Flavour physics
as a test of the standard model
and a probe of new physics

Marcella Bona

School of Physics and Astronomy
Queen Mary University of London

Royal Society Leverhulme Trust
Senior Research Fellow

Online Seminar, Particle Physics Group, Birmingham University
November 10th, 2021
very briefly:
- introduction and motivations
- the tool: the Unitarity Triangle fit

Standard Model fit
- Standard model constraints
- checking for tensions
- Standard Model predictions

Beyond the Standard Model:
- model-independent analysis
- New-physics-specific constraints
- New-physics scale analysis
The CP symmetry is violated in any field theory having in the Lagrangian at least one phase that cannot be re-absorbed. The mass eigenstates are not eigenstates of the weak interaction. This feature of the Standard Model Hamiltonian produces the (unitary) mixing matrix $V_{\text{CKM}}$.

$$
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\approx
\begin{pmatrix}
1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\bar{\rho} - i\bar{\eta}) \\
-\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\
A\lambda^3(1 - \bar{\rho} - i\bar{\eta}) & -A\lambda^2 & 1
\end{pmatrix}
$$
Flavour mixing and CP violation in the Standard Model

- The CP symmetry is violated in any field theory having in the Lagrangian at least one phase that cannot be re-absorbed.
- The **mass eigenstates** are not eigenstates of the weak interaction. This feature of the Standard Model Hamiltonian produces the (unitary) **mixing matrix** $V_{\text{CKM}}$.

$$
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\approx
\begin{pmatrix}
1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix}
$$

- With **three families** of quarks, there is one phase that allows CP violation in the SM. All the flavour mixing processes are related (through the unitarity of the $V_{\text{CKM}}$) to this phase.

**Unitarity Triangle**

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

All the angles are related to the CP asymmetries of specific B decays.
$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$

many observables functions of $\bar{\rho}$ and $\bar{\eta}$: overconstraining

$\alpha = \pi - \beta - \gamma$

normalized:
$\bar{\rho} + i\bar{\eta}$

$\gamma = \text{atan} \left( \frac{\bar{\eta}}{\bar{\rho}} \right)$

$\beta = \text{atan} \left( \frac{\bar{\eta}}{1 - \bar{\rho}} \right)$

$B^0 \rightarrow \pi\pi , \rho\pi$

$B^0 \rightarrow j/\psi K_s$

$B^0 \rightarrow DK$

$V_{ud}V_{ub}^*$

$V_{cd}V_{cb}^*$

$V_{td}V_{tb}^*$
Method and inputs:

\[ f(\bar{\rho}, \bar{\eta}, X|c_1, \ldots, c_m) \sim \prod_{j=1}^{m} f_j(c|\bar{\rho}, \bar{\eta}, X) \prod_{i=1}^{N} f_i(x_i) f_0(\bar{\rho}, \bar{\eta}) \]

Bayes Theorem

\[ X \equiv x_1, \ldots, x_n = m_t, B_K, F_B, \ldots \]

\[ c \equiv c_1, \ldots, c_m = \epsilon, \Delta m_d/\Delta m_s, A_{CP}(J/\psi K_S), \ldots \]

Standard Model + OPE/HQET/Lattice QCD

to go from quarks to hadrons

- \( (b \to u)/(b \to c) \)
- \( \epsilon_K \)
- \( \Delta m_d \)
- \( \Delta m_d/\Delta m_s \)
- \( A_{CP}(J/\psi K_S) \)

- \( \bar{\rho}^2 + \bar{\eta}^2 \)
- \( \bar{\eta}[(1 - \bar{\rho}) + P] \)
- \( (1 - \bar{\rho})^2 + \bar{\eta}^2 \)
- \( \sin 2\beta \)

- \( \bar{\Lambda}, \lambda_1, F(1), \ldots \)
- \( B_K \)
- \( f_B^2 B_B \)
- \( \xi \)

M. Bona et al. (UTfit Collaboration)
M. Bona et al. (UTfit Collaboration)
The LEP-style analysis in the $\bar{\rho}$-$\bar{\eta}$ plane:

\[ |V_{ub}/V_{cb}|, \quad \bar{\rho}^2 + \bar{\eta}^2 \]

\[ \bar{\eta}[(1 - \bar{\rho}) + P] \]

\[ \Delta m_d, \quad (1 - \bar{\rho})^2 + \bar{\eta}^2 \]

\[ \Delta m_s/\Delta m_d \]
The LEP-style analysis in the $\bar{\rho}$-$\bar{\eta}$ plane:

- $|V_{ub}/V_{cb}|$
- $\bar{\eta}[(1 - \bar{\rho}) + P]$
- $\Delta m_d$
- $\Delta m_s/\Delta m_d$

