Max Baak (CERN),
on behalf of the Gfitter group (*)

The global electroweak fit at NNLO
Prospects for LHC and ILC

http://cern.ch/Gfitter

EPJ C 74, 3046 (2014), arXiv:1407.3792

Outline

This presentation:

- Introduction to the Electroweak Fit
  - Inputs to the electroweak fit
    - *Full set of 2-loop calculations and theory uncertainties*
  - After the Higgs: predictions for key observables
  - Modified Higgs couplings
  - Prospects for LHC and ILC
- Conclusion & Outlook
The Gfitter Project – Introduction

A Generic Fitter Project for HEP Model Testing

- Gfitter = state-of-the-art HEP model testing tool
- Latest results always available at: http://cern.ch/Gfitter
  - (Most) results of this presentation: EPJC 74, 3046 (2014)

- Gfitter software and features:
  - Modular, object-oriented C++, relying on ROOT, XML, python, etc.
  - Core package with data-handling, fitting, and statistics tools
  - Independent “plug-in” physics libraries: SM, 2HDM, multiple BSM models, ...
The global electroweak fit of the SM
Idea behind electroweak fits

✓ Observables receive quantum loop corrections from ‘unseen’ virtual effects.

✓ If system is over-constrained, fit for unknown parameters or test the model’s self-consistency.

✓ If precision is better than typical loop factor ($\alpha \approx 1/137$), test the model or try to obtain info on new physics in loops.
  
  • For example, in the past EW fits were used to predict the Higgs mass.
Global EW fits: a long history

- Huge amount of pioneering work by many!
  - Needed to understand importance of loop corrections
    - Important observables (now) known at least at two-loop order, sometimes more.
  - High-precision Standard Model (SM) predictions and measurements required
    - First from LEP/SLC, then Tevatron, now LHC.

- Top mass predictions from loop effects available since ~1990.
  - Official LEPEW fit since 1993.
  - The EW fits have always been able to predict the top mass correctly!
Global EW fits: many fit codes

- EW fits performed by many groups in past and present.
  - D. Bardinet al. (ZFITTER), G. Passarino et al. (TOPAZ0), LEPEW WG (M. Grünewald, et al.), J. Erler (GAP), Bayesian fit (M. Ciuchini et al.), etc …
  - Important results obtained!
- Several groups pursuing global beyond-SM fits, especially SUSY.
- Global SM fits also used at lower energies [CKM-matrix].

- Fits of the different groups agree very well.
- Some differences in treatment of theory errors, which just start to matter.
  - E.g. theoretical and experimental errors added linearly (= conservative) or quadratically.
    - In following: theoretical errors treated as Gaussian (quadratic addition.)
The predictive power of the SM

- As the Z boson couples to all fermions, it is ideal to measure & study both the electroweak and strong interactions.

- Tree level relations for $Z \to f \bar{f}$
  \[ i f \gamma^\mu (g_{V,f} - g_{A,f} \gamma_5) f Z_\mu \]

- Prediction EWSB at tree-level:
  \[ \frac{M_W^2}{M_Z^2 \cos^2 \theta_W} = 1 \]

- The impact of loop corrections
  - Absorbed into EW form factors: $\rho$, $\kappa$, $\Delta r$
  - Effective couplings at the Z-pole
  - Quadratically dependent on $m_t$, logarithmic dependence on $M_H$

\[ g_{V,f} = \sqrt{\rho_Z^f} \left( I_3^f - 2Q_f^f \sin^2 \theta^f_{\text{eff}} \right) \]
\[ g_{A,f} = \sqrt{\rho_Z^f I_3^f} \]
\[ \sin^2 \theta^f_{\text{eff}} = \kappa_Z^f \sin^2 \theta_W \]

\[ M_W^2 = \frac{M_Z^2}{2} \left( 1 + \sqrt{1 - \frac{\sqrt{8\pi\alpha(1 + \Delta r)}}{G_F M_Z^2}} \right) \]
Discovery of Higgs-like boson at LHC
- Cross section, production rate time branching ratios, spin, parity so far compatible with SM Higgs boson.

This talk: assume boson is SM Higgs.

Use in EW fit: $M_H = 125.14 \pm 0.24$ GeV
- ATLAS: $M_H = 125.36 \pm 0.37 \pm 0.18$ GeV
- CMS: $M_H = 125.03 \pm 0.27 \pm 0.14$ GeV
  [arXiv:1406.3827, CMS-PAS-HIG-14-009]

Change in average between fully uncorrelated and fully correlated systematic uncertainties is minor:
$\delta M_H : 0.24 \rightarrow 0.32$ GeV
- EW fit unaffected at this level of precision
Unique situation:

- For first time SM is fully over-constrained.
- And for first time electroweak observables can be unambiguously predicted at loop level.
- Powerful predictions of key observables now possible, much better than w/o $M_H$.

Can now test for:

- Self-consistency of SM.
- Possible contributions from BSM models.

- Part of focus of this talk …
Measurements at the Z-pole (1/2)

- Total cross-section of $e^-e^+\rightarrow Z\rightarrow ff$
  - Expressed in terms of partial decay width of initial and final width:
    \[
    \sigma_{\bar{f}f} = \sigma_{\bar{f}f}^0 \left( s - M_Z^2 \right)^2 + s^2 \frac{\Gamma_Z^2}{M_Z^2} \frac{1}{R_{QED}}
    \]
    
    with
    \[
    \sigma_{\bar{f}f}^0 = \frac{12\pi}{M_Z^2} \frac{\Gamma_{ee}}{\Gamma_Z^2}
    \]
    
  - Full width: $\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{\text{had}} + \Gamma_{\text{inv}}$
  - (Correlated set of measurements.)

- Set of input (width) parameters to EW fit:
  - Z mass and width: $M_Z, \Gamma_Z$
  - Hadronic pole cross section:
    \[
    \sigma_{\text{had}}^0 = 12\pi / M_Z^2 \cdot \Gamma_{ee} \Gamma_{\text{had}} / \Gamma_Z^2
    \]
  - Three leptonic ratios (lepton univ.):
    \[
    R_\ell^0 = R_e^0 = \Gamma_{\text{had}} / \Gamma_{ee} \quad (= R_\mu^0 = R_\tau^0)
    \]
  - Hadronic-width ratios: $R_b^0, R_c^0$

Diagram: X-Section [pb] vs $\sqrt{s}$ [GeV]
Definition of Asymmetry

- Distinguish vector and axial-vector couplings of the Z

\[ A_f = \frac{g_{L,f}^2 - g_{R,f}^2}{g_{L,f}^2 + g_{R,f}^2} = \frac{2g_{V,f} g_{A,f}}{g_{V,f}^2 + g_{A,f}^2} \]

- Directly related to: \[ \sin^2 \theta_{eff}^f = \frac{1}{4Q_f} \left( 1 + \Re \left( \frac{g_{V,f}}{g_{A,f}} \right) \right) \]

Observables

- In case of no beam polarisation (LEP) use final state angular distribution to define forward/backward asymmetry:

\[ A_{FB}^f = \frac{N_f^f - N_B^f}{N_F^f + N_B^f} \quad A_{FB}^{0,f} = \frac{3}{4} A_e A_f \]

- Polarised beams (SLC), define left/right asymmetry:

\[ A_{LR}^f = \frac{N_L^f - N_R^f}{N_L^f + N_R^f} \frac{1}{\langle |P_e| \rangle} \quad A_{LR}^{0} = A_e \]

Measurements: \( A_{FB}^{0,\ell}, A_{FB}^{0,c}, A_{FB}^{0,b}, A_\ell, A_c, A_b \)
Latest averages for $M_W$ and $m_{\text{top}}$

The ElectroWeak fit of Standard Model

Latest Tevatron result from: arXiv:1204.0042


Tevatron+LHC $m_{\text{top}}$ combination - March 2014, $L_{\text{int}} = 3.5$ fb$^{-1}$ - 8.7 fb$^{-1}$

ATLAS + CDF + CMS + D0 Preliminary

CDF Run II, $t$+jets
$M_{\text{top}} = 172.85 \pm 1.12$ (0.52 ± 0.49 ± 0.86)

CDF Run II, di-lepton
$M_{\text{top}} = 170.28 \pm 3.69$ (1.95 ± 3.13)

CDF Run II, all jets
$M_{\text{top}} = 173.93 \pm 1.85$ (1.26 ± 1.05 ± 0.86)

CDF Run II, $E_T^\text{miss}$+jets
$M_{\text{top}} = 174.94 \pm 1.50$ (0.83 ± 0.47 ± 1.16)

D0 Run II, $t$+jets
$M_{\text{top}} = 174.00 \pm 2.79$ (2.36 ± 0.55 ± 1.38)

D0 Run II, di-lepton
$M_{\text{top}} = 172.31 \pm 1.55$ (0.23 ± 0.72 ± 1.35)

