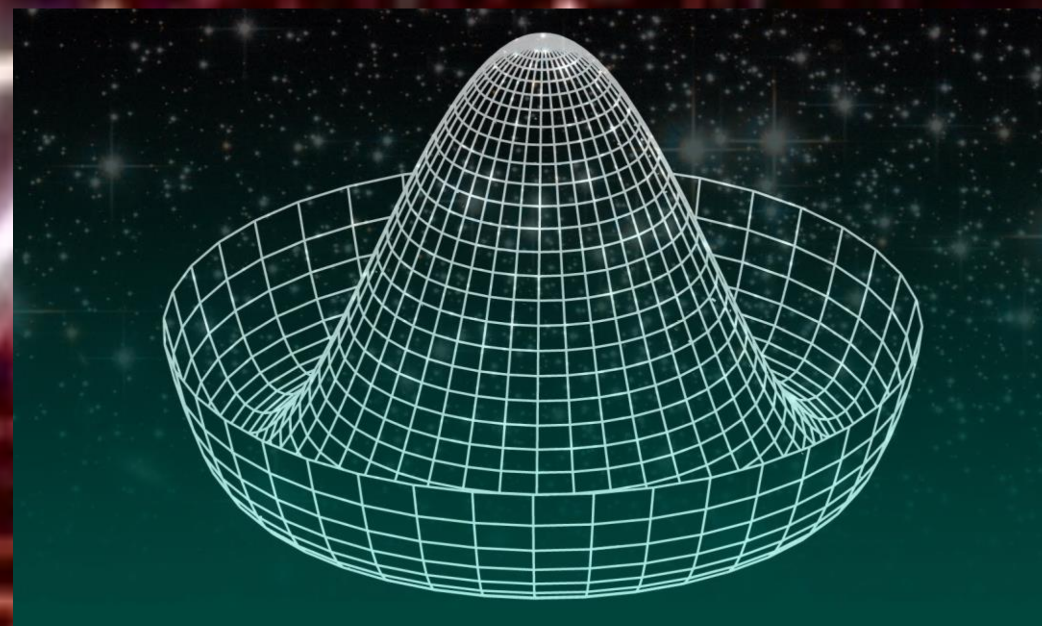
AdS/CFT Correspondence and the Origin of Mass



Nick Evans

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The Origin of Mass

All of the mass in the standard model has a dynamical origin

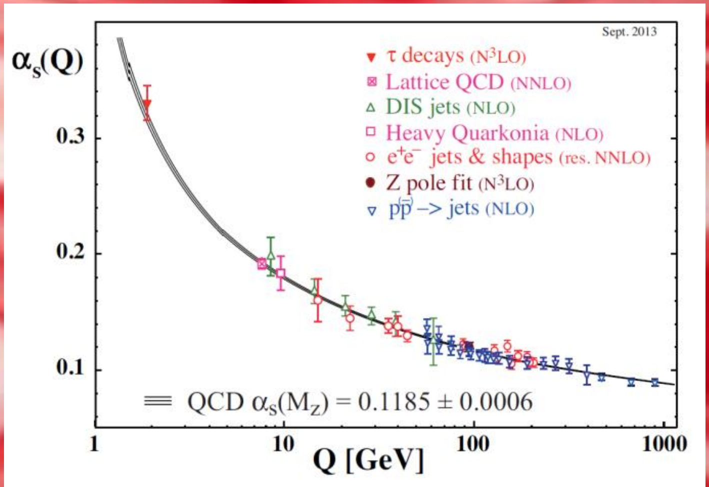
The Proton Mass – The QCD Vacuum

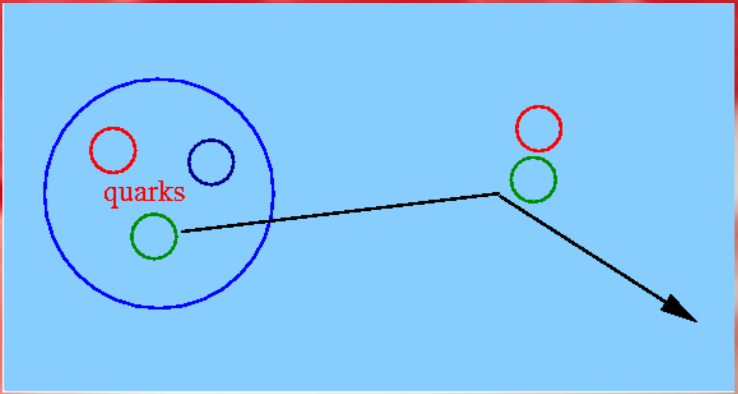
Every so often quantum effects create a quark anti-quark pair.

The attractive force is so strong that

binding energy >> mass energy

The vacuum has lower energy if it fills itself with quark anti-quark pairs!





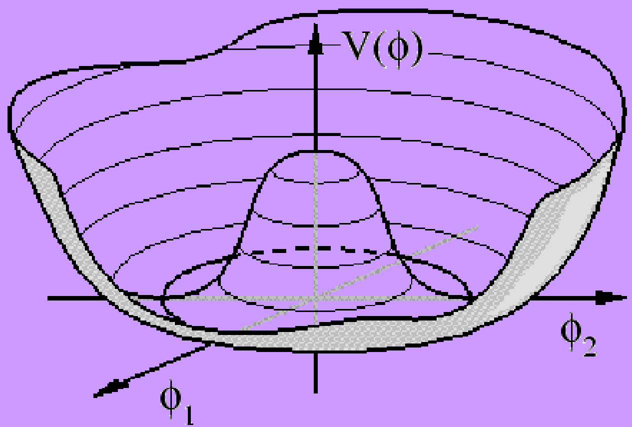
$$SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$$

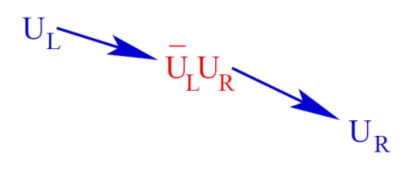
$$m\bar{\psi}\psi = m(\bar{\psi}_L\psi_R + h.c.)$$

$$\bar{u}\gamma^\mu u = \bar{u}_L\gamma^\mu u_L + \bar{u}_R\gamma^\mu u_R$$

Evidence: lack of parity doubling, proton mass, Goldstone pions

$$\langle \bar{u}_L u_R + \bar{d}_L d_R + h.c. \rangle \neq 0$$



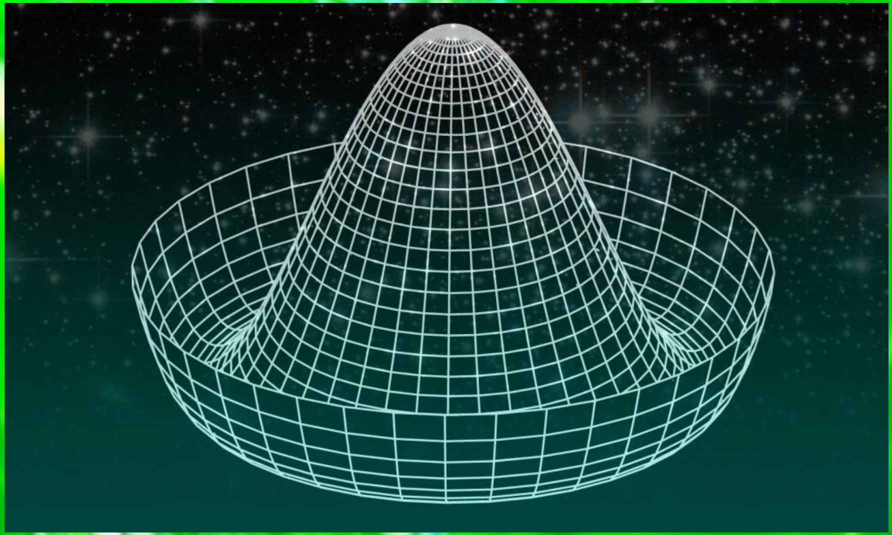


We so far do not know how to do theoretical computations of this dynamics off a supercomputer