From lattice QCD

$$|\varepsilon_K| \sim C_c B_K A^2 \lambda^6 \bar{\eta} \{-\eta S_0(x_c) (1 - \lambda^2/2) + \eta_s S_0(x_c, x_t) + \eta_t S_0(x_t) A^2 \lambda^4 (1 - \bar{\rho})\}$$

$S_0 = \text{Inami-Lim functions for } c-c, c-t, e t-t \text{ contributions}$

From perturbative calculations

$$\varepsilon_K = (2.228 \pm 0.011) \cdot 10^{-3}$$

From perturbative calculations

$$B_K = \frac{\langle K | J^\mu | \bar{K} \rangle}{\langle K | J^\mu | 0 \rangle \langle 0 | J^\mu | \bar{K} \rangle}$$

$B_K = 0.756 \pm 0.016$ (FLAG 2019)
The LEP-style analysis in the $\bar{\rho}$-$\bar{\eta}$ plane:

$$|V_{ub}/V_{cb}|$$

$$\bar{\rho}^2 + \bar{\eta}^2$$

$$\bar{\eta}[(1 - \bar{\rho}) + P]$$

$$\Delta m_d$$

$$\Delta m_s/\Delta m_d$$

$$(1 - \bar{\rho})^2 + \bar{\eta}^2$$

$\Delta m_q$ from $B_q$-$\bar{B}_q$ mixing

$q = d, s$

$\Delta m_d = 0.5065 \pm 0.0019 \text{ ps}^{-1}$

$\Delta m_s = 17.765 \pm 0.006 \text{ ps}^{-1}$

$$\Delta m_d = \frac{G_F^2}{6\pi^2} m_w^2 \eta_b S(x_t) m_{B_d} f_{B_d}^2 \hat{B}_{B_d} |V_{ub}|^2 |V_{td}|^2 =$$

$$= \frac{G_F^2}{6\pi^2} m_w^2 \eta_b S(x_t) m_{B_d} f_{B_d}^2 \hat{B}_{B_d} |V_{ub}|^2 |V_{td}|^2 \lambda^2 ((1 - \bar{\rho})^2 + \bar{\eta}^2)$$

$\Delta m_s \approx \frac{f_{B_s}^2 B_{B_s}}{\xi^2}$

$\Delta m_s \approx f_{B_s}^2 B_{B_s}$

$S = \text{Inami-Lim function for the } t-t \text{ contribution (from perturbative calculations)}$

$B_{B_q}$ and $f_{B_q}$ from lattice QCD
The LEP-style analysis in the $\bar{\rho}-\bar{\eta}$ plane:

- Tree diagrams
  - b → c and b → u transition
  - Negligible new physics contributions
  - Inclusive and exclusive semileptonic B decay branching ratios

QCD corrections to be included
- Inclusive measurements: OPE
- Exclusive measurements: form factors from lattice QCD
**V_{cb} and V_{ub}**

- **|V_{cb}| (excl)** = (39.09 ± 0.68) 10^{-3}
- **|V_{cb}| (incl)** = (42.16 ± 0.50) 10^{-3}
- **|V_{ub}| (excl)** = (3.73 ± 0.14) 10^{-3}
- **|V_{ub}| (incl)** = (4.19 ± 0.17 ± 0.18 [flat]) 10^{-3}

From FLAG 2019 arXiv:1902.08191

- ~2.8σ discrepancy

From Bordone et al. arXiv:2107.00604

- ~1.5σ discrepancy

From GGOU HFLAV 2021

- adding a flat uncertainty covering the spread of central values

**|V_{ub}| / V_{cb}| (LHCb)** = (9.46 ± 0.79) 10^{-2}

**|V_{ub}| / V_{cb}| (LHCb)** = (7.9 ± 0.6) 10^{-2}

From B_s to K at high q^2

From Λ_b, excluded following FLAG guidelines
**$V_{cb}$ and $V_{ub}$**

A-la-D’Agostini two-dimensional average procedure:

$|V_{cb}| = (41.1 \pm 1.0) \times 10^{-3}$

uncertainty $\sim 2.4\%$

$|V_{ub}| = (3.89 \pm 0.21) \times 10^{-3}$

uncertainty $\sim 5.4\%$

From global SM fit

$|V_{cb}| = (41.7 \pm 0.4) \times 10^{-3}$

$|V_{ub}| = (3.70 \pm 0.10) \times 10^{-3}$

**UTfit prediction:**

$|V_{cb}| = (41.9 \pm 0.5) \times 10^{-3}$

$|V_{ub}| = (3.68 \pm 0.10) \times 10^{-3}$
The LEP-style analysis in the $\bar{\rho}$-$\bar{\eta}$ plane:

\[ |\frac{V_{ub}}{V_{cb}}| \]
\[ \bar{\rho}^2 + \bar{\eta}^2 \]

\[ \bar{\eta}[1 - \bar{\rho} + P] \]

\[ (1 - \bar{\rho})^2 + \bar{\eta}^2 \]

levels @ 95% Prob

\[ \bar{\rho} = 0.169 \pm 0.017 \]
\[ \bar{\eta} = 0.383 \pm 0.025 \]