ATLAS 2011, $t$+jets
$M_{\text{top}} = 173.09 \pm 1.63$ (0.64 ± 1.50)

ATLAS 2011, di-lepton
$M_{\text{top}} = 173.49 \pm 1.06$ (0.27 ± 0.33 ± 0.97)

CMS 2011, $t$+jets
$M_{\text{top}} = 172.50 \pm 1.52$ (0.43 ± 1.46)

CMS 2011, di-lepton
$M_{\text{top}} = 173.49 \pm 1.41$ (0.69 ± 1.23)

CMS 2011, all jets
$M_{\text{top}} = 173.49 \pm 1.41$ (0.69 ± 1.23)

World comb. 2014 $\chi^2$/ndf = 310

Lightest $M_{\text{top}}$ (world): arXiv:1457.2682

March 2012

$M_{\text{top}}$ [GeV]

$M_W$ [MeV]

80,000 80,400 80,600

$173.34 \pm 0.76$ GeV/c$^2$ (0.27 ± 0.24 ± 0.67)

$m_{\text{top}}$ [GeV]

165 170 175 180 185

March 2014

$174.34 \pm 0.64$ GeV/c$^2$
The electromagnetic coupling

- The EW fit requires precise knowledge of $\alpha(M_Z)$ – better than 1% level
  - Enters various places: hadr. radiator functions, predictions of $M_W$ and $\sin^2\theta_{eff}$
- Conventionally parametrized as ($\alpha(0) =$ fine structure constant) :
  $$\alpha(s) = \frac{\alpha(0)}{1 - \Delta\alpha(s)}$$

- Evolution with renormalization scale:
  $$\Delta\alpha(s) = \Delta\alpha_{lep}(s) + \Delta\alpha_{had}^{(5)}(s) + \Delta\alpha_{top}(s)$$
The electromagnetic coupling

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- Leptonic term known up to four loops (for $q^2 \gg m_l^2$)
- Top quark contribution known up to 2 loops, small: $-0.7 \times 10^{-4}$

[M. Steinhauser, PLB 429, 158 (1998)]
The electromagnetic coupling

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$$\Delta\alpha(s) = \Delta\alpha_{\text{lep}}(s) + \Delta\alpha_{\text{had}}^{(5)}(s) + \Delta\alpha_{\text{top}}(s)$$

- Hadronic contribution (from the 5 light quarks) completely dominates overall uncertainty on $\alpha(M_Z)$.

- Difficult to calculate, cannot be obtained from pQCD alone.
  - Analysis of low-energy $e^+e^-$ data
  - Usage of pQCD if lack of data

\[\Delta\alpha_{\text{had}}^{(5)}(M_Z) = (274.9 \pm 1.0) \cdot 10^{-4}\]

[Similar analysis to evaluation of hadronic contribution to $(g-2)_\mu$

Theoretical inputs at NNLO

- Radiative corrections are important!
  - E.g. consider tree-level EW unification relation: 
    \[ M^2_{\text{tree-level}} = \frac{M_Z^2}{2} \left( 1 + \sqrt{1 - \frac{3\alpha}{G_F M_Z^2}} \right) \]
  - This predicts: \( M_W = (79.964 \pm 0.005) \) GeV
  - Experiment: \( M_W = (80.385 \pm 0.015) \) GeV

- Without loop corrections: shift of 400 MeV, 27\( \sigma \) discrepancy!
Theoretical inputs at NNLO

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1. Experimental precision (<1%), better than typical loop factor ($\alpha \approx 1/137$)
   → Requires radiative corrections at 2-loop level.

   → After: inclusion of all relevant theoretical uncertainties.

(Part of focus of this talk …)
Theoretical inputs at NNLO

- Radiative corrections are important!
  - E.g. consider tree-level EW unification relation:
    \[
    M_{\text{tree-level}}^2 = \frac{M_Z^2}{2} \left(1 + \sqrt{\frac{8\pi\alpha}{G_F M_Z^2}}\right)
    \]
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    - Experiment: \(M_W = (80.385 \pm 0.015) \text{ GeV}\)

- Without loop corrections: shift of 400 MeV, 27\(\sigma\) discrepancy!

- In EW fit with Gfitter we use state-of-the-art calculations:
  - \(\sin^2\theta_{\text{eff}}\): Effective weak mixing angle
    - Full two-loop + leading beyond-two-loop form factor corrections
  - \(M_W\): Mass of the W boson
    \[\text{[M. Awramik et al., arXiv:0311148v2]}\]
    - Full two-loop + leading beyond-two-loop \(+\) 4-loop QCD correction
      \[\text{[Kuhn et al., hep-hp/0504055,0605201,0606232]}\]

- \(\Gamma_{\text{had}}\): QCD Adler functions at \(N^3\)LO
  \[\text{[P. A. Baikov et al., PRL108, 222003 (2012)]}\]
  - \(N^3\)LO prediction of the hadronic cross section

- \(\Gamma_i\): Partial Z decay widths
  \[\text{[A. Freitas, JHEP04, 070 (2014)]}\]

- \textbf{New: all EWPOs\(^(*)\) now described at 2-loop level or better!}
### Theory uncertainties from unknown H.O. terms

**Most important observables:**

<table>
<thead>
<tr>
<th>Observable</th>
<th>Exp. error</th>
<th>Theo. error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_W$</td>
<td>15 MeV</td>
<td>4 MeV</td>
</tr>
<tr>
<td>$\sin^2 \theta_{\text{eff}}$</td>
<td>$1.6 \cdot 10^{-4}$</td>
<td>$0.5 \cdot 10^{-4}$</td>
</tr>
<tr>
<td>$\Gamma_Z$</td>
<td>2.3 MeV</td>
<td>0.5 MeV</td>
</tr>
<tr>
<td>$\sigma^0_{\text{had}} = \sigma[e^+e^- \rightarrow Z \rightarrow \text{had.}]$</td>
<td>37 pb</td>
<td>6 pb</td>
</tr>
<tr>
<td>$R^0_b = \Gamma[Z \rightarrow b\bar{b}] / \Gamma[Z \rightarrow \text{had.}]$</td>
<td>$6.6 \cdot 10^{-4}$</td>
<td>$1.5 \cdot 10^{-4}$</td>
</tr>
</tbody>
</table>

**Theory uncertainties accounted for in EW fit (w/ Gauss constraints):**

- **Old setup:** two nuisance pars for theoretical uncertainties:
  - $\delta M_W$ (4 MeV), $\delta \sin^2 \theta_{\text{eff}}$ (4.7x10^{-5})

- **Newly included in EW fit setup:**
  - Full fermionic 2-loop corrections of partial Z decay widths (A. Freitas)
  - 6 corresponding nuisance parameters. ($\delta \Gamma_Z = 0.5$ MeV)
  - $\Gamma_{\text{had}}$ QCD Adler functions at N^3LO
  - 2 nuisance parameters.
  - Top quark mass: conversion from measurement to pole to MS-bar mass
  - Agnostic value used here: $\delta_{\text{theo}} m_t = 0.5$ GeV. *(more later)*
Latest experimental inputs:

- **Z-pole observables**: from LEP / SLC
  [ADLO+SLD, Phys. Rept. 427, 257 (2006)]

- \( M_W \) and \( \Gamma_W \) from LEP/Tevatron

- \( m_{\text{top}} \) latest avg from Tevatron+LHC
  [arXiv:1403.4427]

- \( m_c, m_b \) world averages (PDG)

- \( \Delta \alpha_{\text{had}}^{(5)}(M_Z^2) \) including \( \alpha_S \) dependency
  [Davier et al., EPJC 71, 1515 (2011)]

- \( M_H \) from LHC
  [arXiv:1406.3827, CMS-PAS-HIG-14-009]

7 (+10) free fit parameters:

- \( M_H, M_Z, \alpha_S(M_Z^2), \Delta \alpha_{\text{had}}^{(5)}(M_Z^2), m_t, m_c, m_b \)

- 10 theory nuisance parameters
  - e.g. \( \delta M_W \) (4 MeV), \( \delta \sin^2 \theta_{\text{eff}} \) (4.7x10^{-5})
### Electroweak Fit – SM Fit Results