Standard Model Higgs

We need an extra process to generate the electron/top mass…

The potential is adhoc – imposed dynamics…



Technicolour

It is possible to replace the higgs with a copy of QCD where

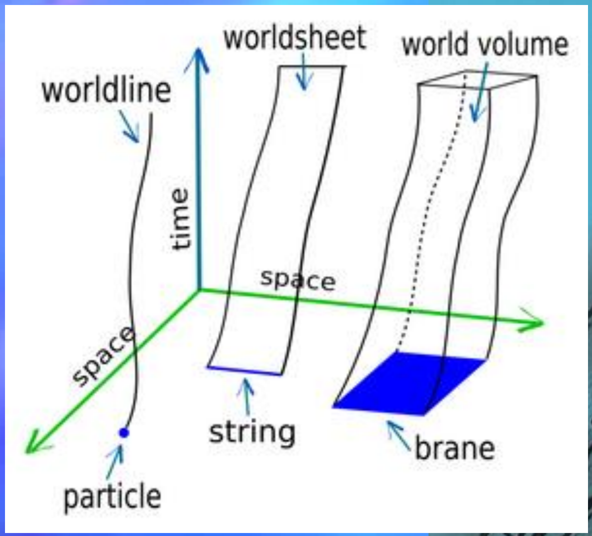
$$SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$$

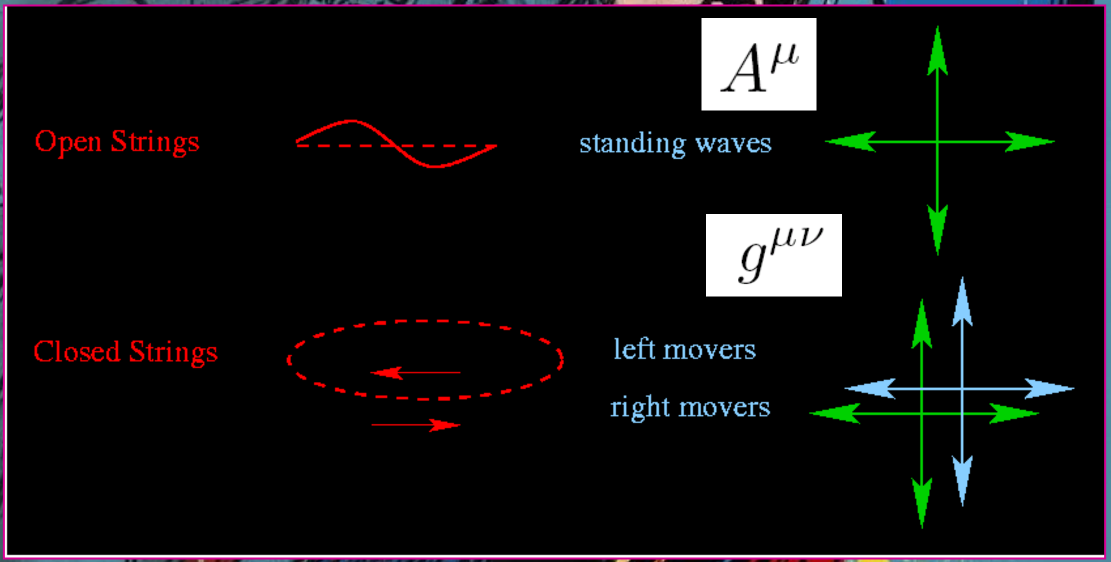
In a new strongly coupled sector – the higgs is a techni-quark meson

BUT why is it the lightest state? How could it possibly mimic the SM higgs so well

We can't use a supercomputer for all possible BSM models yet – even a simple model of dynamics nice

Strings – a potted history

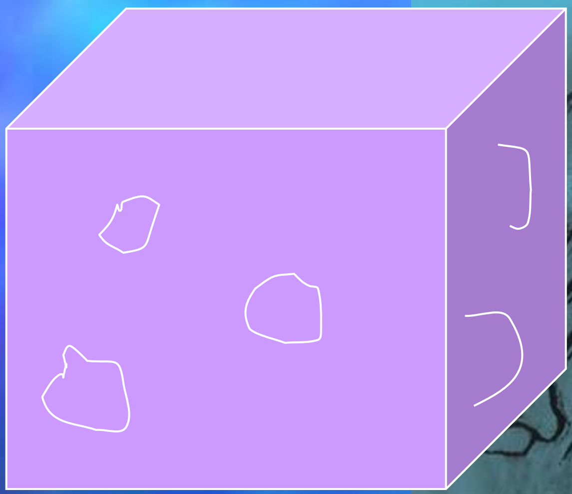


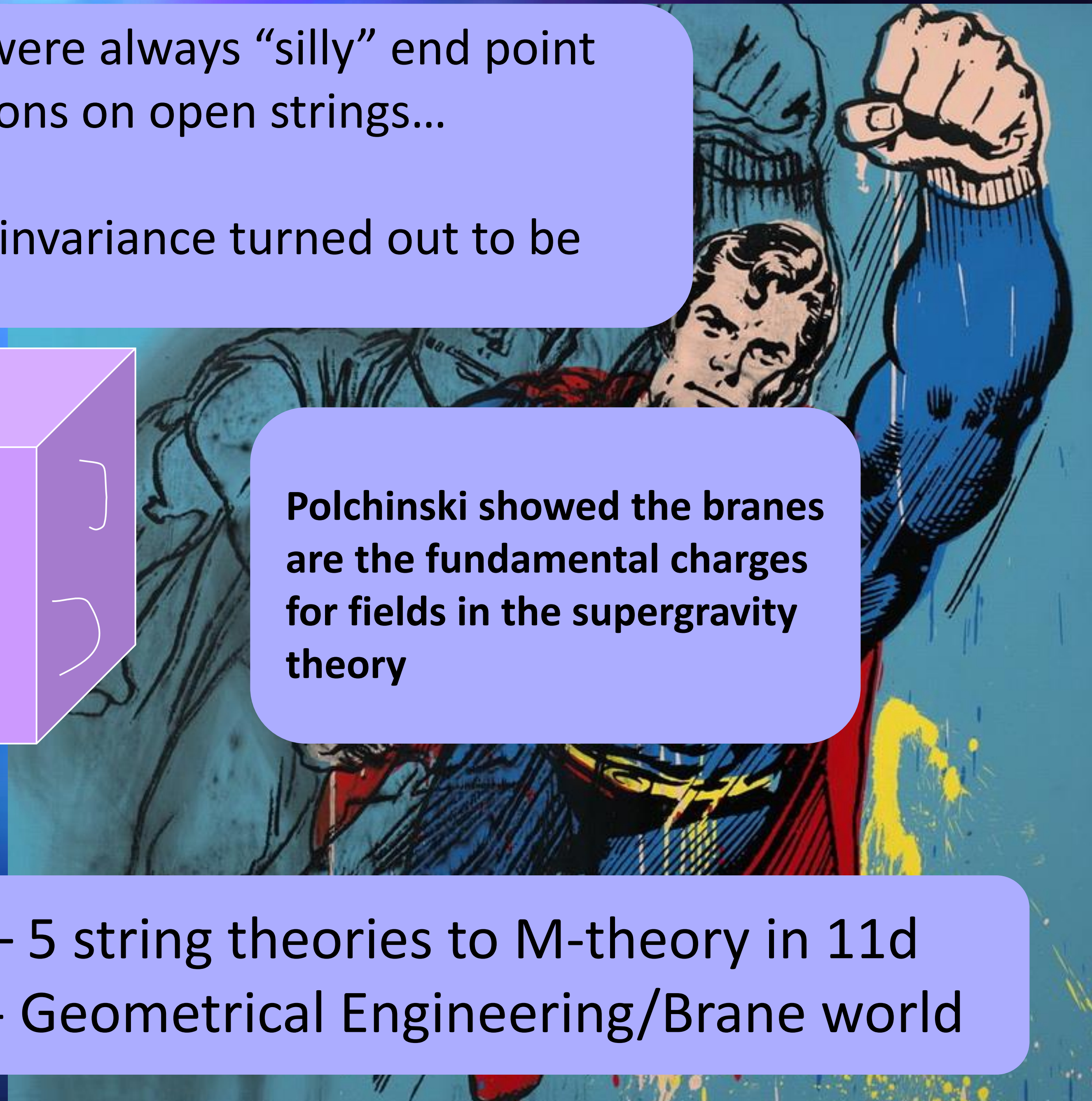


1980s – a possible theory of everything… in 10 dimensions… with supersymmetry

D-branes: there were always "silly" end point boundary conditions on open strings…

Breaking Lorentz invariance turned out to be important…





Polchinski showed the branes are the fundamental charges for fields in the supergravity theory

1990s – 5 string theories to M-theory in 11d
- Geometrical Engineering/Brane world

The AdS/CFT Correspondence

In the limit where T-> infinity.. Gravity and gauge theory decouple…

Yet share the same symmetries…

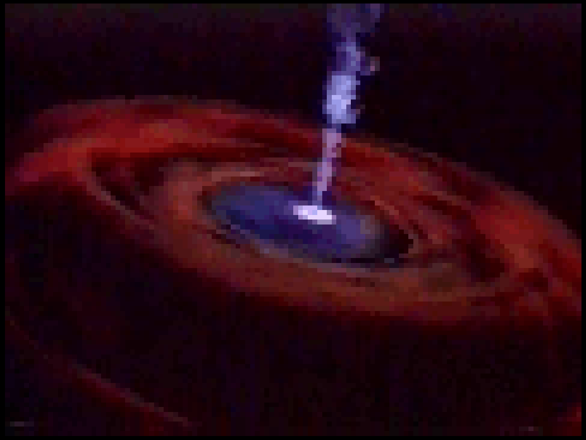
Oh…

Maldacena proposed the two theories (10=5+5d gravity/string & 4d YM) were dual – mathematically equivalent.