~10%

~7%
angle constraints in the $\bar{\rho}$-$\bar{\eta}$ plane:

B factories + LHCb
angle constraints in the $\bar{\rho}$-$\bar{\eta}$ plane:

$\sin 2\beta$ from time-dependent $A_{\text{CP}}$ in $B \to J/\psi K$

\[
\alpha_{f_{\text{CP}}}(t) = \frac{\text{Prob}(B^0(t) \to f_{\text{CP}}) - \text{Prob}(B^0(t) \to \overline{f}_{\text{CP}})}{\text{Prob}(B^0(t) \to f_{\text{CP}}) + \text{Prob}(B^0(t) \to \overline{f}_{\text{CP}})} = C_f \cos \Delta m_t t + S_f \sin \Delta m_t t
\]

\[
\alpha_{f_{\text{CP}}}(t) = -\eta_{\text{CP}} \sin \Delta m_t \Delta t \sin 2\beta
\]
Latest sin2\(\beta\) results:

\[
\sin(2\beta) = \sin(2\phi_1)
\]

**HFLAV**

\[
\sin^2(\beta) = 0.688 \pm 0.020
\]

**UTfit input**

\[
\sin^2(\beta)(J/\psi K^0) = 0.698 \pm 0.017
\]

raw asymmetry as function of \(\Delta t\)

\[
\Delta S = -0.01 \pm 0.01
\]

M.Ciuchini, M.Pierini, L.Silvestrini
angle constraints in the $\bar{\rho}$-$\bar{\eta}$ plane:

$\alpha$: CP violation in $B^0 \rightarrow \pi^+\pi^-$

- considering the tree (T) only:
  \[ \lambda_{\pi\pi} = e^{2i\alpha} \]
  \[ C_{\pi\pi} = 0 \]
  \[ S_{\pi\pi} = \sin(2\alpha) \]

- adding the penguins (P):
  \[ \lambda_{\pi\pi} = e^{2i\alpha} \frac{1 + |P/T|e^{i\delta}e^{i\gamma}}{1 + |P/T|e^{i\delta}e^{-i\gamma}} \]
  \[ C_{\pi\pi} \propto \sin(\delta) \]
  \[ S_{\pi\pi} = \sqrt{1 - C_{\pi\pi}^2} \sin(2\alpha_{\text{eff}}) \]
angle constraints in the $\bar{\rho}$-$\bar{\eta}$ plane:

from $\alpha_{\text{eff}}$ to $\alpha$: isospin analysis

- $B \to \pi^+\pi^-, \pi^+\pi^0, \pi^0\pi^0$ decays are connected from isospin relations
- $\pi\pi$ states can have $I = 2$ or $I = 0$
- the gluonic penguins contribute only to the $I = 0$ state ($\Delta I = 1/2$)
- $\pi^+\pi^0$ is a pure $I = 2$ state ($\Delta I = 3/2$) and it gets contribution only from the tree diagram
- triangular relations allow for the determination of the phase difference induced on $\alpha$:

$$2\alpha_{\text{eff}} = 2\alpha + \kappa_{\pi\pi}$$

$$|A^+|^2 = |A^-|^2$$

Both $\text{BR}(B^0)$ and $\text{BR}(\bar{B}^0)$ have to be measured in all the $\pi\pi$ channels
angle constraints in the $\bar{\rho}-\bar{\eta}$ plane:

$\alpha$ result for $\pi^+\pi^-$ and $\rho^+\rho^-$

$\pi^+ \pi^- \ S_{\text{CP}}$ vs $C_{\text{CP}}$

$\rho^+ \rho^- \ S_{\text{CP}}$ vs $C_{\text{CP}}$

$C_{\pi\pi} = -0.311 \pm 0.030$
$S_{\pi\pi} = -0.666 \pm 0.029$

$C_{\rho\rho} = 0.00 \pm 0.09$
$S_{\rho\rho} = -0.14 \pm 0.13$
angle constraints in the $\bar{\rho}$-$\bar{\eta}$ plane:

$\gamma$ and DK trees

- $D^{(*)}K^{(*)}$ decays: from BRs and BR ratios, no time-dependent analysis, just rates
- the phase $\gamma$ is measured exploiting interferences: two amplitudes leading to the same final states
- some rates can be really small: $\sim 10^{-7}$