#### From the Gfitter group:
www.cern.ch/gfitter

#### Left: full fit result

#### Middle: fit excluding the row

#### Right: not incl. theory errors

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Input value</th>
<th>Free in fit</th>
<th>Fit Result</th>
<th>w/o exp. input in line</th>
<th>w/o exp. input in line, no theo. unc</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_H$ [GeV]$^{(\odot)}$</td>
<td>125.14 ± 0.24</td>
<td>yes</td>
<td>125.14 ± 0.24</td>
<td>$93^{+25}_{-21}$</td>
<td>$93^{+24}_{-20}$</td>
</tr>
<tr>
<td>$M_W$ [GeV]</td>
<td>80.385 ± 0.015</td>
<td>–</td>
<td>80.364 ± 0.007</td>
<td>80.358 ± 0.008</td>
<td>80.358 ± 0.006</td>
</tr>
<tr>
<td>$\Gamma_W$ [GeV]</td>
<td>2.085 ± 0.042</td>
<td>–</td>
<td>2.091 ± 0.001</td>
<td>2.091 ± 0.001</td>
<td>2.091 ± 0.001</td>
</tr>
<tr>
<td>$M_Z$ [GeV]</td>
<td>91.1875 ± 0.0021</td>
<td>yes</td>
<td>91.1880 ± 0.0021</td>
<td>91.200 ± 0.011</td>
<td>91.2000 ± 0.010</td>
</tr>
<tr>
<td>$\Gamma_Z$ [GeV]</td>
<td>2.4952 ± 0.0023</td>
<td>–</td>
<td>2.4950 ± 0.0014</td>
<td>2.4946 ± 0.0016</td>
<td>2.4945 ± 0.0016</td>
</tr>
<tr>
<td>$\sigma^0_{had}$ [nb]</td>
<td>41.540 ± 0.037</td>
<td>–</td>
<td>41.484 ± 0.015</td>
<td>41.475 ± 0.016</td>
<td>41.474 ± 0.015</td>
</tr>
<tr>
<td>$R^0_\ell$</td>
<td>20.767 ± 0.025</td>
<td>–</td>
<td>20.743 ± 0.017</td>
<td>20.722 ± 0.026</td>
<td>20.721 ± 0.026</td>
</tr>
<tr>
<td>$A_{FB}^{0,\ell}$</td>
<td>0.0171 ± 0.0010</td>
<td>–</td>
<td>0.01626 ± 0.0001</td>
<td>0.01625 ± 0.0001</td>
<td>0.01625 ± 0.0001</td>
</tr>
<tr>
<td>$A_{\ell}^{(*)}$</td>
<td>0.1499 ± 0.0018</td>
<td>–</td>
<td>0.1472 ± 0.0005</td>
<td>0.1472 ± 0.0005</td>
<td>0.1472 ± 0.0004</td>
</tr>
<tr>
<td>$\sin^2\theta_{eff}(Q_{FB})$</td>
<td>0.2324 ± 0.0012</td>
<td>–</td>
<td>0.23150 ± 0.00006</td>
<td>0.23149 ± 0.00007</td>
<td>0.23150 ± 0.00005</td>
</tr>
<tr>
<td>$A_c$</td>
<td>0.670 ± 0.027</td>
<td>–</td>
<td>0.6680 ± 0.00022</td>
<td>0.6680 ± 0.00022</td>
<td>0.6680 ± 0.00016</td>
</tr>
<tr>
<td>$A_b$</td>
<td>0.923 ± 0.020</td>
<td>–</td>
<td>0.93463 ± 0.00004</td>
<td>0.93463 ± 0.00004</td>
<td>0.93463 ± 0.00003</td>
</tr>
<tr>
<td>$A_{FB}^{0,c}$</td>
<td>0.0707 ± 0.0035</td>
<td>–</td>
<td>0.0738 ± 0.0003</td>
<td>0.0738 ± 0.0003</td>
<td>0.0738 ± 0.0002</td>
</tr>
<tr>
<td>$A_{FB}^{0,b}$</td>
<td>0.0992 ± 0.0016</td>
<td>–</td>
<td>0.1032 ± 0.0004</td>
<td>0.1034 ± 0.0004</td>
<td>0.1033 ± 0.0003</td>
</tr>
<tr>
<td>$R_c^0$</td>
<td>0.1721 ± 0.0030</td>
<td>–</td>
<td>0.17226 $^{+0.00009}_{-0.00008}$</td>
<td>0.17226 ± 0.00008</td>
<td>0.17226 ± 0.00006</td>
</tr>
<tr>
<td>$R_b^0$</td>
<td>0.21629 ± 0.00066</td>
<td>–</td>
<td>0.21578 ± 0.00011</td>
<td>0.21577 ± 0.00011</td>
<td>0.21577 ± 0.00004</td>
</tr>
<tr>
<td>$m_c$ [GeV]</td>
<td>$1.27^{+0.07}_{-0.11}$</td>
<td>yes</td>
<td>$1.27^{+0.07}_{-0.11}$</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$m_b$ [GeV]</td>
<td>$4.20^{+0.17}_{-0.07}$</td>
<td>yes</td>
<td>$4.20^{+0.17}_{-0.07}$</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$m_t$ [GeV]</td>
<td>173.34 ± 0.76</td>
<td>yes</td>
<td>173.81 ± 0.85 $^{(\triangledown)}$</td>
<td>177.0 $^{+2.3}_{-2.4}$ $^{(\triangledown)}$</td>
<td>177.0 ± 2.3</td>
</tr>
<tr>
<td>$\Delta c^0_{had}(M_Z^2)^{(\dagger\Delta)}$</td>
<td>2757 ± 10</td>
<td>yes</td>
<td>2756 ± 10</td>
<td>2723 ± 44</td>
<td>2722 ± 42</td>
</tr>
<tr>
<td>$\alpha_s(M_Z^2)$</td>
<td>–</td>
<td>yes</td>
<td>0.1196 ± 0.0030</td>
<td>0.1196 ± 0.0030</td>
<td>0.1196 ± 0.0028</td>
</tr>
</tbody>
</table>
Results drawn as *pull values*: → deviations to the *indirect* determinations, divided by *total error*.

Total error: *error of direct measurement plus error from indirect determination*.

- Black: direct measurement (data)
- Orange: full fit
- Light-blue: fit excluding input from the row

*The prediction (light blue) is often more precise than the measurement!*
Electroweak Fit – SM Fit Results

- Results drawn as *pull values*: → deviations to the *indirect* determinations, divided by *total error*.

- Total error: *error of direct measurement plus error from indirect determination*.

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- *The prediction (light blue) is often more precise than the measurement!*
Electroweak Fit – SM Fit Results

- No individual value exceeds $3\sigma$
- Largest deviations in b-sector: $A^{0,b}_{FB}$ with $2.5\sigma$
  - $\rightarrow$ largest contribution to $\chi^2$
- Small pulls for $M_H$, $M_Z$, $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$, $\overline{m}_c$, $\overline{m}_b$ indicate that input accuracies exceed fit requirements
- Small changes from switching between 1 and 2-loop calc. for partial Z widths and small $M_W$ correction.
  - $\chi^2_{\text{min}}$(complete setup) = 17.8
  - $\chi^2_{\text{min}}$(1-loop Z width) = 18.0
  - $\chi^2_{\text{min}}$(no $M_W$ correction) = 17.4
  - $\chi^2_{\text{min}}$(no extra theory errors) = 18.2
Goodness of Fit

- Toy analysis: p-value for wrongly rejecting the SM = $21 \pm 2 \text{ (theo)} \%$
  - p-value is equivalent to $0.8\sigma$
  - Evaluated with 20k pseudo experiments – follows $\chi^2$ with 14 d.o.f.
  - For comparison: $\chi^2_{\text{min}} = 17.8 \rightarrow \text{Prob}(\chi^2_{\text{min}}, 14) = 21 \%$
- Large value of $\chi^2_{\text{min}}$ not due to inclusion of $M_H$ measurement.
  - Without $M_H$ measurement: $\chi^2_{\text{min}} = 16.3 \rightarrow \text{Prob}(\chi^2_{\text{min}}, 13) = 23\%$
Higgs results of the EW fit

- **Scan of $\Delta \chi^2$ profile versus $M_H$**
  - Grey band: fit w/o $M_H$ measurement
  - Blue line: full SM fit, with $M_H$ meas.
  - Fit w/o $M_H$ measurement gives: $M_H = 93^{+25}_{-21}$ GeV
  - Consistent at 1.3$\sigma$ with LHC measurements.