Large N strong coupling → weakly coupled gravity

Holography

General Relativity predicts BLACK HOLES



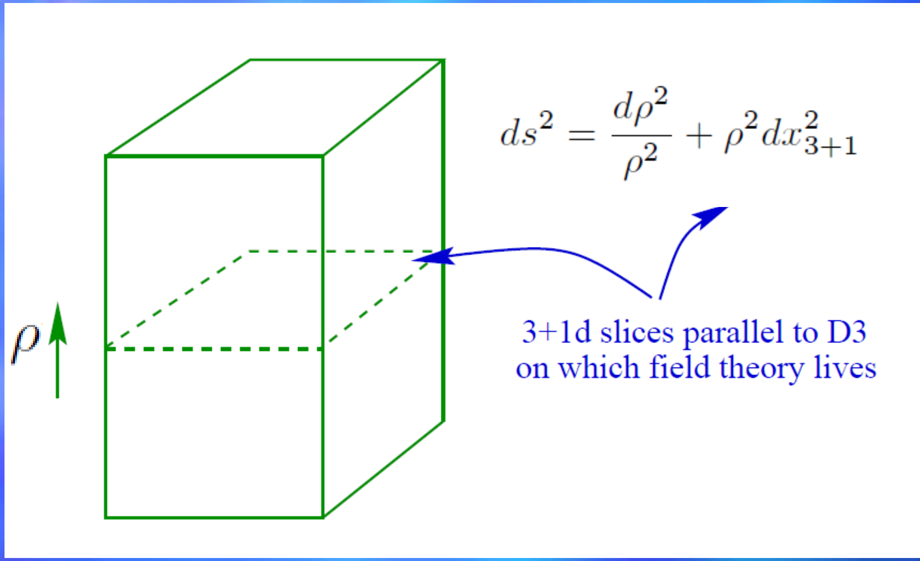
'tHooft argued that any information dropped into a black hole must be

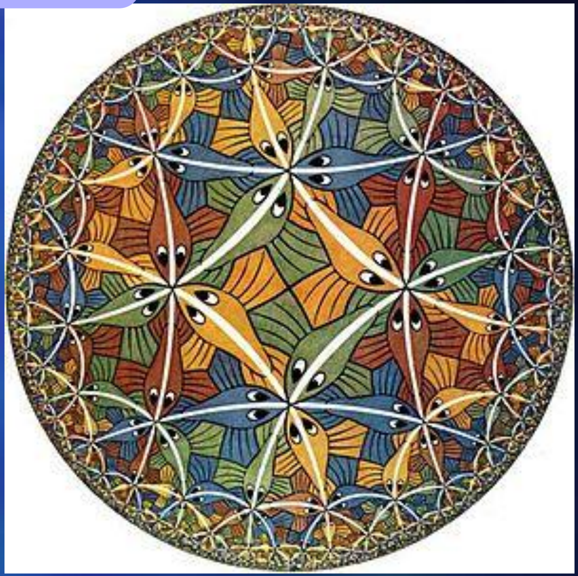
- lost to our Universe (breaking QM unitarity) (Hawking radiation is purely thermal)

- spread over the surface (Hawking radiation is non-thermal)

If the surface can contain all the information of the contents the real theory of the Universe must be 2 + 1 dimensional!

How Does AdS/CFT Work?





Dilatations

$$\int d^4x \; \partial^\mu \phi \partial_\mu \phi, \qquad x \rightarrow e^{-\alpha} x, \qquad \phi \rightarrow e^{\alpha} \phi$$

Become spacetime symmetry of AdS

$$\rho \rightarrow e^{\alpha} \rho$$

is a continuous mass dimension

$\rho \rightarrow$ RG Scale

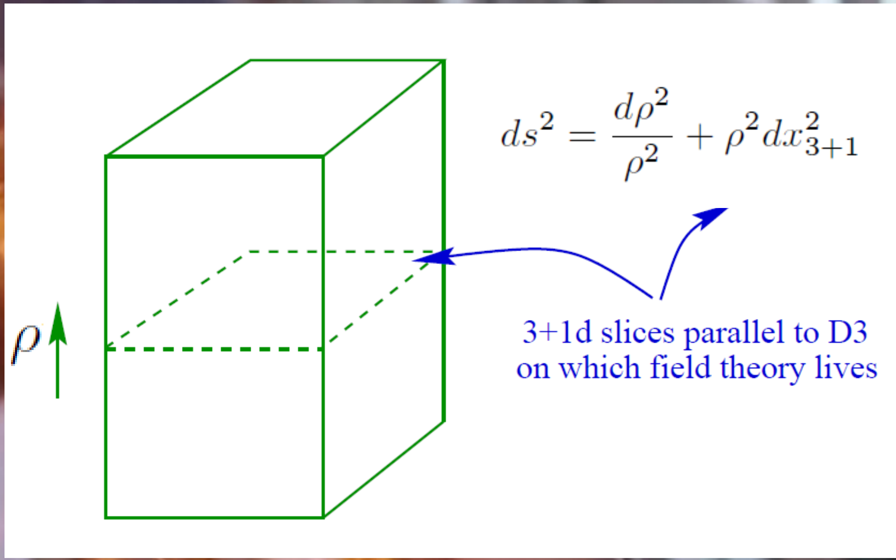
The Magic

The SUGRA fields can be coupled in symmetry invariant ways to gauge theory fields at any $u = const$ slice

$$\int d^4x\ \Phi_{SUGRA}(u_0)\lambda\lambda$$

SUGRA field looks like a SOURCE

eg fermion mass

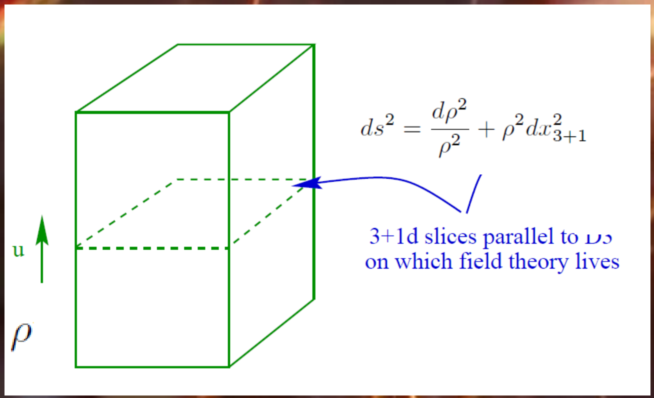


To be consistent with RG flow interpretation of u $\Phi_{SUGRA}(u)$ must give RG flow of source...

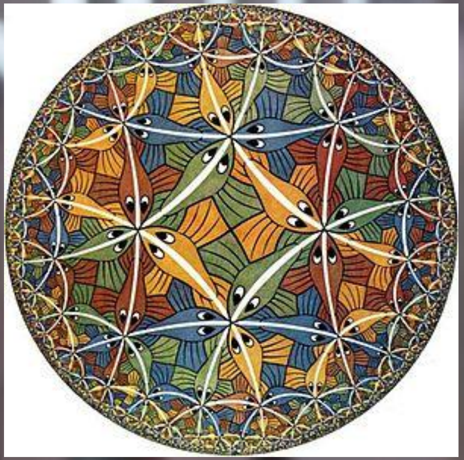
CLAIM: The classical SUGRA EoM give non-perturbative RG flow of the field theory!

In N=4 SYM Witten first checked there were supergravity fields to play the role of all operators/sources… people have since used integrability checks valid at all coupling strengths to prove the relations for many observables.