$B \to D^{(*)0} (\bar{D}^{(*)0}) K^{(*)}$ decays can proceed both through $V_{cb}$ and $V_{ub}$ amplitudes
sensitivity to $\gamma$: the ratio $r_B$

\[ V_{ub} = |V_{ub}| e^{-i\gamma} (\sim \lambda^3) \]

\[ \delta_B = \text{strong phase diff.} \]

\[ A(B^- \rightarrow D^0 K^-) = A_B \]
\[ A(B^+ \rightarrow \bar{D}^0 K^+) = A_B \]

\[ A(B^- \rightarrow \bar{D}^0 K^-) = A_B r_B e^{i(\delta_B - \gamma)} \]
\[ A(B^+ \rightarrow D^0 K^+) = A_B r_B e^{i(\delta_B + \gamma)} \]

\[ r_B = \frac{|B^- \rightarrow \bar{D}^0 K^-|}{|B^- \rightarrow D^0 K^-|} = \sqrt{\eta^2 + \rho^2} \times F_{CS} \]

\[ \sim 0.36 \quad \text{hadronic contribution channel-dependent} \]

- in $B^+ \rightarrow D^{(*)0} K^+$: $r_B$ is $\sim 0.1$
- while in $B^0$ to $D^{(*)0} K^0$ $r_B$ is $\sim 0.25$
- Also measured: $r_B(\bar{D}K)$, $r^*_B(D^*K)$ and $r^s_B(DK^*)$
angle constraints in the $\bar{\rho}$-$\bar{\eta}$ plane:

$\gamma$ and DK trees

<table>
<thead>
<tr>
<th>Parameter: $\gamma \equiv \phi_3$ from all $B \to DK$ and similar $b \to cu$-bar $s$ &amp; $b \to uc$-bar $s$ modes</th>
<th>( (66.2 \pm 3.4 - 3.6) )°</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_B(DK^+) = 0.0996 \pm 0.0026$</td>
<td>$\delta_B(DK^+) = (128.0 \pm 3.8 - 4.0)\text{°}$</td>
</tr>
<tr>
<td>$r_B(D^*K^+) = 0.104 + 0.013 - 0.014$</td>
<td>$\delta_B(D^*K^+) = (314.9 \pm 7.8 - 10.0)\text{°}$</td>
</tr>
<tr>
<td>$r_B(DK^{*+}) = 0.101 + 0.016 - 0.037$</td>
<td>$\delta_B(DK^{*+}) = (49 + 61 - 16)\text{°}$</td>
</tr>
<tr>
<td>$r_B(DK^{*0}) = 0.257 + 0.021 - 0.022$</td>
<td>$\delta_B(DK^{*0}) = (194 + 9.5 - 8.8)\text{°}$</td>
</tr>
</tbody>
</table>

\[ 1-\text{CL} \]

- **HFLAV PDG 2021**
- $B^+ \to D^0 K^+, D^0 \to h\pi^0/h3\pi$
- $B^+ \to D^0 K^+, D^0 \to K_S^0 hh$
- $B^+ \to D^0 K^+, D^0 \to h^+h^-/K_S^0 \pi^0/K_S^0 \omega/K_S^0 \phi$
- All $B^+ \to D^0 K^+$ modes
- World Average

\[ r_{\bar{\rho}}^\gamma \]

- **HFLAV PDG 2021**
- $B^+ \to D^0 K^+, D^0 \to h\pi^0/h3\pi$
- $B^+ \to D^0 K^+, D^0 \to K_S^0 hh$
- $B^+ \to D^0 K^+, D^0 \to h^+h^-/K_S^0 \pi^0/K_S^0 \omega/K_S^0 \phi$
- All $B^+ \to D^0 K^+$ modes
- World Average

\[ 1-\text{CL} \]

- **HFLAV PDG 2021**
- $B^+ \to D^0 K^+, D^0 \to h\pi^0/h3\pi$
- $B^+ \to D^0 K^+, D^0 \to K_S^0 hh$
- $B^+ \to D^0 K^+, D^0 \to h^+h^-/K_S^0 \pi^0/K_S^0 \omega/K_S^0 \phi$
- All $B^+ \to D^0 K^+$ modes
- World Average
**sin2\(\alpha\) \((\phi_2)\) and \(\gamma\) \((\phi_3)\)**

\(\alpha\) updated with latest \(\pi\pi/\rho\rho\) BR and C/S results

\[\sin^2\alpha (f_2)\] and \(g (f_3)\) from B into DK decays:

- **HFLAV**: \((66.1 \pm 3.5)^\circ\)
- **UTfit prediction**: \((66.1 \pm 2.1)^\circ\)

\(\alpha\) from HFLAV: \(85.5 \pm 4.6\)

\(\gamma\) updated with all the latest results (LHCb)
angle constraints in the $\bar{\rho}$-$\bar{\eta}$ plane:

- $\bar{\rho} = 0.156 \pm 0.018$
- $\bar{\eta} = 0.335 \pm 0.018$

levels @ 95% Prob

$\alpha$ ~12%

$\beta$ ~5%
Unitarity Triangle analysis in the SM:

- \[ \bar{\rho} = 0.157 \pm 0.012 \]
- \[ \bar{\eta} = 0.350 \pm 0.010 \]

Levels @ 95% Prob
Unitarity Triangle analysis in the SM:

zoomed in..

levels @ 95% Prob

\[ \rho = 0.157 \pm 0.012 \]
\[ \eta = 0.350 \pm 0.010 \]

~8%

~3%
Unitarity Triangle analysis in the SM:

2021

2004
Some interesting configurations

Universal Unitary Triangle

\[ \overline{\rho} = 0.162 \pm 0.017 \]
\[ \overline{\eta} = 0.341 \pm 0.011 \]

\( \sim 10\% \)

\( \sim 3\% \)

Tree-level processes:
Semileptonic and DK B decays

\( \rightarrow \) reference for model building

\( \overline{\rho} = \pm 0.165 \pm 0.025 \)
\[ \overline{\eta} = \pm 0.373 \pm 0.025 \]

\( \sim 15\% \)

\( \sim 7\% \)

Sides and \( \varepsilon_K \)
compatibility plots

A way to “measure” the agreement of a single measurement with the indirect determination from the fit using all the other inputs: test for the SM description of the flavour physics

Color code: agreement between the predicted values and the measurements at better than 1, 2, ...nσ

The cross has the coordinates (x,y)=(central value, error) of the direct measurement

\[ \alpha_{\text{exp}} = (93.6 \pm 4.2)^\circ \]
\[ \alpha_{\text{UTfit}} = (90.5 \pm 2.1)^\circ \]

\[ \gamma_{\text{exp}} = (66.1 \pm 3.5)^\circ \]
\[ \gamma_{\text{UTfit}} = (66.1 \pm 2.1)^\circ \]
Checking the usual *tensions*..

\[ \sin^2 \beta_{\text{exp}} = 0.688 \pm 0.020 \]

\[ \sin^2 \beta_{\text{UTfit}} = 0.751 \pm 0.027 \]

\(~1.4\sigma~\)
Checking the usual tensions..

$$|V_{ub}| (excl) = (3.73 \pm 0.14) \cdot 10^{-3}$$

$$|V_{ub}| (incl) = (4.19 \pm 0.20) \cdot 10^{-3}$$

$$|V_{cb}| (excl) = (39.09 \pm 0.68) \cdot 10^{-3}$$

$$|V_{cb}| (incl) = (42.16 \pm 0.50) \cdot 10^{-3}$$

$$V_{ub}^{exp} = (3.89 \pm 0.21) \cdot 10^{-3}$$

$$V_{ub}^{UTfit} = (3.68 \pm 0.10) \cdot 10^{-3}$$

$$V_{cb}^{exp} = (41.1 \pm 1.0) \cdot 10^{-3}$$

$$V_{cb}^{UTfit} = (41.9 \pm 0.5) \cdot 10^{-3}$$
## Unitarity Triangle analysis in the SM:

<table>
<thead>
<tr>
<th>Observables</th>
<th>Measurement</th>
<th>Prediction</th>
<th>Pull (#σ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>sin2(β)</td>
<td>0.688 ± 0.020</td>
<td>0.751 ± 0.027</td>
<td>~ 1.4</td>
</tr>
<tr>
<td>(γ)</td>
<td>66.1 ± 3.5</td>
<td>66.1 ± 2.1</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>(α)</td>
<td>93.6 ± 4.2</td>
<td>90.5 ± 2.1</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>(ε_K \cdot 10^3)</td>
<td>2.228 ± 0.001</td>
<td>2.05 ± 0.13</td>
<td>~ 1.4</td>
</tr>
<tr>
<td>(</td>
<td>V_{cb}</td>
<td>\cdot 10^3)</td>
<td>40.4 ± 1.3</td>
</tr>
<tr>
<td>(</td>
<td>V_{cb}</td>
<td>\cdot 10^3) (incl)</td>
<td>42.16 0.50</td>
</tr>
<tr>
<td>(</td>
<td>V_{cb}</td>
<td>\cdot 10^3) (excl)</td>
<td>39.09 0.68</td>
</tr>
<tr>
<td>(</td>
<td>V_{ub}</td>
<td>\cdot 10^3)</td>
<td>3.89 ± 0.21</td>
</tr>
<tr>
<td>(</td>
<td>V_{ub}</td>
<td>\cdot 10^3) (incl)</td>
<td>4.19 ± 0.20</td>
</tr>
<tr>
<td>(</td>
<td>V_{ub}</td>
<td>\cdot 10^3) (excl)</td>
<td>3.73 ± 0.14</td>
</tr>
<tr>
<td>BR(B → (τν))[10^{-4}]</td>
<td>1.09 ± 0.24</td>
<td>0.87 ± 0.05</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>(A_{SL}^{d} \cdot 10^3)</td>
<td>-2.1 ± 1.7</td>
<td>-0.32 ± 0.03</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>(A_{SL}^{s} \cdot 10^3)</td>
<td>-0.6 ± 2.8</td>
<td>0.014 ± 0.001</td>
<td>&lt; 1</td>
</tr>
</tbody>
</table>