- **Bottom plot: impact of other most sensitive Higgs observables**
  - Determination of $M_H$ removing all sensitive observables except the given one.
  - Known tension (2.5$\sigma$) between $A_l$(SLD), $A^{0,b}_{FB}$, and $M_W$ clearly visible.
History of Higgs mass predictions

- The EW fits have always been able to predict the Higgs mass correctly!
Prediction of $W$ mass

- Scan of $\Delta \chi^2$ profile versus $M_W$
  - Also shown: SM fit with minimal inputs:
    $M_Z$, $G_F$, $\Delta \alpha_{\text{had}}^{(5)}(M_Z)$, $\alpha_s(M_Z)$, $M_H$, and fermion masses
  - Good consistency between total fit and SM w/ minimal inputs

- $M_H$ measurement allows for precise constraint on $M_W$
  - Agreement at 1.4$\sigma$
  - Fit result for indirect determination of $M_W$ (full fit w/o $M_W$):

$$M_W = 80.3584 \pm 0.0046_{m_t} \pm 0.0030_{\delta_{\text{theo}} m_t} \pm 0.0026_{M_Z} \pm 0.0018_{\Delta \alpha_{\text{had}}}$$
$$\pm 0.0020_{\alpha_s} \pm 0.0001_{M_H} \pm 0.0040_{\delta_{\text{theo}} M_W} \text{GeV},$$

$$= 80.358 \pm 0.008_{\text{tot}} \text{ GeV}.$$  

- More precise estimate of $M_W$ than the direct measurements!
  - Uncertainty on world average measurement: 15 MeV

Obtained with simple error propagation
Prediction of effective weak mixing angle

- Right: scan of $\Delta \chi^2$ profile versus $\sin^2 \theta^l_{\text{eff}}$
  - All sensitive measurements removed from the SM fit.
  - Also shown: SM fit with minimal inputs

- $M_H$ measurement allows for very precise constraint on $\sin^2 \theta^l_{\text{eff}}$

- Fit result for indirect determination of $\sin^2 \theta^l_{\text{eff}}$:

$$\sin^2 \theta^l_{\text{eff}} = 0.231488 \pm 0.000024_{m_t} \pm 0.000016_{\delta_{\text{theo}} m_t} \pm 0.000015_{M_Z} \pm 0.000035_{\Delta \alpha_{\text{had}}} \pm 0.000047_{\delta_{\text{theo}} \sin^2 \theta^f_{\text{eff}}} \pm 0.000010_{\alpha_S} \pm 0.000001_{M_H}$$

$$= 0.23149 \pm 0.00007_{\text{tot}},$$

- More precise than direct determination (from LEP/SLD)!
  - Uncertainty on LEP/SLD average: $1.6 \times 10^{-4}$

Obtained with simple error propagation
Prediction of top mass

- Shown: scan of $\Delta \chi^2$ profile versus $m_t$ (without $m_t$ measurement)
  - $M_H$ measurement allows for significant better constraint of $m_t$
  - Indirect determination consistent with direct measurements
    - Remember: fully obtained from radiative corrections!

- Indirect result: $m_t = 177.0^{+2.3}_{-2.4}$ GeV

Tevatron+LHC: $173.34 \pm 0.76$ GeV
new Tevatron-only: $174.34 \pm 0.64$ GeV
Scan of $M_W$ vs $m_t$, with the direct measurements excluded from the fit.

Results from Higgs measurement significantly reduces allowed indirect parameter space → corners the SM!

- Observed agreement demonstrates impressive consistency of the SM!
- Scan of $M_H$ vs $m_{top}$ (left) and $M_W$ vs $\sin^2\theta^l_{\text{eff}}$ (right), with direct measurements excluded from the fit.
- Again, significant reduction allowed indirect parameter space from Higgs mass measurement.

**Observables from radiative corrections**

- $M_W$ and $\sin^2\theta^l_{\text{eff}}$ have become *the* sensitive probes of new physics!
- Reason: both are ‘tree-level’ SM predictions.
Theoretical uncertainty on $m_{\text{top}}$

- $\delta_{\text{theo}} m_t$ : uncer. on conversion of measured top mass to MS-bar mass
  - Sources: ambiguity top mass definition, fragmentation process, pole→MS conv.
  - Predictions for $\delta_{\text{theo}} m_t$ : between 0.25 – 0.9 GeV or greater.
  [Moch etal, aX:1405.4781, Mangano: TOP'12, Buckley etal, aX:1101.2599, Juste etal: aX:1310.0799]
  - $\delta_{\text{theo}} m_t$ varied here between 0 and 1.5 GeV, in steps of 0.5 GeV.

Better assessment of $\delta_{\text{theo}} m_t$ of relevance for the EW fit. (see also backup)
Prediction for $\alpha_s(M_Z)$ from $Z\to$hadrons

- Scan of $\Delta\chi^2$ versus $\alpha_s$
  - Also shown: SM fit with minimal inputs: $M_Z$, $G_F$, $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$, $\alpha_s(M_Z)$, $M_H$, and fermion masses

- Determination of $\alpha_s$ at full $N^2$LO and partial $N^3$LO.
  - Most sensitive through total hadronic cross-section $\sigma^0_{\text{had}}$ and partial leptonic width $R^0_{\text{l}}$

$$\alpha_s(M^2_Z) = 0.1196 \pm 0.0028_{\text{exp}} \pm 0.0006\delta_{\text{theo}} R_{V,A} \pm 0.0006\delta_{\text{theo}} \Gamma_i \pm 0.0002\delta_{\text{theo}} \sigma^0_{\text{had}}$$

Most affected by new theory uncertainties
Before: $\delta_{\text{theo}} = 0.0001$

- In good agreement with value from $\tau$ decays, at $N^3$LO, and with WA.
  - (Improvements in precision only expected with ILC/GigaZ. See later.)
Beyond the SM

95% CL contours
w/o $M_W$ and $\kappa_V$ measurement

- $\lambda = 10$ TeV
- $\lambda = 5$ TeV
- $\lambda = 1$ TeV

$M_W$ world comb. ± 1σ

68% and 95% CL contours for
- direct $M_W$ and $\kappa_V$ measurements

$\kappa_V$ private LHC average ± 1σ

$$\Lambda = \frac{\lambda}{|1 - \kappa_V^2|}$$
Constraints on BSM models

- If energy scale of NP is high, BSM physics could appear dominantly through vacuum polarization corrections
  - Aka, “oblique corrections”

- Oblique corrections reabsorbed into electroweak form factors
  - $\Delta \rho$, $\Delta \kappa$, $\Delta r$ parameters, appearing in: $M_W^2$, $\sin^2 \theta_{\text{eff}}$, $G_F$, $\alpha$, etc.

- Electroweak fit sensitive to BSM physics through oblique corrections
  - Similar to sensitivity to top and Higgs loop corrections.

- Also implemented: extended parameters ($VWX$), correction to $Z \rightarrow \text{bb}$ couplings.

\[
O_{\text{meas}} = O_{\text{SM,REF}}(m_H, m_t) + c_S S + c_T T + c_U U
\]

- Oblique corrections from New Physics described through STU parametrization

- $S$: New Physics contributions to neutral currents
- $T$: Difference between neutral and charged current processes – sensitive to weak isospin violation
- $U$: (+$S$) New Physics contributions to charged currents. U only sensitive to $W$ mass and width, usually very small in NP models (often: $U=0$)

- Also implemented: extended parameters ($VWX$), correction to $Z \rightarrow \text{bb}$ couplings.

The ElectroWeak fit of Standard Model
Fit results for S, T, U

- S, T, U parameters obtained directly from fit to the EW observables.

- SM: $M_H = 125 \text{ GeV}, m_t = 173 \text{ GeV}$
  - This defines $(S, T, U) = (0,0,0)$
- S, T depend logarithmically on $M_H$

Fit result (with U floating):

<table>
<thead>
<tr>
<th></th>
<th>S</th>
<th>T</th>
<th>U</th>
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</thead>
<tbody>
<tr>
<td>S</td>
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<td>+0.90</td>
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<tr>
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<td></td>
</tr>
<tr>
<td>U</td>
<td>1</td>
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</tr>
</tbody>
</table>

- Also results for $Z \rightarrow b\bar{b}$ correction (see backup)

- Stronger constraints with U=0.

- No indication for new physics.
- Use this to constrain 4th gen, Ex-Dim, T-C, Higgs couplings (in backup)
Modified Higgs couplings

- Study of potential deviations of Higgs couplings from SM.
- BSM modeled as extension of SM through effective Lagrangian.
  - Consider leading corrections only.

- Popular benchmark model:
  - Scaling of Higgs-vector boson \((\kappa_V)\) and Higgs-fermion couplings \((\kappa_F)\) with no invisible/undetectable width
  - (Custodial symmetry is assumed.)
  - “Kappa parametrization”

- Main effect on EWPO due to modified Higgs coupling to gauge bosons \((\kappa_V)\)
  - Involving the longitudinal d.o.f.

- Most BSM models: \(\kappa_V < 1\)
  - Additional Higgses typically give \textit{positive} contribution to \(M_W\).
Modified Higgs couplings

- Main effect on EWPO due to Higgs coupling to gauge bosons ($\kappa_V$).
  
  \[
  S = \frac{1}{12\pi} \left(1 - \kappa_V^2\right) \log \left(\frac{\Lambda^2}{M_H^2}\right), \quad T = -\frac{3}{16\pi c_W^2} \left(1 - \kappa_V^2\right) \log \left(\frac{\Lambda^2}{M_H^2}\right), \quad \Lambda = \frac{\lambda}{\sqrt{|1 - \kappa_V^2|}}
  \]

- Formulas from: Espinosa et al [arXiv:1202.3697]

- Cut-off scale $\Lambda$ represents mass scale of new states that unitarize longitudinal gauge-boson scattering.
  - (As required in this model.)