How Does AdS/CFT Work?



$$\sqrt{-Detg} = Det\left[-\begin{pmatrix} -\rho^2 & 0 & 0 & 0 & 0 \\ 0 & \rho^2 & 0 & 0 & 0 \\ 0 & 0 & \rho^2 & 0 & 0 \\ 0 & 0 & 0 & \rho^2 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\rho^2} \end{pmatrix}\right]^{1/2} = \rho^3$$



A simple example is to just work out the Klein Gordon equation in AdS.. For example to describe the RG flow of a mass term:

$$\int d^4x \; m \; \bar{\psi}\psi$$

m is the quark mass
c is the quark condensate

This pairing of solutions is standard…

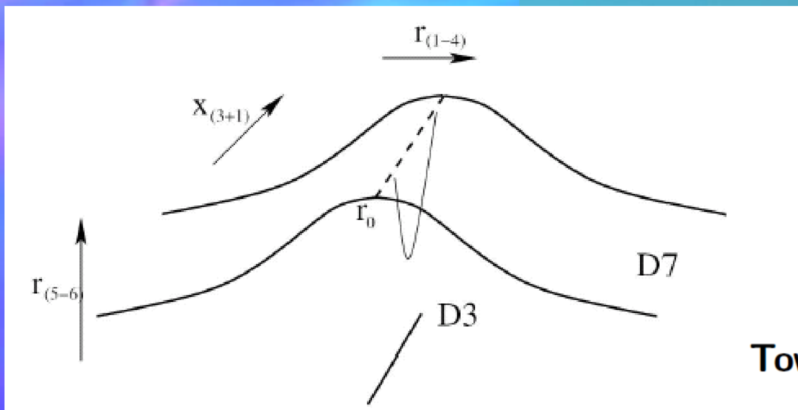
A field for the mass,

$$S = \int d^4x \int d\rho \, \frac{1}{2}\rho^3(\partial_\rho L)^2$$

$$\partial_\rho \left[\rho^3 \partial_\rho L\right] = 0$$

$$L = m + \frac{c}{\rho^2}$$

N=4 SYM is Conformal: Top-Down Models of Chiral Symmetry Breaking and Confinement



Chiral Symmetry Breaking and Pions in Non-Supersymmetric Gauge/Gravity Duals

J. Babington [a], J. Erdmenger [a], N. Evans [b], Z. Guralnik [a] and I. Kirsch [a*]

Towards a holographic dual of large-N_c QCD

Martín Kruczenski,[a] David Mateos,[b] Robert C. Myers[b,c] and David J. Winters[b,d]

Mesons in Gauge/Gravity Duals
A Review

Johanna Erdmenger [a], Nick Evans [bc], Ingo Kirsch [d] and Ed Threlfall [b *]

Flavoured Large N Gauge Theory in an External Magnetic Field

Veselin G. Filev[*], Clifford V. Johnson[*], R. C. Rashkov[†1] and K. S. Viswanathan[†]

Towards a Holographic Model of the QCD Phase Diagram

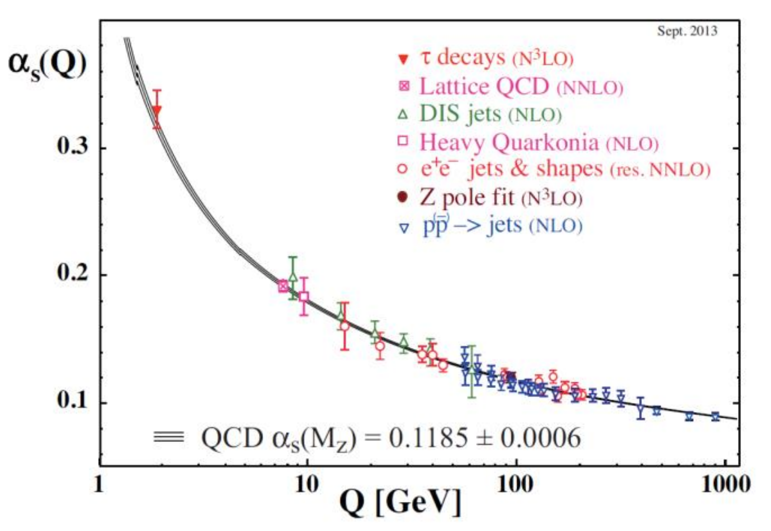
Nick Evans,[*] Astrid Gebauer,[†] and Maria Magou[‡]
School of Physics and Astronomy, University of Southampton, Southampton, SO17 1BJ, UK

Keun-Young Kim[§]

Low Energy Hadron Physics in Holographic QCD

Tadakatsu SAKAI[1,*)] and Shigeki SUGIMOTO[2,**)]

Simple Holographic Models of QCD Dynamics



$$\mu \frac{d\alpha}{d\mu} = -b_0 \alpha^2, \qquad b_0 = \frac{1}{6\pi}(11N_c - 2N_F),$$

$$\gamma = \frac{3C_2}{2\pi}\alpha = \frac{3(N_c^2-1)}{4N_c\pi}\alpha\,.$$

$$\Delta\mathcal{L} = m\,\bar{\psi}\psi$$

m has dimension $1 + \gamma$ condensate dimension $3 - \gamma$

The RG scale where $\gamma = 1$ is special and gap equations suggest the point of condensation…

Running Dimensions in Holography

Raul Alvares, NE, Keun-Young arXiv:1204.2474 [hep-ph]; Matti Jarvinen, Elias Kiritsis arXiv:1112.1261 [hep-ph]

Holographically we can change the dimension of our operator by adding a mass term

$$\partial_\rho[\rho^3\partial_\rho L] - \rho\Delta m^2 L = 0\,.$$

$$L = \frac{m_{FP}}{\rho^\gamma} + \frac{c_{FP}}{\rho^{2-\gamma}}, \qquad \gamma(\gamma-2) = \Delta m^2$$

$\Delta m^2 = -1$ corresponds to $\gamma = 1$ and is again special – the Breitenlohner Freedman bound instability…

DUALITY between strong dynamics condensation and a 5th dimensional higgs mechanism

So we can include a running coupling by a running mass squared for the scalar.

$$\Delta m^2 = -2\gamma = -\frac{3(N_c^2-1)}{2N_c\pi}\alpha$$

The only free parameters are Nc, Nf, m, Λ

Formation of the Chiral Condensate

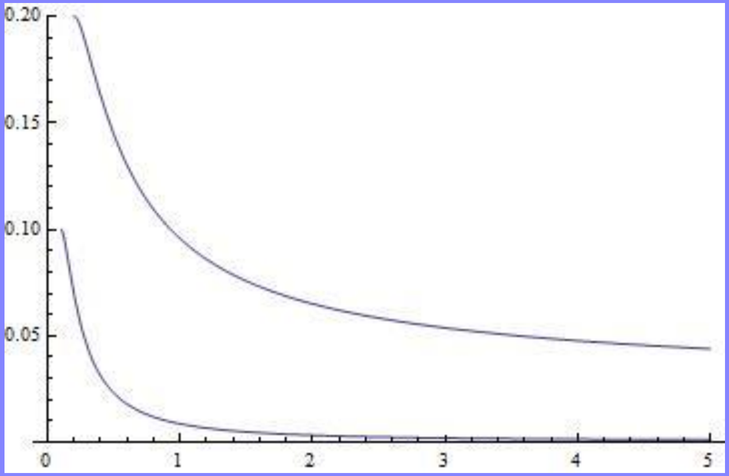
We solve for the vacuum configuration of L

$$\partial_\rho[\rho^3 \partial_\rho L] - \rho \Delta m^2 L = 0\,.$$

Shoot out with

L'(ρ=L) = 0

This is a string theory inspired on-shell IR boundary condition



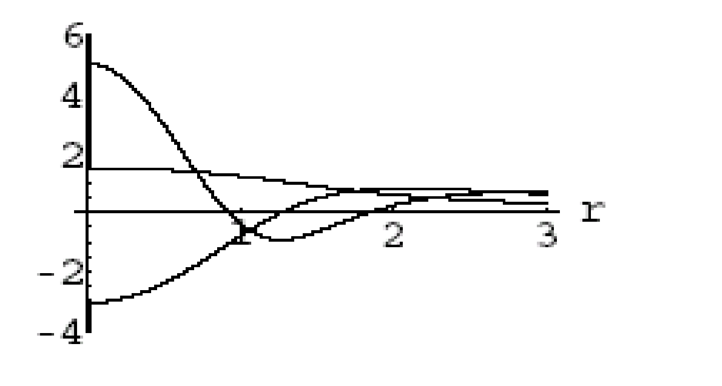
Read off m and qq in the UV…

Meson Fluctuations

$$S = \int d^4x\, d\rho\, \mathrm{Tr}\, \rho^3 \left[\frac{1}{\rho^2 + |X|^2} |DX|^2 + \frac{\Delta m^2}{\rho^2} |X|^2 + \frac{1}{2\kappa^2} (F_V^2 + F_A^2) \right]$$

$$L = L_0 + \delta(\rho) e^{ikx} \qquad k^2 = -M^2$$

$$\partial_\rho(\rho^3 \delta') - \Delta m^2 \rho \delta - \rho L_0 \delta \left. \frac{\partial \Delta m^2}{\partial L} \right|_{L_0} + M^2 R^4 \frac{\rho^3}{(L_0^2+\rho^2)^2} \delta = 0\,.$$