obtained excluding the given constraint from the fit
Consider for example $B_s$ mixing process. Given the SM amplitude, we can define

$$C_{B_s} e^{-2i\phi_{B_s}} = \frac{\langle B_s | H_{\text{eff}}^{\text{SM}} + H_{\text{eff}}^{\text{NP}} | B_s \rangle}{\langle B_s | H_{\text{eff}}^{\text{SM}} | B_s \rangle} = 1 + \frac{A_{NP} e^{-2i\phi_{NP}}}{A_{SM} e^{-2i\beta_s}}$$

All NP effects can be parameterized in terms of one complex parameter for each meson mixing, to be determined in a simultaneous fit with the CKM parameters (now there are enough experimental constraints to do so).

For kaons we use $\text{Re}$ and $\text{Im}$, since the two exp. constraints $\varepsilon_K$ and $\Delta m_K$ are directly related to them (with distinct theoretical issues)

$$C_{\varepsilon_K} = \frac{\text{Im}\langle K^0 | H_{\text{eff}}^{\text{full}} | \bar{K}^0 \rangle}{\text{Im}\langle K^0 | H_{\text{eff}}^{\text{SM}} | \bar{K}^0 \rangle}$$

$$C_{\Delta m_K} = \frac{\text{Re}\langle K^0 | H_{\text{eff}}^{\text{full}} | \bar{K}^0 \rangle}{\text{Re}\langle K^0 | H_{\text{eff}}^{\text{SM}} | \bar{K}^0 \rangle}$$
UT analysis including new physics

fit simultaneously for the CKM and the NP parameters (generalized UT fit)

- add most general loop NP to all sectors
- use all available experimental info
- extract posteriors on NP contributions to $\Delta F=2$ transitions

$B_d$ and $B_s$ mixing amplitudes

(2+2 real parameters):

$$A_q = C_{B_q} e^{2i\phi_{B_q}} A_q^{SM} e^{2i\phi_{q}^{SM}} = \left( 1 + \frac{A_q^{NP}}{A_q^{SM}} e^{2i(\phi_{q}^{NP} - \phi_{q}^{SM})} \right) A_q^{SM} e^{2i\phi_{q}^{SM}}$$

$$\Delta m_{q/K} = C_{B_q/\Delta m_{K}} (\Delta m_{q/K})^{SM}$$

$$A_{CP}^{B_d \rightarrow J/ψ K_s} = \sin 2 (\beta + \phi_{B_d})$$

$$A_{SL}^q = \text{Im} \left( \frac{\Gamma_{12}^q}{A_q} \right)$$

$$\varepsilon_K = C_{\varepsilon} \varepsilon_{K}^{SM}$$

$$A_{CP}^{B_s \rightarrow J/ψ} \sim \sin 2 (-\beta_s + \phi_{B_s})$$

$$\Delta \Gamma^q/\Delta m_q = \text{Re} \left( \frac{\Gamma_{12}^q}{A_q} \right)$$

m.bona
new-physics-specific constraints

\[ A^{s}_{SL} \equiv \frac{\Gamma(\bar{B}_s \to \ell^+ X) - \Gamma(B_s \to \ell^- X)}{\Gamma(\bar{B}_s \to \ell^+ X) + \Gamma(B_s \to \ell^- X)} = \text{Im}\left( \frac{\Gamma_{12}^{s}}{A^{s}_{full}} \right) \]

semileptonic asymmetries in $B^0$ and $B_s$: sensitive to NP effects in both size and phase. Taken from the latest HFLAV.

same-side dilepton charge asymmetry: admixture of $B_s$ and $B_d$ so sensitive to NP effects in both.

\[ A^{\mu\mu} \times 10^3 = -7.9 \pm 2.0 \]

lifetime $\tau^{FS}$ in flavour-specific final states: average lifetime is a function to the width and the width difference

\[ \tau^{FS}(B_s) = 1.527 \pm 0.011 \text{ ps} \]

$\phi_s = 2\beta_s$ vs $\Delta \Gamma_s$ from $B_s \to J/\psi \phi$

angular analysis as a function of proper time and b-tagging

\[ \phi_s = -0.050 \pm 0.019 \text{ rad} \]
NP analysis results

\[ r = 0.175 \pm 0.027 \]
\[ h = 0.380 \pm 0.026 \]

SM is

\[ \bar{\rho} = 0.157 \pm 0.012 \]
\[ \bar{\eta} = 0.350 \pm 0.010 \]