- $\lambda$ is varied between 1 and 10 TeV, nominally fixed to 3 TeV ($4\pi v$).
Approximate reproduction of ATLAS/CMS results within limited public-info available.
Higgs coupling results

- Private LHC combination:
  - \( \kappa_V = 1.026^{+0.043}_{-0.043} \)
  - \( \kappa_F = 0.88^{+0.10}_{-0.09} \)

- Result from stand-alone EW fit:
  - \( \kappa_V = 1.03 \pm 0.02 \) (using \( \lambda = 3 \) TeV)
  - Implies NP-scale of \( \Lambda \gtrsim 13 \) TeV.

- Some dependency for \( \kappa_V \) in central value [1.02-1.04] and error [0.02-0.03] on cut-off scale \( \lambda \) [1-10 TeV].
  1. EW fit sofar more precise result for \( \kappa_V \) than current LHC experiments.
  2. EW fit has positive deviation of \( \kappa_V \) from 1.0.
     - (Many BSM models: \( \kappa_V < 1 \))
Prospects for the Standard Model fit
Two prospects scenarios: LHC, ILC/GigaZ

Prospects of EW fit tested for two (three) scenarios:

1. LHC Phase-1 = *before HL upgrade*
2. ILC *with GigaZ* (*)
3. (FCC-ee in backup)

(*) GigaZ:

- Operation of ILC at lower energies like Z-pole or WW threshold.
  - Allows to perform precision measurements of EW sector of the SM.
- At Z-pole, several billion Z’s can be studied within ~1-2 months.
  - Physics of LEP1 and SLC can be revisited with few days of data.

*In following studies:*

central values of input measurements adjusted to $M_H = 125$ GeV.

- *(Except where indicated.*)
Future Linear Collider can improve precision of EWPO’s tremendously.

- **WW threshold scan + kinematic reconstruction, to obtain $M_W$**
  - From threshold scan: $\delta M_W : 15 \rightarrow 5$ MeV

- **$t\bar{t}$ threshold scan, to obtain $m_t$**
  - Obtain $m_t$ indirectly from production cross section: $\delta m_t : 0.8 \rightarrow 0.1$ GeV
    - Dominated by conversion from threshold to MSbar mass.

- **Z pole measurements**
  - High statistics: $10^9$ Z decays: $\delta R^0_{\text{lep}} : 2.5 \cdot 10^{-2} \rightarrow 4 \cdot 10^{-3}$
  - With polarized beams, uncertainty on $\delta A^{0,f}_{LR} : 10^{-3} \rightarrow 10^{-4}$, which translates to $\delta \sin^2 \theta^\text{eff} : 1.6 \cdot 10^{-4} \rightarrow 1.3 \cdot 10^{-5}$

- **$H\rightarrow ZZ$ and $H\rightarrow WW$ couplings**: measured at 1% precision.

ILC prospects: from ILC TDR (Vol-2).
Prospects of EW fit for: LHC Phase-1

**LHC Phase-1 (300/fb)**

- *W mass measurement*: $\delta M_W : 15 \rightarrow 8$ MeV
- *Final top mass measurement* $m_t : \delta m_t : 0.8 \rightarrow 0.6$ GeV
- *$H\rightarrow ZZ$ and $H\rightarrow WW$ couplings*: measured at 3% precision.

LHC prospects: possibly optimistic scenario, but not impossible.
Prospects of EW fit

**LHC Phase-1 (300/fb)**
- *W mass measurement*: $\delta M_W : 15 \rightarrow 8$ MeV
- *Final top mass measurement* $m_t : \delta m_t : 0.8 \rightarrow 0.6$ GeV
- *$H \rightarrow ZZ$ and $H \rightarrow WW$ couplings*: measured at 3% precision.

*For both LHC and ILC:*
- Low-energy data results to improve $\Delta\alpha_{\text{had}}$:
  - ISR-based (BABAR), KLOE-II, VEPP-2000 (at energy below cc resonance), and BESIII $e^+e^-$ cross-section measurements (around cc resonance).
  - Plus: improved $\alpha_s$ (from reliable Lattice predictions): $\Delta\alpha_{\text{had}}: 10^{-4} \rightarrow \mathbf{5 \cdot 10^{-5}}$

- Assuming $\sim 25\%$ of today’s theoretical uncertainties on $M_W$ and $\sin^2\theta_{\text{eff}}$
  - *Implies ambitions three-loop electroweak calculations!*
    - $\delta M_W (4 \rightarrow 1$ MeV), $\delta \sin^2\theta_{\text{eff}} (4.7 \times 10^{-5} \rightarrow 1 \times 10^{-5})$ (from Snowmass report)
  - Partial Z decay widths at 3-loop level: factor 4 improvement
  - LHC: top quark mass theo uncertainty: $0.50 \rightarrow 0.25$ GeV
Indirect prediction $M_H$ dominated by experimental uncertainties.

- Present: $\sigma(M_H) = +31_{-26}^{+10_{-8}} \text{(exp)} +10_{-8} \text{(theo)} \text{ GeV}$
- LHC: $\sigma(M_H) = +20_{-18}^{+3.9_{-3.8}} \text{(exp)} +3.9_{-3.8} \text{(theo)} \text{ GeV}$
- ILC: $\sigma(M_H) = +6.9_{-6.6}^{+2.5_{-2.3}} \text{(exp)} +2.5_{-2.3} \text{(theo)} \text{ GeV}$

Logarithmic dependency on $M_H \rightarrow \text{cannot compete with direct } M_H \text{ meas.}$

If EWP-data central values unchanged, i.e. keep favoring low value of Higgs mass (93 GeV), $\sim 5\sigma$ discrepancy with measured Higgs mass.
Huge reduction of uncertainty on indirect determinations of $m_t$, $m_W$, and $\sin^2\theta_{\text{eff}}$, by a factor of 3 or more.

Assuming central values of $m_t$ and $M_W$ do not change, (at ILC) a deviation between the SM prediction and the direct measurements would be prominently visible.
### Impact of individual uncertainties

- Breakdown of individual contributions to errors of $M_W$ and $\sin^2\theta_{\text{eff}}$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\delta_{\text{meas}}$</th>
<th>$\delta_{\text{fit}}^{\text{tot}}$</th>
<th>$\delta_{\text{fit}}^{\text{theo}}$</th>
<th>$\delta_{\text{fit}}^{\text{exp}}$</th>
<th>$\delta M_W$</th>
<th>$\delta M_Z$</th>
<th>$\delta m_t$</th>
<th>$\delta \sin^2\theta_{\text{eff}}^f$</th>
<th>$\delta \Delta \alpha_{\text{had}}$</th>
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<td>6.0</td>
<td>-</td>
<td>2.5</td>
<td>4.3</td>
<td>5.1</td>
<td>1.6</td>
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<tr>
<td>$\sin^2\theta_{\text{eff}}^{(o)}$</td>
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<td>6.6</td>
<td>4.9</td>
<td>4.5</td>
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<td>1.2</td>
<td>2.0</td>
<td>-</td>
<td>3.4</td>
<td>1.2</td>
</tr>
</tbody>
</table>

**Present uncertainties**

| $M_W$ [MeV]     | 8                       | 5.5                                 | 1.8                                 | 5.2                                 | -            | 2.5          | 3.5          | 4.8                           | 0.8              | 2.6         |
| $\sin^2\theta_{\text{eff}}^{(o)}$ | 16                      | 3.0                                 | 1.1                                 | 2.8                                 | 2.5          | 1.1          | 1.4          | -                             | 1.5              | 0.9         |
| $m_t$ [GeV]     | 0.6                     | 1.5                                 | 0.2                                 | 1.5                                 | 1.3          | 0.4          | -            | 1.2                           | 0.2              | 0.5         |

**LHC prospects**

| $M_W$ [MeV]     | 5                       | 2.3                                 | 1.3                                 | 1.9                                 | -            | 1.7          | 0.3          | 1.3                           | 0.7              | 0.3         |
| $\sin^2\theta_{\text{eff}}^{(o)}$ | 1.3                     | 2.3                                 | 1.0                                 | 2.0                                 | 1.7          | 1.2          | 0.2          | -                             | 1.5              | 0.1         |
| $M_Z$ [MeV]     | 2.1                     | 2.7                                 | 1.0                                 | 2.6                                 | 2.5          | -            | 0.4          | 1.3                           | 1.9              | 0.2         |

**ILC/GigaZ prospects**

| $M_W$ [MeV]     | 8                       | 5.5                                 | 1.8                                 | 5.2                                 | -            | 1.7          | 0.3          | 1.3                           | 0.7              | 0.3         |
| $\sin^2\theta_{\text{eff}}^{(o)}$ | 1.3                     | 2.3                                 | 1.0                                 | 2.0                                 | 1.7          | 1.2          | 0.2          | -                             | 1.5              | 0.1         |
| $M_Z$ [MeV]     | 2.1                     | 2.7                                 | 1.0                                 | 2.6                                 | 2.5          | -            | 0.4          | 1.3                           | 1.9              | 0.2         |

\(^{(o)}\) In units of $10^{-5}$.