The normalizable solutions pick out particular mass states… the σ and its radial excited states…

The gauge fields let us also study the operators and states

$$\bar{q}\gamma^\mu q \to \rho \text{ meson}$$

$$\bar{q}\gamma^\mu\gamma^5 q \to \text{a meson}$$

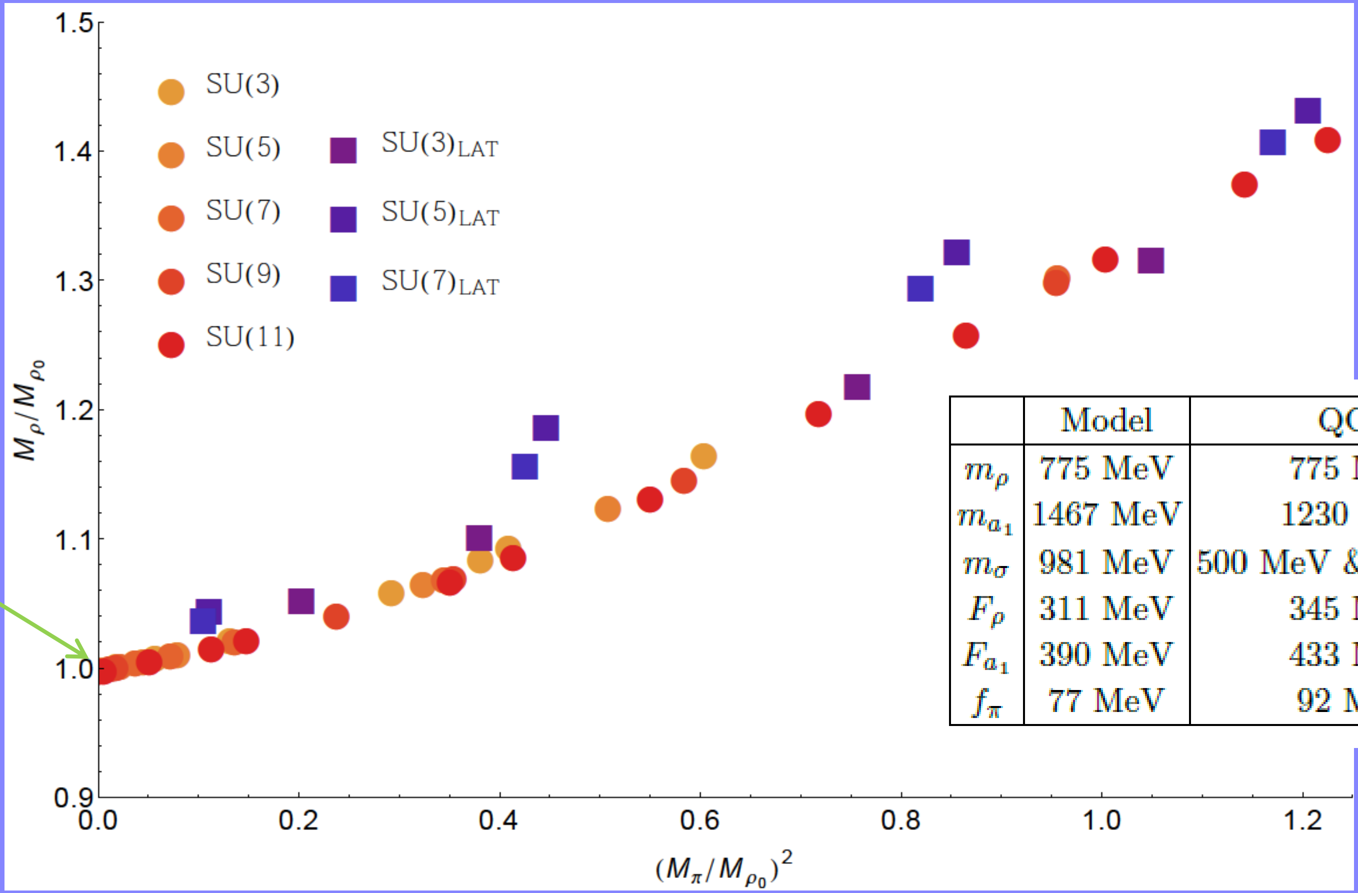
SU(Nc) gauge + 3 quarks

NE, Erdmenger & Mark Scott

arXiv:1412.3165 [hep-ph]

M_rho = 1 defines Λ

Real QCD lies here



	Model	QCD
m_ρ	775 MeV	775 MeV
m_{a_1}	1467 MeV	1230 MeV
m_σ	981 MeV	500 MeV & 980 MeV
F_ρ	311 MeV	345 MeV
F_{a_1}	390 MeV	433 MeV
f_π	77 MeV	92 MeV

There is very little Nc dependence – basically quenched…

Hence comparison to quenched lattice data (Bali et al… arXiv1304.4437)

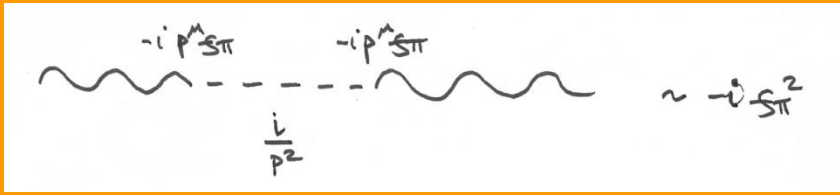
All of these models lie within 10% on any point....

Electroweak Scale – Technicolour

The base idea is to repeat QCD

$$SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$$

The scale is set by v = fπ…



There is no scalar below this scale in QCD… dead?

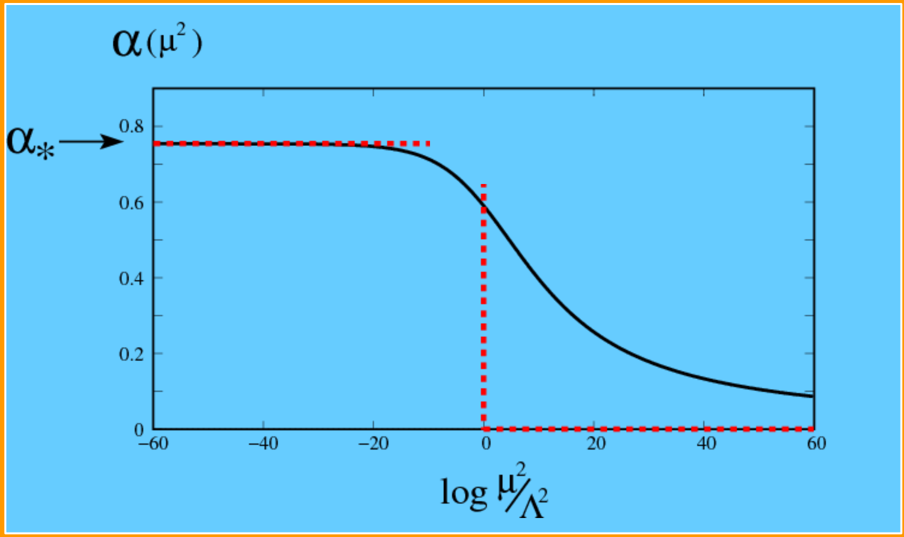
Adding extra electroweak singlet quarks changes the running of the technicolour gauge group…

