lead to neutrino/antineutrino mixing & oscillations

M Sinha & B. Mukhopadhyay
arXiv: 0704.2593
mass eigenstates $\nu_1$ and $\nu_2$ as

$$|\nu_1\rangle = \frac{1}{N} \left\{ \left( B_0 + \sqrt{B_0^2 + m^2} \right) |\psi^c\rangle + m |\psi\rangle \right\}$$

$$|\nu_2\rangle = \frac{1}{N} \left\{ -m |\psi^c\rangle + \left( B_0 + \sqrt{B_0^2 + m^2} \right) |\psi\rangle \right\}.$$

$$m_{1,2} = \mp \sqrt{B_0^2 + m^2}.$$

$$|\nu_1\rangle = \cos \theta |\psi^c\rangle + \sin \theta |\psi\rangle$$

$$|\nu_2\rangle = -\sin \theta |\psi^c\rangle + \cos \theta |\psi\rangle$$

$$m_{1,2} = \mp \sqrt{B_0^2 + m^2}.$$
mass eigenstates $\nu_1$ and $\nu_2$ as

$$\langle \nu_1 \rangle = \frac{1}{N} \left\{ \left( B_0 + \sqrt{B_0^2 + m^2} \right) \langle \psi^c \rangle + m \langle \psi \rangle \right\}$$

$$\langle \nu_2 \rangle = \frac{1}{N} \left\{ -m \langle \psi^c \rangle + \left( B_0 + \sqrt{B_0^2 + m^2} \right) \langle \psi \rangle \right\}.$$ 

$$m_{1,2} = \mp \sqrt{B_0^2 + m^2}.$$ 

$$\langle \nu_1 \rangle = \cos \theta \langle \psi^c \rangle + \sin \theta \langle \psi \rangle$$

$$\langle \nu_2 \rangle = -\sin \theta \langle \psi^c \rangle + \cos \theta \langle \psi \rangle$$

$$\tan \theta = \frac{m}{B_0 + \sqrt{B_0^2 + m^2}}.$$ 

$$\langle \psi^c \rangle = \cos \theta \langle \nu_1 \rangle - \sin \theta \langle \nu_2 \rangle$$

$$\langle \psi \rangle = \sin \theta \langle \nu_1 \rangle + \cos \theta \langle \nu_2 \rangle.$$ 

$$\lambda = \frac{\pi}{B_0 - |\vec{B}|}.$$ 

$${\mathcal P}(t) = \frac{m^2}{B_0^2 + m^2} \sin^2 \left\{ (B_0 - |\vec{B}|) t \right\} \sin^2 2\theta \sin^2 \delta(t)$$

$$\delta(t) = \frac{|E_\nu - E_\nu^c| t}{2}.$$ 

**NB:** neutrino CPTV mass shifts

neutrino/antineutrino mixing

oscillations

oscillation length
Modifications in Neutrinoless 2Beta decay rate in 2 flavour mixing (due to CPTV modified effective mass)

Majorana neutrino

Amplitude $A \propto \sqrt{B_{0}^{2} + m_{e}^{2}}$. 

ignore neutrino/antineutrino mixing here:

CPTV Effects of different Space-Time-Curvature/Spin couplings between $\nu$, $\bar{\nu}$ in Bianchi Cosmologies

B. Mukhopadhyay, U. Debnath, N. Dadhich, M. Sinha Lambiase, Mohanty

Assumption: Neutrinos non clustering properties, dominant species in early Universe

$$ds^2 = -dt^2 + S(t)^2 \, dx^2 + R(t)^2 \, [dy^2 + f(y)^2 \, dz^2] - S(t)^2 \, h(y) \, [2dx - h(y) \, dz] \, dz$$

Bianchi II, VIII and IX models, respectively $f(y)$ and $h(y)$ are given as

$$f(y) = \{y, \sinh y, \sin y\}, \quad h(y) = \{-y^2/2, - \cosh y, \cos y\}.$$  

$$B^0 = \frac{S[-f^2 R^2 (hf'R + Sh') + h^2 S^2 (hf'R + Sh') + 2fh \, RS (R f' - hh'S)]}{f^4 R^4 + f^2 h^2 R^2 S^2}$$

$$B^2 = \frac{h[-f^2 R^2 + 2fRS + h^2 S^2][RS' - R'S]}{f^3 R^4 + fh^2 R^2 S^2}.$$  

$$B^3 = B^1 = 0$$
Consider the metric of a Kerr (rotating) black hole

\[
\begin{align*}
    ds^2 &= \eta_{ij} \, dx^i \, dx^j - \left[ \frac{2\alpha}{\rho} \, s_i \, v_j + \alpha^2 \, v_i \, v_j \right] \, dx^i \, dx^j \\
    \alpha &= \frac{\sqrt{2Mr}}{\rho}, \quad \rho^2 = r^2 + \frac{a^2 z^2}{r^2} \quad v_i = \left( 1, \frac{ay}{a^2 + r^2}, \frac{-ax}{a^2 + r^2}, 0 \right) \\
    s_i &= \left( 0, \frac{rx}{\sqrt{r^2 + a^2}}, \frac{ry}{\sqrt{r^2 + a^2}}, \frac{z \sqrt{r^2 + a^2}}{r} \right)
\end{align*}
\]

and we have

\[
r^- - r^- \left( x^- + y^- + z^- - a^- \right) - a^2 z^2 = 0
\]
Modified Neutrino dispersion relations due to locally induced metric

\[ p^\mu p^\nu g_{\mu\nu} = -m^2 \Rightarrow \quad E = \vec{p} \cdot \vec{u} \pm \sqrt{p^2 + m^2 + (\vec{p} \cdot \vec{u})^2} \]

Interpret (Dirac hole theory) negative energies as corresponding to anti-particles ↔ Fermions, exclusion principle

\[ \ll E \rr = \ll \vec{p} \cdot \vec{u} \rr \pm \ll \sqrt{p^2 + m^2 + (\vec{p} \cdot \vec{u})^2} \rr \]

\[ \ll E \rr \approx \pm \sqrt{p^2 + m^2} \left( 1 + \frac{1}{2} \sigma^2 \right), \quad p \gg m \]

Momentum-Energy conservation during ν scattering with D-particles

\[ \ll \vec{p}_1 + \vec{p}_2 \rr = \frac{M_s}{g_s} \quad \ll \vec{u} \rr = 0 \]

\[ \ll E_1 \rr = \ll E_2 \rr + \frac{1}{2} \frac{M_s}{g_s} \ll u^2 \rr \quad \Rightarrow \]

\[ \ll E_2 \rr = \pm \sqrt{p^2 + m^2} \left( 1 + \frac{1}{2} \sigma^2 \right) - \frac{1}{2} \frac{M_s}{g_s} \sigma^2 \]

NEM, Sarkar, Tarantino
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