- $M_W$ and $\sin^2\theta_{\text{eff}}$ are sensitive probes of new physics! For all scenarios.
- At ILC/GigaZ, precision of $M_Z$ will become important again.
For STU parameters, **improvement of factor of >3** is possible at ILC.

Again, at ILC a deviation between the SM predictions and direct measurements would be prominently visible.

Competitive results between EW fit and Higgs coupling measurements!

- (At level of 1%).
Conclusion and Today’s prospects

- Including $M_H$ measurement, for first time SM is fully over-constrained!
  - $M_H$ consistent at $1.3\sigma$ with indirect prediction from EW fit.
  - p-Value of global electroweak fit of SM: 21% (pseudo-experiments)
- New: $N^2$LO calcs and theo. uncertainties for all relevant observables.
  - $\delta_{\text{theo}} m_t$ starting to become relevant.

Knowledge of $M_H$ dramatically improves SM prediction of key observables
- $M_W$ ($28\rightarrow 8$ MeV), $\sin^2\theta_{\text{eff}}$ ($2.3\times 10^{-5}\rightarrow 0.7\times 10^{-5}$), $m_t$ ($6.2\rightarrow 2.5$ GeV)
- Improved accuracies set benchmark for new direct measurements!

- $\delta M_W$ (indirect) = 8 MeV
  - Large contributions to $\delta M_W$ from top and unknown higher-order EW corrections
- $\delta M_W$ (direct) = 15 MeV

- Including new data electroweak fits remain very interesting in the next years!
- Latest results always available at: http://cern.ch/Gfitter

Thanks!
A Generic Fitter Project for HEP Model Testing
### Input correlation coefficients between Z pole measurements

<table>
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<th>$M_Z$</th>
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<th>$\sigma_{\text{had}}^0$</th>
<th>$R_{\ell}^0$</th>
<th>$A_{\text{FB}}^{0,\ell}$</th>
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<th>$A_{\text{FB}}^{0,b}$</th>
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<th>$A_b$</th>
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<td>-0.18</td>
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</table>

Table 2: Correlation matrices for observables determined by the $Z$ lineshape fit (left), and by heavy flavour analyses at the $Z$ pole (right) [56].
Radiator Functions

- Partial widths are defined inclusively: contain both QCD and QED contributions.
- Corrections expressed as so-called radiator functions $R_{A,f}$ and $R_{V,f}$

\[ \Gamma_{f \bar{f}} = N_c^f \frac{G_F M_Z^3}{6\sqrt{2}\pi} \left( |g_{A,f}|^2 R_{A,f} + |g_{V,f}|^2 R_{V,f} \right)^2 \]

- High sensitivity to the strong coupling $\alpha_s$
- Recently, full four-loop calculation of QCD Adler function became available ($N^3\text{LO}$)
- Much-reduced scale dependence!
- *Theoretical uncertainty of* $\sim0.15$ MeV, compared with experimental uncertainty of 2.0 MeV.


Calculation of $M_W$

- Full EW one- and two-loop calculation of fermionic and bosonic contributions.
- One- and two-loop QCD corrections and leading terms of higher order corrections.
- Results for $\Delta r$ include terms of order $O(\alpha)$, $O(\alpha \alpha_s)$, $O(\alpha^2)$, $O(\alpha^2_{\text{ferm}})$, $O(\alpha^2_{\text{bos}})$, $O(\alpha^2 \alpha_s m_t^4)$, $O(\alpha^3 m_t^6)$
- Uncertainty estimate:
  - Missing terms of order $O(\alpha^2 \alpha_s)$: about 3 MeV (from $O(\alpha^2 \alpha_s m_t^4)$)
  - Electroweak three-loop correction $O(\alpha^3)$: < 2 MeV
  - Three-loop QCD corrections $O(\alpha_s^3)$: < 2 MeV
- Total: $\delta M_W \approx 4$ MeV

[M Awramik et al., Phys. Rev. D69, 053006 (2004)]
Calculation of $\sin^2(\theta^l_{\text{eff}})$

- **Effective mixing angle:**

  \[
  \sin^2 \theta^l_{\text{eff}} = \left(1 - \frac{M_W^2}{M_Z^2}\right) \left(1 + \Delta \kappa\right)
  \]

- Two-loop EW and QCD correction to $\Delta \kappa$ known, leading terms of higher order QCD corrections.

- Fermionic two-loop correction about $10^{-3}$, whereas bosonic one $10^{-5}$.

- Uncertainty estimate obtained with different methods, geometric progression, leading to total of:
  \[\delta \sin^2(\theta^l_{\text{eff}}) = 4.7 \times 10^{-5}\]
Uncertainty in Top mass definition

- Difficult to define a pole mass for heavy, unstable and colored particle.
  - Single top decays before hadronizing. To have colorless final states, additional quarks needed.
  - Non-perturb. color-reconnection effects in fragmentation → biases in simulation.
  - ‘Renormalon’ ambiguity in top mass definition.
    - For pole mass, not for MS-bar scheme.
  - Impact of finite top width effects.

- Result: \(m_t^{\text{exp}} \neq m_t^{\text{pole}}\), and event-dependent.

- The top mass extracted in hadron collisions is not well defined below a precision of \(O(\Gamma_t) \sim 1\) GeV

- Hard to estimate additional theo. uncertainties. With 0.5 GeV on \(m_t\):
  - \(M_H = 90^{+34}_{-21}\) GeV, \(M_W = 80.359\pm0.013\) GeV, \(\sin^2\theta^\text{eff} = 0.23148\pm0.00010\).
  - → Sofar only small deterioration in precision.
Interesting Top pole mass measurement

- From: ATLAS-CONF-2014-053:
  “top-quark pole mass measurement from ttbar+1jet events”

- Through study of inverse of invariant mass of ttbar+1jet system (quantity: $\rho_S$).

- Free of MC→pole mass conversion uncertainty.

\[ \Rightarrow m_t^{\text{pole}} = 173.7 \pm 1.5 \text{(stat)} \pm 1.4 \text{(syst)} +1.0^{+0.5}_{-0.5} \text{(theo)} \text{ GeV} \]

- Great to see these efforts ongoing!
  - Similar measurements / tests ongoing at CMS.
**Moriond 2011: Prediction for Higgs mass**

- **LEP + Tevatron (Fall 2010):**
  - CL\(^{2s}_{s+b}\) central value \(\pm 1\sigma\): \(M_H = 120.2^{+17.9}_{-5.2}\) GeV
  - \(2\sigma\) interval:
    - \(-2\ln Q\): \([115,152]\) GeV
    - \(CL_{s+b}^{2\text{-sided}}\): \([114,155]\) GeV

- **LEP + Tevatron (Moriond 2011):**
  - CL\(^{2s}_{s+b}\) central value \(\pm 1\sigma\): \(M_H = 120.2^{+12.3}_{-4.7}\) GeV
  - \(2\sigma\) interval:
    - \(-2\ln Q\): \([115,138]\) GeV
    - \(CL_{s+b}^{2\text{-sided}}\): \([114,149] \cup [152,155]\) GeV

- **Fit with LEP + Tevatron + LHC (H\(\rightarrow\)WW) searches (Moriond 2011):**
  - Central value unchanged
  - \(2\sigma\) interval:
    - \(-2\ln Q\): \([115,137]\) GeV
    - \(CL_{s+b}^{2\text{-sided}}\): \([114,14?]\) GeV
Low energy observables with interesting precision will soon become available.