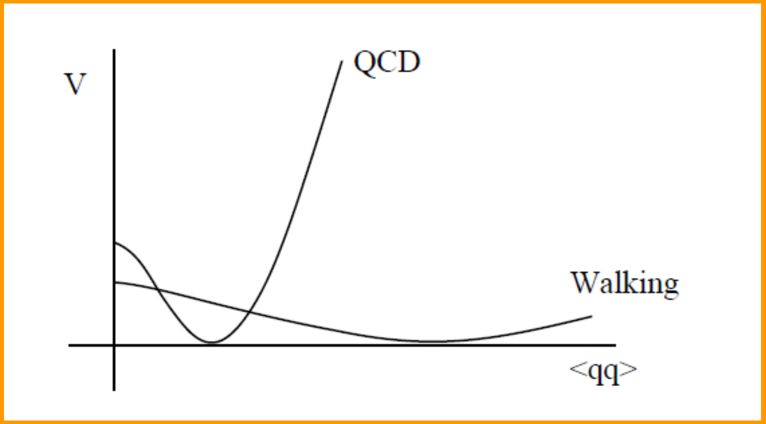
Walking Dynamics Holdom

It's feasible to have much slower running at strong coupling and change the dimension of qq in the IR

$$\langle \bar{q}q \rangle_{UV} \sim \Lambda_{UV} \langle \bar{q}q \rangle_{IR} \sim \Lambda_{UV} \Lambda_{IR}^2$$

$$f_\pi \sim \Lambda_{IR}$$

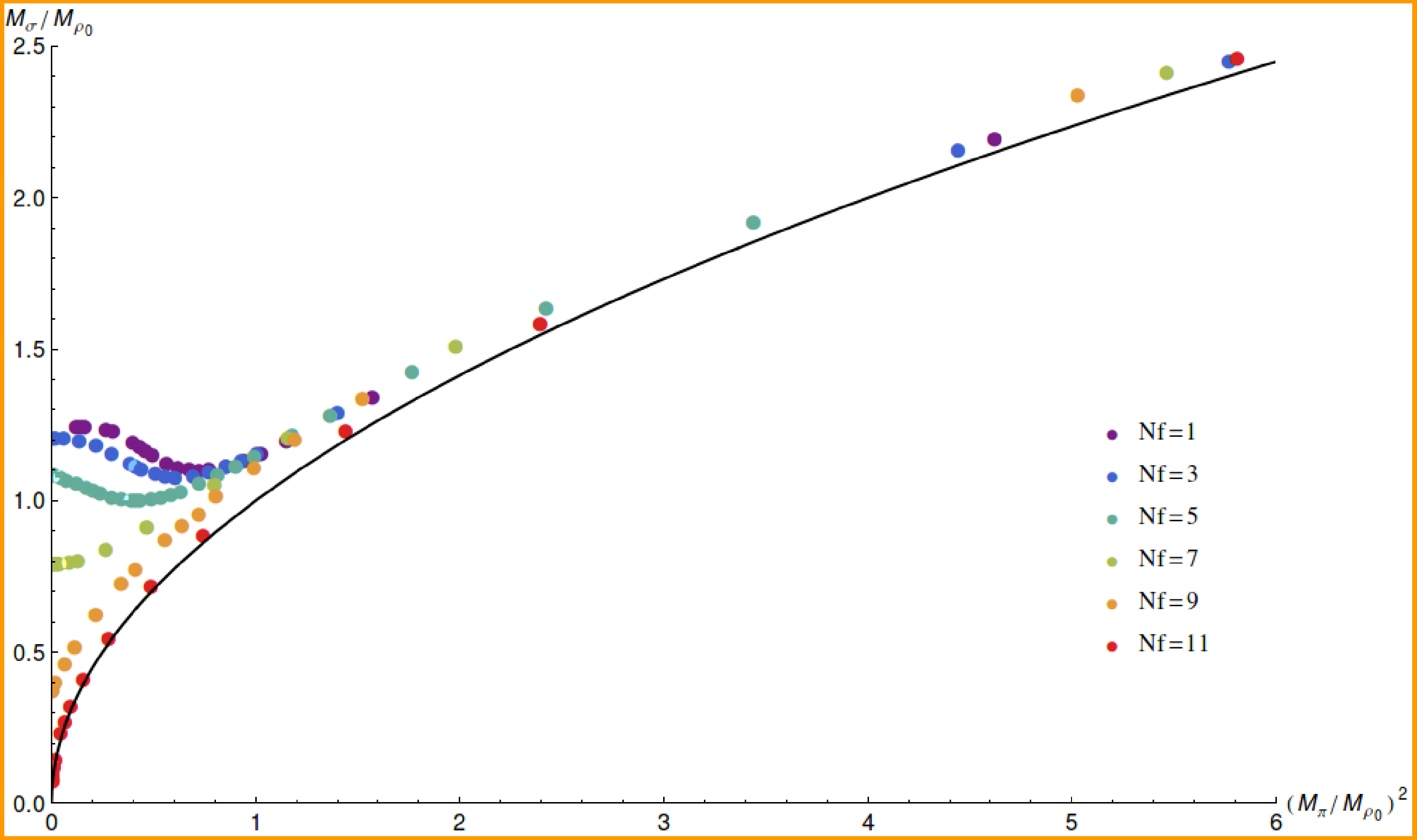




- Is the sigma particle light – a techni-dilaton?
- Is the higgs such a technicolor state?

SU(3) gauge theory + Nf quarks: Holography

The QCD point is not right for the f0(500) but about right for the f0(980) – is the f0(500) odd eg a molecule ???



We indeed see a light sigma relative to the rho… cf higgs

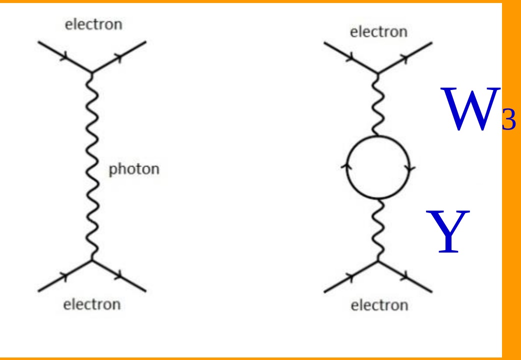
Yes we are **fine tuning** – here by choice of Nf….

Technicolour Exclusions

S broken gauge theories have non-decoupling effects

$$\left.\frac{d\Pi_{3Y}}{dq^2}\right|_{q^2=0}$$

Counts the number of electroweak doublets



Low energy computation:

$$S = 4\pi\left[\frac{F_V^2}{M_V^2} - \frac{F_A^2}{M_A^2}\right]$$

$S_{QCD} = 0.3$

It has been suggested that as one approaches the critical Nf at the edge of the conformal window V-A symmetry is restored and S-> 0

$$S = \int d^4x\, d\rho\, \mathrm{Tr}\, \rho^3 \left[\frac{1}{\rho^2 + |X|^2}|DX|^2 + \frac{\Delta m^2}{\rho^2}|X|^2 + \frac{1}{2\kappa^2}(F_V^2 + F_A^2)\right]$$

V-A symmetry is restored holographically by κ -> 0
(no Nf prediction)

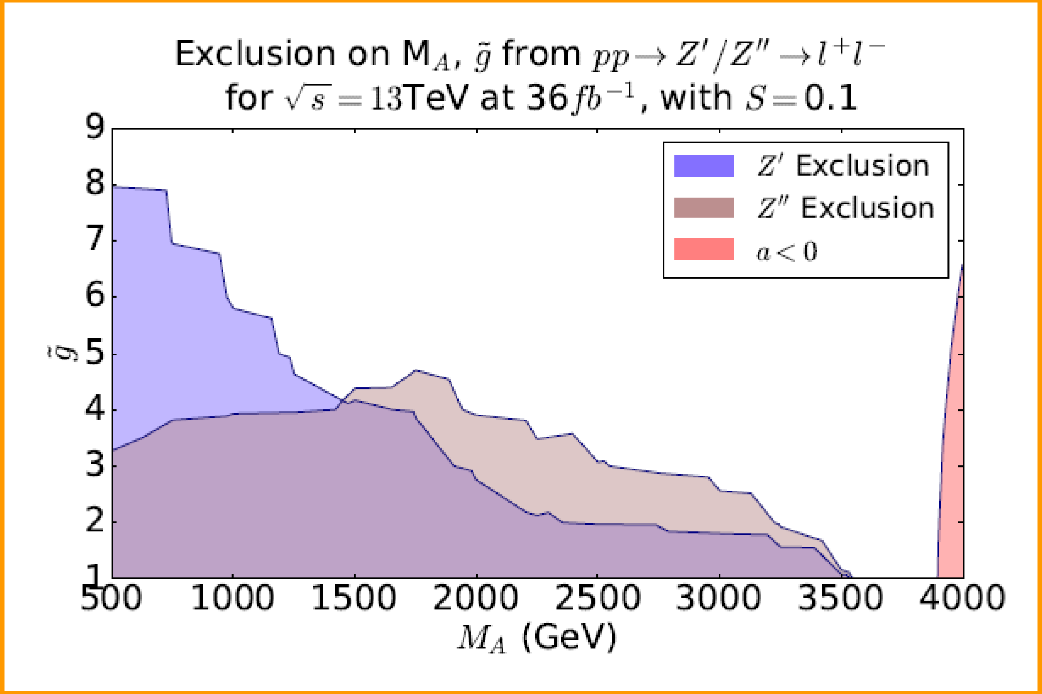
Pheno Exculsions

Walking Technicolor in the light of Z' searches at the LHC

Alexander Belyaev[a,c] , Azaria Coupe[a] , Mads Frandsen[b] , Emmanuel Olaiya[c] , Claire Shepherd-Themistocleous[c]