Only shown the constraints unaffected by NP levels @ 95% Prob

\[ \bar{\rho} = 0.175 \pm 0.027 \]
\[ \bar{\eta} = 0.380 \pm 0.026 \]
NP parameter results

\[ A_q = C_{B_q} e^{2i\phi_{B_q}} A_q^{SM} e^{2i\phi_{q}^{SM}} \]

- dark: 68%
- light: 95%
- SM: red cross

**K system**

\[ C_{eK} = 1.05 \pm 0.10 \]

**CBd vs \(\phi_{Bd}\)**

\[ C_{Bd} = 1.03 \pm 0.10 \]
\[ \phi_{Bd} = (-3.1 \pm 1.8)^\circ \]

**CBS vs \(\phi_{Bs}\)**

\[ C_{Bs} = 1.04 \pm 0.07 \]
\[ \phi_{Bs} = (-0.3 \pm 0.5)^\circ \]
NP parameter results

The ratio of NP/SM amplitudes is:

$A_q = \left(1 + \frac{A_q^{NP}}{A_q^{SM}} e^{2i(\phi_q^{NP} - \phi_q^{SM})}\right) A_q^{SM} e^{2i\phi_q^{SM}}$

< 18% @68% prob. (30% @95%) in $B_d$ mixing

< 10% @68% prob. (18% @95%) in $B_s$ mixing
At the high scale
new physics enters according to its specific features

At the low scale
use OPE to write the most general effective Hamiltonian.
the operators have different chiralities than the SM
NP effects are in the Wilson Coefficients C

\[ \mathcal{H}^{\Delta B=2} = \sum_{i=1}^{5} C_i Q_i^{bq} + \sum_{i=1}^{3} \tilde{C}_i \tilde{Q}_i^{bq} \]

\[ Q_1^{q_i q_j} = \bar{q}_{jL}^\alpha \gamma_\mu q_{iL}^\alpha \bar{q}_{jL}^\beta \gamma_\mu q_{iL}^\beta , \]
\[ Q_2^{q_i q_j} = \bar{q}_{jR}^\alpha q_{iL}^\alpha \bar{q}_{jR}^\beta q_{iL}^\beta , \]
\[ Q_3^{q_i q_j} = \bar{q}_{jR}^\alpha q_{iL}^\beta \bar{q}_{jR}^\beta q_{iL}^\alpha , \]
\[ Q_4^{q_i q_j} = \bar{q}_{jR}^\alpha q_{iL}^\alpha \bar{q}_{jL}^\beta q_{iR}^\beta , \]
\[ Q_5^{q_i q_j} = \bar{q}_{jR}^\alpha q_{iL}^\beta \bar{q}_{jL}^\beta q_{iR}^\alpha . \]

function of the NP flavour couplings
loop factor (in NP models with no tree-level FCNC)
NP scale (typical mass of new particles mediating \( \Delta F=2 \) processes)
The dependence of \( C \) on \( \Lambda \) changes depending on the flavour structure. We can consider different flavour scenarios:

- **Generic**: \( C(\Lambda) = \alpha/\Lambda^2 \)  
  \( F_i \sim 1 \), arbitrary phase

- **NMFV**: \( C(\Lambda) = \alpha \times |F_{SM}|/\Lambda^2 \)  
  \( F_i \sim |F_{SM}| \), arbitrary phase

- **MFV**: \( C(\Lambda) = \alpha \times |F_{SM}|/\Lambda^2 \)  
  \( F_1 \sim |F_{SM}|, F_{\neq 1} \sim 0 \), SM phase

\( \alpha(L_i) \) is the coupling among NP and SM

- \( \alpha \sim 1 \) for strongly coupled NP
- \( \alpha \sim \alpha_W (\alpha_s) \) in case of loop coupling through weak (strong) interactions

If no NP effect is seen, lower bound on NP scale \( \Lambda \)

\( F \) is the flavour coupling and so \( F_{SM} \) is the combination of CKM factors for the considered process.
results from the Wilson coefficients

**Generic:** \( C(\Lambda) = \alpha/\Lambda^2, \)
\[ F \sim 1, \text{ arbitrary phase} \]
\[ \alpha \sim 1 \text{ for strongly coupled NP} \]

\( \Lambda > 4.3 \times 10^5 \text{ TeV} \)

\( \alpha \sim \alpha_W \) in case of loop coupling through weak interactions

\( \Lambda > 1.3 \times 10^4 \text{ TeV} \)

**NMFV:** \( C(\Lambda) = \alpha \times |F_{SM}|/\Lambda^2, \)
\[ F \sim |F_{SM}|, \text{ arbitrary phase} \]

\( \Lambda > 89 \text{ TeV} \)