The ElectroWeak fit of Standard Model
Two prospects scenarios: LHC, ILC/GigaZ

- Uncertainty estimates used:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Experimental input $[\pm 1\sigma_{\text{exp}}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_H$ [GeV]</td>
<td>Present &lt; 0.1 LHC &lt; 0.1 ILC/GigaZ</td>
</tr>
<tr>
<td>$M_W$ [MeV]</td>
<td>15 8 5</td>
</tr>
<tr>
<td>$M_Z$ [MeV]</td>
<td>2.1 2.1 2.1</td>
</tr>
<tr>
<td>$m_t$ [GeV]</td>
<td>0.8 0.6 0.1</td>
</tr>
<tr>
<td>$\sin^2 \theta_{\text{eff}} [10^{-5}]$</td>
<td>16 16 1.3</td>
</tr>
<tr>
<td>$\Delta \alpha_{\text{had}}^5 (M_Z^2) [10^{-5}]$</td>
<td>10 4.7 4.7</td>
</tr>
<tr>
<td>$R^0_l [10^{-3}]$</td>
<td>25 25 4</td>
</tr>
<tr>
<td>$\alpha_S (M_Z^2) [10^{-4}]$</td>
<td>– – –</td>
</tr>
</tbody>
</table>

- ILC prospects from: ILC TDR (Vol-2).
- Theoretical uncertainty estimates from recent Snowmass report

- Central values of input measurements adjusted to $M_H = 126$ GeV.
FCC-ee prospects

- From TLEP prospects: arXiv:1308.6176

Table 9: Selected set of precision measurements at TLEP. The statistical errors have been determined with (i) a one-year scan of the Z resonance with 50% data at the peak, leading to $71^{+10}_{-9}$ Z visible decays, with resonant depolarization of single bunches for energy calibration at $0(20\text{min})$ intervals; (ii) one year at the Z peak with 40% longitudinally-polarized beams and a luminosity reduced to 20% of the nominal luminosity; (iii) a one-year scan of the WW threshold (around 161 GeV), with resonant depolarization of single bunches for energy calibration at $0(20\text{min})$ intervals; and (iv) a five-years scan of the $t\bar{t}$ threshold (around 346 GeV). The systematic uncertainties indicated below are only a “first look” estimate and will be revisited in the course of the design study.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Physics</th>
<th>Present precision</th>
<th>Statistical uncertainty</th>
<th>Systematic uncertainty</th>
<th>Key</th>
<th>Challenge</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_Z$ (keV)</td>
<td>Input</td>
<td>91187500 ± 2100</td>
<td>Z Line shape scan</td>
<td>5 keV</td>
<td>$E_{\text{beam}}$ calibration</td>
<td>QED corrections</td>
</tr>
<tr>
<td>$\Gamma_Z$ (keV)</td>
<td>$\Delta \rho$ (not $\Delta \alpha_{\text{had}}$)</td>
<td>2495200 ± 2300</td>
<td>Z Line shape scan</td>
<td>8 keV</td>
<td>$E_{\text{beam}}$ calibration</td>
<td>QED corrections</td>
</tr>
<tr>
<td>$R_t$</td>
<td>$\alpha_t, \alpha_b$</td>
<td>20.767 ± 0.025</td>
<td>Z Peak</td>
<td>0.0001</td>
<td>Statistics</td>
<td>QED corrections</td>
</tr>
<tr>
<td>$N_\nu$</td>
<td>PMNS Unitarity, ...</td>
<td>2.984 ± 0.008</td>
<td>Z Peak</td>
<td>0.00008</td>
<td>&lt; 0.001</td>
<td>Bhabha scatter</td>
</tr>
<tr>
<td>$N_\nu$</td>
<td>... and sterile $\nu$'s</td>
<td>2.92 ± 0.05</td>
<td>$Z$ Peak, 161 GeV</td>
<td>0.001</td>
<td>&lt; 0.001</td>
<td>Statistics</td>
</tr>
<tr>
<td>$R_b$</td>
<td>$\delta_b$</td>
<td>0.21929 ± 0.00006</td>
<td>Z Peak</td>
<td>0.000003</td>
<td>&lt; 0.000006</td>
<td>Statistics, small IP</td>
</tr>
<tr>
<td>$A_{LR}$</td>
<td>$\Delta \rho, \delta_1, \delta_2, \Delta \alpha_{\text{had}}$</td>
<td>0.1514 ± 0.0022</td>
<td>Z peak, polarized</td>
<td>0.00015</td>
<td>&lt; 0.00015</td>
<td>4 bunch scheme, 2exp</td>
</tr>
<tr>
<td>$m_W$ (MeV)</td>
<td>$\Delta \rho, \delta_1, \delta_2$</td>
<td>80385 ± 15</td>
<td>WW threshold scan</td>
<td>0.3 MeV</td>
<td>&lt; 0.5 MeV</td>
<td>$E_{\text{beam}}$, Statistics</td>
</tr>
<tr>
<td>$m_{t\bar{t}}$ (MeV)</td>
<td>Input</td>
<td>173200 ± 900</td>
<td>tt threshold scan</td>
<td>10 MeV</td>
<td>&lt; 10 MeV</td>
<td>Statistics</td>
</tr>
</tbody>
</table>
Experimental inputs – Predicted uncertainties

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Present</th>
<th>LHC</th>
<th>ILC/GigaZ</th>
<th>TLEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_H$ [GeV]</td>
<td>$0.4 \Rightarrow 0.1$</td>
<td>$&lt; 0.1$</td>
<td>$&lt; 0.1$</td>
<td>$&lt; 0.1$</td>
</tr>
<tr>
<td>$M_W$ [MeV]</td>
<td>$15 \Rightarrow 8$</td>
<td>$5 \Rightarrow 1.3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_Z$ [MeV]</td>
<td>$2.1$</td>
<td>$2.1$</td>
<td>$2.1 \Rightarrow 0.1$</td>
<td></td>
</tr>
<tr>
<td>$m_t$ [GeV]</td>
<td>$0.9 \Rightarrow 0.6$</td>
<td>$0.1 \Rightarrow 0.08$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Gamma_Z$ [MeV]</td>
<td>$2.3 \Rightarrow 2.3$</td>
<td>$0.8 \Rightarrow 0.1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sin^2 \theta^l_{\text{eff}}$ [\cdot 10^{-5}]</td>
<td>$16 \Rightarrow 16$</td>
<td>$1.3 \Rightarrow 0.3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^0_l$ [\cdot 10^{-3}]</td>
<td>$25 \Rightarrow 25$</td>
<td>$4 \Rightarrow 1.3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta \alpha^5_{\text{had}}(M_Z^2)$ [\cdot 10^{-5}]</td>
<td>$10 \Rightarrow 4.7$</td>
<td>$4.7 \Rightarrow 4.7$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_S(M_Z^2)$ [\cdot 10^{-4}]</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$\delta_{\text{th}} M_W$ [MeV]</td>
<td>$4 \Rightarrow 1$</td>
<td>$1 \Rightarrow 1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta_{\text{th}} \sin^2 \theta^l_{\text{eff}}$ [\cdot 10^{-5}]</td>
<td>$4.7 \Rightarrow 1$</td>
<td>$1 \Rightarrow 1$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

FCC-ee scenario:

- **Preliminary estimates**
- Clearly not the same level of understanding as LHC or ILC.
- Uncertainties may turn out completely different.
  - From arXiv:1308.6176,
  - and Snowmass report.
    - Of these two, we take most conservative estimate.
- Note: top mass dominated by theoretical uncertainty.
- Higher statistics
- From beam energy precision: improved $M_Z$ and $\Gamma_Z$
Prospects of the EW fit: Higgs mass (126 GeV)

- Logarithmic dependency on $M_H$ → cannot compete with direct $M_H$ meas.

- Indirect prediction $M_H$ dominated by theory uncertainties.
  - ILC with (without) theory errors: $M_H = 126^{+10\,-9}(\pm7)$ GeV
  - ILC with present-day theory uncertainties: $M_H = 126^{+20\,-17}$ GeV
  - FCC-ee with (without) theory errors: $M_H = 126 \pm 5 (\pm3)$ GeV
If EWP-data central values are unchanged, i.e. they keep favoring low value of Higgs mass (94 GeV), >5σ discrepancy with measured Higgs mass.

- In both ILC and FCC-ee scenarios.
Prospects of the EW fit: W mass and $\sin^2\theta_{\text{eff}}$

Present / LHC / ILC

FCC-ee scenario

Future scenario: $\delta M_Z = 0.1$ GeV, $\delta M_W = 0.1$ MeV, $\delta \Gamma_Z = 0.1$ MeV, $\delta M_H = 1.3$ MeV, $\delta m_t = 80$ MeV, $\delta \sin^2(\theta_{\text{eff}}) = 3 \times 10^{-6}$, $\delta R^{\text{lep}}_b = 1.25 \times 10^{-3}$, $\delta \Delta\alpha_{\text{had}} = 4.7 \times 10^{-5}$.

Future scenario:

Prospects for ILC/GigaZ ($\delta_{\text{theo}} = \text{Gauss}$)

Direct measurement (present / future)

Prospects for LHC

Prospects for ILC/GigaZ ($\delta_{\text{theo}} = R^{\text{fit}}$)

Direct measurement (present / LHC / ILC)

Prospects for FCC-ee scenario

Direct measurement (present / future)

Present SM fit

Prospects for Present / LHC / ILC

Direct measurement (present / future)

Future scenario:

Prospects for Present SM fit

Direct measurement (present / future)

Prospects for ILC/GigaZ ($\delta_{\text{theo}} = R^{\text{fit}}$)

Direct measurement (present / future)

Direct measurement (present / future)

Max Baak (CERN)

The ElectroWeak fit of Standard Model and Beyond
Prospects of the EW fit: W mass versus $\sin^2\theta^l_{\text{eff}}$

- **Huge reduction of uncertainty on indirect determinations of $m_W$, and $\sin^2\theta^l_{\text{eff}}$, by a factor of $\gtrsim 3$ ($\gtrsim 4-5$) at ILC (FCC-ee).**

- **Assuming central values of $M_W$ and $\sin^2\theta^l_{\text{eff}}$ do not change, a deviation between the SM prediction and the direct measurements would be prominently visible, at both ILC and FCC-ee.**
  - But also in LHC-300 scenario, from improved theory uncertainties.
### Confrontation of measurement and prediction

- Breakdown of individual contributions to errors of $M_W$ and $\sin^2\theta_{\text{eff}}$
- Parametric uncertainties (not the full fit).