$$\mathcal{L}_{boson} = -\frac{1}{2}\mathrm{Tr}[\tilde{W}_{\mu\nu}\tilde{W}^{\mu\nu}] - \frac{1}{4}\tilde{B}_{\mu\nu}\tilde{B}^{\mu\nu} - \frac{1}{2}\mathrm{Tr}[F_{L\mu\nu}F_L^{\mu\nu} + F_{R\mu\nu}F_R^{\mu\nu}] + m^2\mathrm{Tr}[C_{L\mu}^2 + C_{R\mu}^2] + \frac{1}{2}\mathrm{Tr}[D_\mu M D^\mu M^\dagger] - \tilde{g}^2 r_2 \mathrm{Tr}[C_{L\mu} M C_R^\mu M^\dagger] - \frac{i\tilde{g}r_3}{4}\mathrm{Tr}[C_{L\mu}(MD^\mu M^\dagger - D^\mu M M^\dagger) + C_{R\mu}(M^\dagger D^\mu M - D^\mu M^\dagger M)] + \frac{\tilde{g}^2 s}{4}\mathrm{Tr}[C_{L\mu}^2 + C_{R\mu}^2]\mathrm{Tr}[MM^\dagger] + \frac{\mu^2}{2}\mathrm{Tr}[MM^\dagger] - \frac{\lambda}{4}\mathrm{Tr}[MM^\dagger]^2,$$

They generically model SM + higgs + rho and a mesons



from Z' and Z'' DY processes

Looks like pretty good reach and exclusion

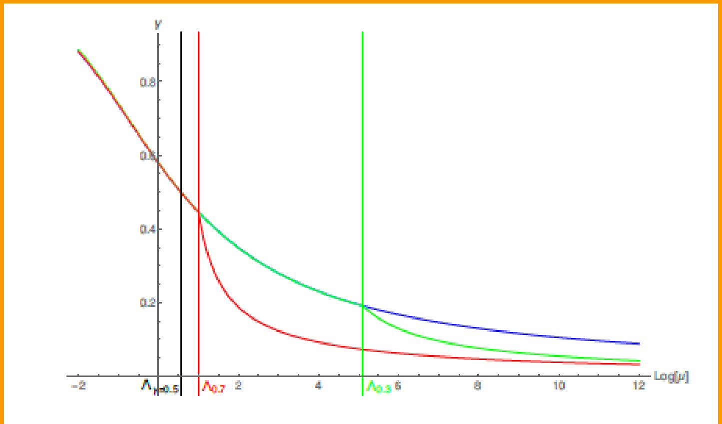
But where do real models lie in the space?

Giving TC a last chance…

Most likely there is no choice of Nc Nf that will realize the physical S and mh…

But let's imagine we get lucky… because we don't know the IR running of the gauge coupling we don't know which Nc Nf combination to pick…

So lets holographically describe all Nc Nf pairs:

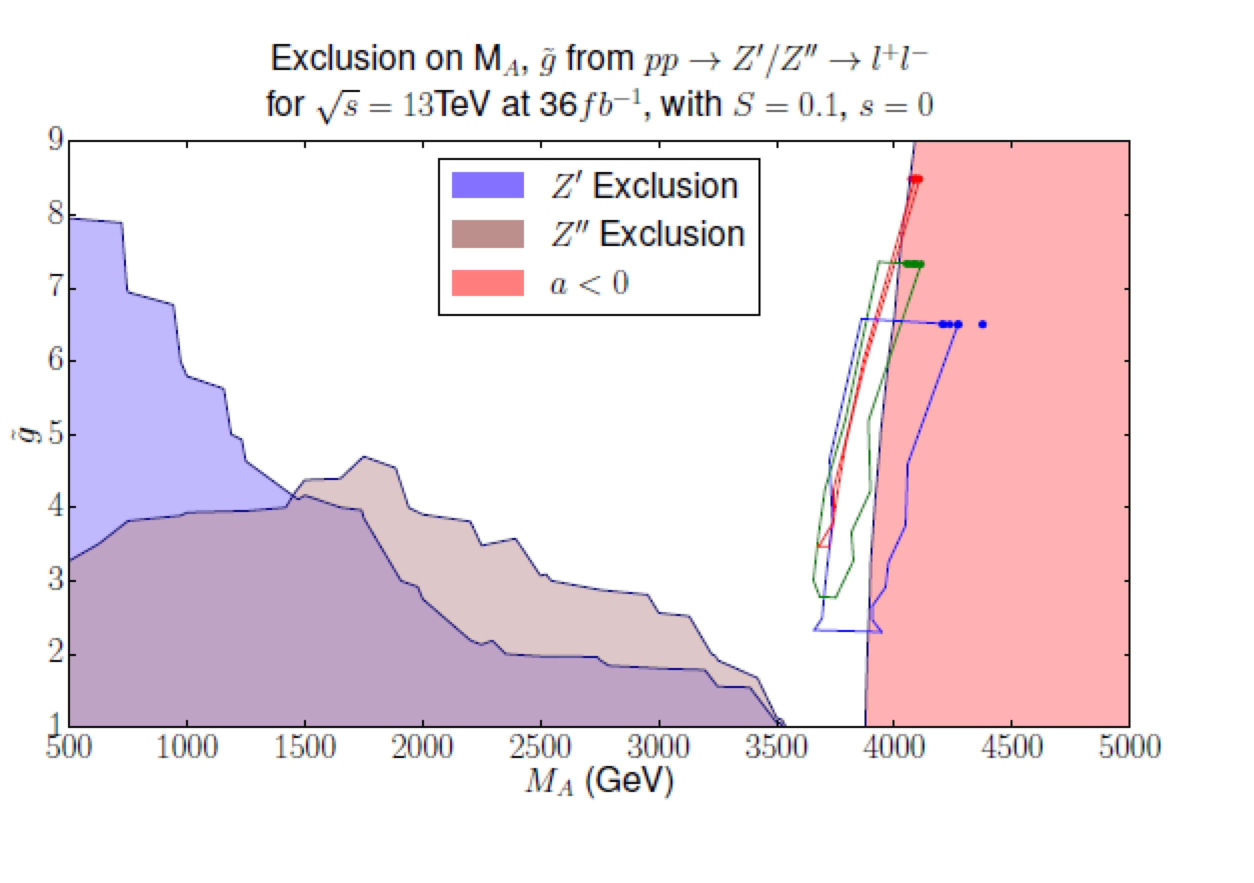


tune k to give S=0.1…

Change the IR running (NfIR) to give mh = fp/2

Most likely the spectrum is in every case wrong! BUT if there is one theory that works we hope to have captured it… lets rule it out!

With Belyaev and Coupe



Red is Nc = 3

Green Nc = 4

Blue Nc = 5

Moving downwards is adding electroweak doublets

The a=0 line is where the rho and a become degenerate in mass and decay constant – elsewhere conspiracies balance S= 0.1

Yukawa Couplings

$$y_t \phi \psi_L t_R \rightarrow \frac{g^2}{M^2} \bar{U}_R \Psi_R \bar{\psi}_L t_R$$

Making the top mass is very hard... if M ~ 5 TeV then you need

walking to grow the techniquark condensate

strong g

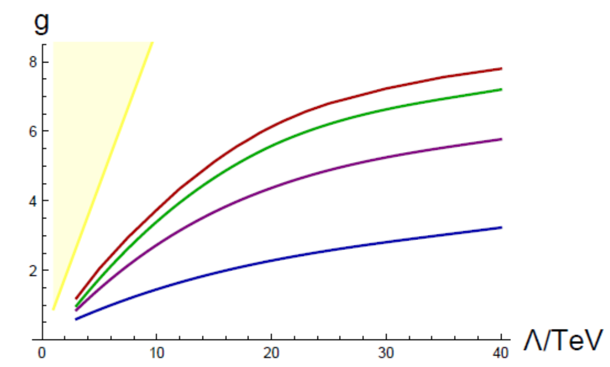


FIG. 7: g vs UV cut off Λ for consistent solutions with the physical top mass on the TC dominated branch for $N_c = 3$, $N_f = 2, 4, 8, 11$ from the top down. The shaded region is excluded by the two loop $\delta\rho$ contribution.