\( \alpha \sim \alpha_W \) in case of loop coupling through weak interactions

\( \Lambda > 2.7 \text{ TeV} \)

for lower bound for loop-mediated contributions, simply multiply by \( \alpha_s (\sim 0.1) \) or by \( \alpha_W (\sim 0.03). \)
conclusions

- SM analysis displays very good (improved) overall consistency
- Still open discussion on semileptonic inclusive vs exclusive: exclusive fit shows tension, $V_{cb}$ now showing the biggest discrepancy.
- UTA provides determination of NP contributions to $\Delta F=2$ amplitudes. It currently leaves space for NP at the level of 20-30%
- So the scale analysis points to high scales for the generic scenario and at the limit of LHC reach for weak coupling. Indirect searches are not only complementary to direct searches, but they might be the main way to glimpse at new physics.
Back up slides
lattice QCD inputs

updated in early 2020

<table>
<thead>
<tr>
<th>Observable</th>
<th>Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_K$</td>
<td>$0.756 \pm 0.016$</td>
</tr>
<tr>
<td>$f_{Bs}$</td>
<td>$0.2301 \pm 0.0012$</td>
</tr>
<tr>
<td>$f_{Bs}/f_{Bd}$</td>
<td>$1.208 \pm 0.005$</td>
</tr>
<tr>
<td>$B_{Bs}/B_{Bd}$</td>
<td>$1.032 \pm 0.038$</td>
</tr>
<tr>
<td>$B_{Bs}$</td>
<td>$1.35 \pm 0.06$</td>
</tr>
</tbody>
</table>

FLAG 2019 suggests to take the most precise between the Nf=2+1+1 and Nf=2+1 averages. We quote, instead, the weighted average of the Nf=2+1+1 and Nf=2+1 results with the error rescaled when chi2/dof > 1, as done by FLAG for the Nf=2+1+1 and Nf=2+1 averages separately.
analytic expression for the contribution to the mixing amplitudes
\[
\langle \bar{B}_q | \mathcal{H}^{\Delta B=2}_{\text{eff}} | B_q \rangle_i = \sum_{j=1}^{5} \sum_{r=1}^{5} (b_j^{(r,i)} + \eta c_j^{(r,i)}) \eta^{-a_j} C_i(\Lambda) \langle \bar{B}_q | Q_r^{bq} | B_q \rangle
\]

arXiv:0707.0636: for "magic numbers" a,b and c, \( \eta = \alpha_s(\Lambda)/\alpha_s(m_t) \)

analogously for the K system
\[
\langle \bar{K}^0 | \mathcal{H}^{\Delta S=2}_{\text{eff}} | K^0 \rangle_i = \sum_{j=1}^{5} \sum_{r=1}^{5} (b_j^{(r,i)} + \eta c_j^{(r,i)}) \eta^{-a_j} C_i(\Lambda) R_r \langle \bar{K}^0 | Q_1^{sd} | K^0 \rangle
\]

to obtain the p.d.f. for the Wilson coefficients \( C_i(\Lambda) \) at the new-physics scale, we switch on one coefficient at a time in each sector and calculate its value from the result of the NP analysis.
some old plots coming back to fashion:

As NA62 and KOTO are analysing data:

$\text{BR}(K^+ \to \pi^+\nu\bar{\nu})$

2007 global fit area

$\text{SM central value}$

projection 100 events

including $\text{BR}(K^0 \to \pi^0\nu\bar{\nu})$

$\text{SM central value}$

projection 100 events

$\text{E949 central value}$

7 events
Look at the near future

**future I** scenario:
- errors from Belle II at 5/ab
- + LHCb at 10/fb

\[
\begin{align*}
\rho &= \pm 0.015 \\
\eta &= \pm 0.015
\end{align*}
\]

\[
\begin{align*}
\bar{\rho} &= 0.154 \pm 0.015 \\
\bar{\eta} &= 0.344 \pm 0.013
\end{align*}
\]

current sensitivity

\[
\begin{align*}
\bar{\rho} &= 0.150 \pm 0.027 \\
\bar{\eta} &= 0.363 \pm 0.027
\end{align*}
\]

\[
\begin{align*}
\rho &= \pm 0.016 \\
\eta &= \pm 0.019
\end{align*}
\]

\[
\begin{align*}
\bar{\rho} &= 0.154 \pm 0.015 \\
\bar{\eta} &= 0.344 \pm 0.013
\end{align*}
\]
Theory error on $\sin 2\beta$:

1) Fit the amplitudes in the SU(3)-related decay $J/\psi \pi^0$ and keep solution compatible with $J/\psi K$

2) Obtain the upper limit on the penguin amplitude and add 100% error for SU(3) breaking

3) Fit the amplitudes in $J/\psi K^0$ imposing the upper bound on the CKM suppressed amplitude and extract the error on $\sin 2\beta$

$\Delta S = 0.000 \pm 0.012$

M. Ciuchini, M. Pierini, L. Silvestrini