#### Table: Error due to uncertainty

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Scenario</th>
<th>$\delta_{\text{meas}}$</th>
<th>$\delta_{\text{pred}}$</th>
<th>$\delta_{\exp}$</th>
<th>$\delta_{M_Z}$</th>
<th>$\delta_{m_t}$</th>
<th>$\delta_{\Delta\alpha_{\text{had}}}$</th>
<th>$\delta_{\alpha_s}$</th>
<th>$\delta_{\text{theo}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_W$ [MeV]</td>
<td>Present</td>
<td>15</td>
<td>10.4</td>
<td>6.4</td>
<td>2.6</td>
<td>5.2</td>
<td>1.8</td>
<td>1.7</td>
<td>4.0</td>
</tr>
<tr>
<td></td>
<td>LHC</td>
<td>8</td>
<td>5.8</td>
<td>4.8</td>
<td>2.6</td>
<td>3.6</td>
<td>0.9</td>
<td>1.7</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>ILC</td>
<td>5</td>
<td>3.8</td>
<td>2.8</td>
<td>2.6</td>
<td>0.6</td>
<td>0.9</td>
<td>0.4</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>Future</td>
<td>1.3</td>
<td>2.0</td>
<td>1.0</td>
<td>0.1</td>
<td>0.5</td>
<td>0.9</td>
<td>0.3</td>
<td>1.0</td>
</tr>
<tr>
<td>$\sin^2\theta_{\text{eff}}$</td>
<td>Present</td>
<td>16</td>
<td>9.5</td>
<td>4.8</td>
<td>1.5</td>
<td>2.8</td>
<td>3.5</td>
<td>1.0</td>
<td>4.7</td>
</tr>
<tr>
<td></td>
<td>LHC</td>
<td>16</td>
<td>4.1</td>
<td>3.1</td>
<td>1.5</td>
<td>1.9</td>
<td>1.6</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>ILC</td>
<td>1.3</td>
<td>3.2</td>
<td>2.2</td>
<td>1.5</td>
<td>0.3</td>
<td>1.6</td>
<td>0.2</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>Future</td>
<td>0.3</td>
<td>2.7</td>
<td>1.7</td>
<td>0.1</td>
<td>0.3</td>
<td>1.6</td>
<td>0.2</td>
<td>1.0</td>
</tr>
</tbody>
</table>

($^o$)In units of $10^{-5}$.

- $M_W$ and $\sin^2\theta_{\text{eff}}$ are sensitive probes of new physics! In all scenarios.
- At ILC/GigaZ, precision of $M_Z$ will become important again.
- At FCC-ee (‘Future’), limited by external inputs: theory errors and $\Delta\alpha_{\text{had}}$. 
Prospects of the EW fit: W versus top mass

- Huge reduction of uncertainty on indirect determinations of \( m_t \) and \( m_W \) by a factor of \( \approx 3 \) (\( \approx 5 \)) at ILC (FCC-ee).

- Assuming central values of \( m_t \) and \( M_W \) do not change, a deviation between the SM prediction and the direct measurements would be prominently visible.
Prospects of EW fit: S versus T

For STU parameters, improvement of factor of $\geq 4$ ($\geq 10$) is possible at ILC (FCC-ee).

Again, at both ILC and FCC-ee a deviation between the SM predictions and direct measurements would be prominently visible.
Predicted uncertainties from EW fit

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\delta_{\text{meas}}$</th>
<th>$\delta_{\text{fit}}^\text{tot}$</th>
<th>$\delta_{\text{fit}}^\text{exp}$</th>
<th>$\delta_{\text{fit}}^\text{theo}$</th>
<th>$\delta M_W$</th>
<th>$\delta M_Z$</th>
<th>$\delta m_t$</th>
<th>$\delta \sin^2\theta^\ell_{\text{eff}}^{(c)}$</th>
<th>$\delta \Delta \alpha_{\text{had}}^{(c)}$</th>
<th>$\delta \alpha_S^{(\Delta)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_H$ [GeV]</td>
<td>&lt; 0.1</td>
<td>+0.9</td>
<td>-0.6</td>
<td>2.7</td>
<td>4.2</td>
<td>4.4</td>
<td>0.9</td>
<td>3.1</td>
<td>4.2</td>
<td>0.6</td>
</tr>
<tr>
<td>$M_W$ [MeV]</td>
<td>5</td>
<td>3.6</td>
<td>1.9</td>
<td>1.7</td>
<td>1.7</td>
<td>0.3</td>
<td>1.2</td>
<td>0.7</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>$M_Z$ [MeV]</td>
<td>2.1</td>
<td>3.7</td>
<td>2.6</td>
<td>1.1</td>
<td>2.4</td>
<td>-</td>
<td>0.5</td>
<td>1.3</td>
<td>1.9</td>
<td>0.3</td>
</tr>
<tr>
<td>$m_t$ [GeV]</td>
<td>0.1</td>
<td>1.0</td>
<td>0.7</td>
<td>0.3</td>
<td>0.5</td>
<td>-</td>
<td>0.3</td>
<td>0.4</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\sin^2\theta^\ell_{\text{eff}}^{(c)}$</td>
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<td>3.2</td>
<td>2.0</td>
<td>1.2</td>
<td>1.7</td>
<td>1.2</td>
<td>0.2</td>
<td>-</td>
<td>1.5</td>
<td>0.1</td>
</tr>
<tr>
<td>$\Delta \alpha_{\text{had}}^{(c)}$</td>
<td>4.7</td>
<td>8.6</td>
<td>5.7</td>
<td>2.9</td>
<td>2.5</td>
<td>4.2</td>
<td>0.8</td>
<td>3.9</td>
<td>-</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Future prospects

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\delta_{\text{meas}}$</th>
<th>$\delta_{\text{fit}}^\text{tot}$</th>
<th>$\delta_{\text{fit}}^\text{exp}$</th>
<th>$\delta_{\text{fit}}^\text{theo}$</th>
<th>$\delta M_W$</th>
<th>$\delta M_Z$</th>
<th>$\delta m_t$</th>
<th>$\delta \sin^2\theta^\ell_{\text{eff}}^{(c)}$</th>
<th>$\delta \Delta \alpha_{\text{had}}^{(c)}$</th>
<th>$\delta \alpha_S^{(\Delta)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_H$ [GeV]</td>
<td>&lt; 0.1</td>
<td>5.3</td>
<td>3.3</td>
<td>2.0</td>
<td>3.0</td>
<td>0.3</td>
<td>1.0</td>
<td>$^{+0.0}_{-1.2}$</td>
<td>3.2</td>
<td>0.6</td>
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<tr>
<td>$M_W$ [MeV]</td>
<td>1.3</td>
<td>1.9</td>
<td>0.4</td>
<td>1.5</td>
<td>-</td>
<td>0.1</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$M_Z$ [MeV]</td>
<td>0.1</td>
<td>1.5</td>
<td>1.0</td>
<td>0.5</td>
<td>1.0</td>
<td>-</td>
<td>0.3</td>
<td>0.9</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>$m_t$ [GeV]</td>
<td>0.08</td>
<td>0.38</td>
<td>0.24</td>
<td>0.14</td>
<td>0.24</td>
<td>0.03</td>
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<td>0.22</td>
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<tr>
<td>$\sin^2\theta^\ell_{\text{eff}}^{(c)}$</td>
<td>0.3</td>
<td>$^{+2.8}_{-2.4}$</td>
<td>1.4</td>
<td>$^{+1.5}_{-1.1}$</td>
<td>1.2</td>
<td>0.1</td>
<td>-</td>
<td>1.3</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>$\Delta \alpha_{\text{had}}^{(c)}$</td>
<td>4.7</td>
<td>0.4</td>
<td>0.1</td>
<td>0.3</td>
<td>0.1</td>
<td>0.1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

(°) In units of $10^{-5}$. (Δ) In units of $10^{-4}$

- Breakdown of uncertainties derived from EW fit. (Note: correlated errors.)
- Compared to parametric breakdown: reduced experimental, but increased theory errors. Slightly smaller total errors.
Hunt for the Higgs

- $M_H$ was last missing input parameter of the electroweak fit
- Indirect determination from EW fit (2012): $M_H = 96^{+31}_{-24}$ GeV
  - With direct limits incorporated in the EW fit: $M_H = 120^{+12}_{-5}$ GeV