Holographic computation by Evans & Clemens

Superconducting Instability

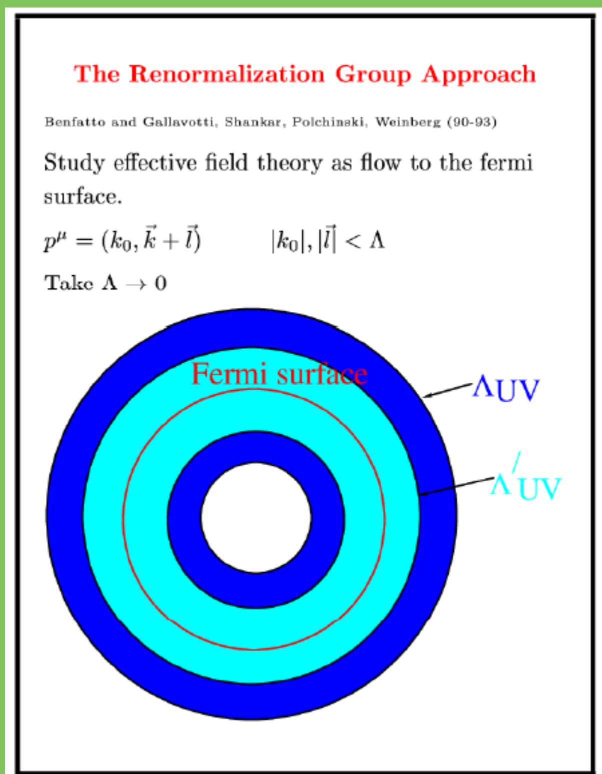
In the presence of a Fermi surface any attractive interaction will cause Cooper pair formation

$$\mu \bar{q} \gamma^0 q$$

A chemical potential sources fermion density

$$\langle A^0 \rangle \bar{q} \gamma^0 q$$

A vev for a temporal U(1) "number" gauge field plays the same role



μ generates a Fermi surface – fermion states filled out to momentum μ

Interactions between fermions at k and –k become very strong and bind, then condense, Cooper pairs…

The Simplest Holographic Superconductor

Sean Hartnoll, Chris Herzog, Gary Horowitz, arXiv:0803.3295 [hep-th]; NE, Michela Petrini, hep-th/0108052

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + |\partial\psi - iBA\psi|^2 + 3\psi^2$$

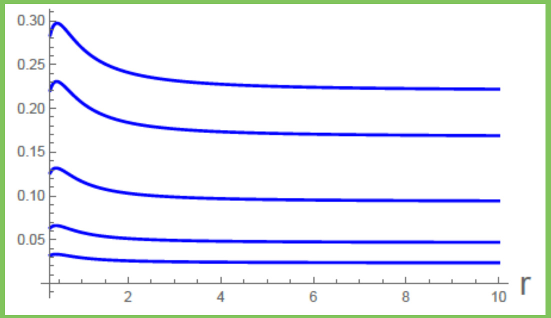
An A vev is a negative mass squared for the scalar L - describing some dimension 1/3 J & <O>…

If it gets big enough it violates the BF bound and causes condensation…

The final state of the instability is unclear at T=0

If introduce a temperature by a black hole horizon in AdS a stable ground state does exist

L



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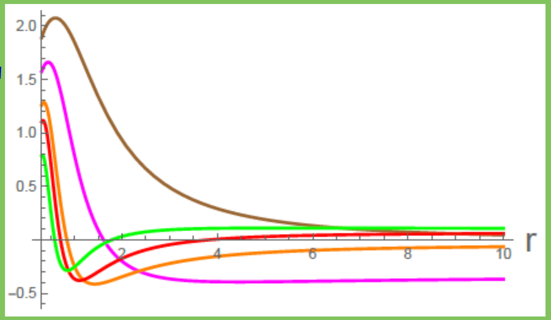

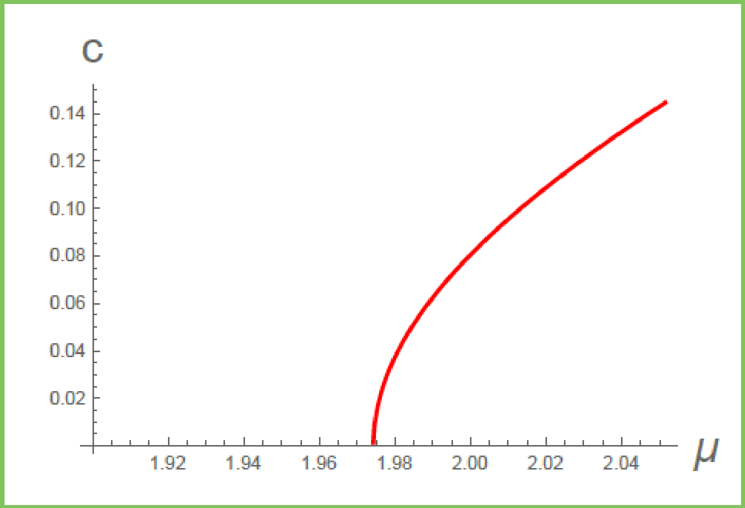

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - |\partial\psi - iGBA\psi|^2 + 3/L^2\psi^2$$

Psi represents the vev of the di-quark condensing operator….

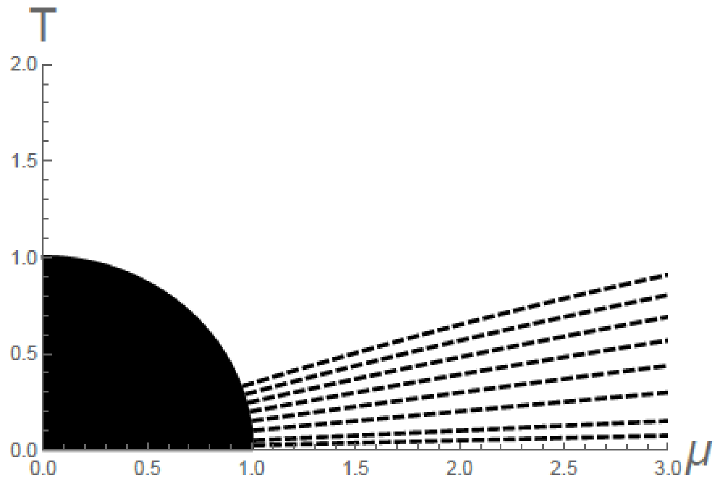

FIG. 5: Plot of the superconducting phase boundary at different $G = 0.5, 1, 2, 3, 4, 5, 6, 7$ from bottom to top in the T - μ plane. The black region is expected to be the chirally symmetric phase below a scale of $\mu^2 + T^2 = 1$.

$$G^2 = \frac{\kappa}{b\ln(T^2+\mu^2)/\Lambda_c^2}, \qquad b = 11N_c/3 - 2N_f/3,$$

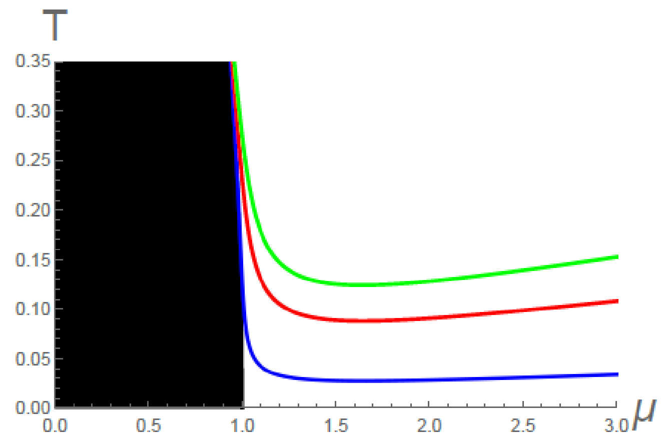

FIG. 6: QCD phase diagram: the blacked out area is below Λ_c where chiral symmetry breaking is expected. The remaining phase edges shows where the CFL phase is present for the choices of $\kappa = 1, 10, 20$ from bottom to top.

Conclusions

- The origin of mass remains a key element of the SM
- Can we do a better job of computing at strong coupling; is the higgs potential dynamical in origin?
- Holography is a technique for computing at strong coupling from string theory
- remarkably it can be brought, at least in toy form, all the way to the theories in question
- Duality between strong dynamics symmetry breaking and fifth dimension higgs mechanism with a corresponding potential
- We can mimic QCD
- We can explore simply technicolour like models
- Superconductivity is naturally reproduced also