Hard Jets and Higgs Bosons
HEJ: All-Order Perturbative Corrections to Hard Multi-Jet Processes

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Overview of Talk

Elements of Proton Collisions

Hard scattering, shower, matching to fixed order
multiple interactions, underlying event. . .

Jets to the rescue!

Multi-Jet Predictions

Why we must care about HO corrections (in some situations). . .
A new approach to multiple, wide-angle emissions from the hard scattering:

High Energy Jets


Theory vs. Data

Results of first data compared to HEJ
Hard, higher order effects beyond NLO
$pp \rightarrow \mu^+ \mu^- + 3 \text{ jets}$
Depending on the question we want to answer, we may not need to describe all the stages of the collision.

The notion of jets allow us to compare pure perturbation theory (few partons) to experimental observation (many hadrons)

Transverse Momentum
Rapidity: \( y = \frac{1}{2} \ln \frac{E+p_z}{E-p_z} \)

still need to ensure (relative) insensitivity to underlying event, multiple interactions... ask questions only about relatively hard jets (\( p_\perp > 30 \text{ GeV} \))
Jet (algorithms) to the Rescue!

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The notion of jets allow us to **compare pure perturbation theory** (few partons) to **experimental observation** (many hadrons).

**Transverse Momentum**

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still need to ensure (relative) insensitivity to underlying event, multiple interactions... ask questions only about relatively hard jets ($p_\perp > 30 \text{ GeV}$?)

Obviously need the jet algorithms to be well-defined both experimentally (many discrete hits) and theoretically (probing singularity structure). Use fastjet!
The Perturbative Description
Why Study Multi-jet Observables?

We don’t have a choice!

1. Many BSM (e.g. SUSY) particles will have decay chains involving the production of jets (e.g. 4 jets + $p_T$). Calculation of signal is easy (one process), SM contribution is very hard (several processes).

2. All LHC processes involves QCD-charged particles; sometimes the (n+1)-jet cross section is as large as the n-jet cross section!

3. It is a challenge we cannot ignore!
The age old hunt. . .

Effects beyond NLO DGLAP?

. . . apart from the obvious soft and collinear regions (shower profile)
Do we need more than NLO DGLAP to describe the hard jet events at the LHC?

The News

The data collected in 2010 already show effects beyond NLO DGLAP. . .

1 for some observables based on hard jets
2 in certain regions of phase space
Will not discuss several interesting effects:

- jet broadening (shower profiles)
- impact of underlying event on the jet energy

These are (well?) described by a tunable shower MC.

Will instead focus on the description of the hard event, and in particular on observables not well described by NLO DGLAP.

Which regions of phase space receive large corrections from hard perturbative corrections (= additional jet activity)

Compare the description of hard jet activity from NLO, NLO+shower, High Energy Jets. Dijets, W+Dijets, H+Dijets; Similarities in Jet Activity
Multiple ($\geq 2$) hard jets...

Smaller number of jets solved satisfactory (?) already... (POWHEG, MC@NLO, NNLO,...)

**Special radiation pattern** from current-current scattering

Look into **higher order corrections beyond** “inclusive $K$-factor”

Concentrate on the **hard, perturbative corrections** relevant for a description of the final state **in terms of jets**.

**Goal**

Build framework for **all-order summation** (virtual+real emissions). Exact in another limit than the usual soft&collinear. Better suited for describing radiation relevant for multi-jet production.

**Insight**

Can use the insight gained from studying the relevant limit to **guide and improve** analyses: $CP$-properties of the Higgs-boson couplings
Drivers of Emission

1. Collinear (jet profile)
2. Soft ($p_t$-hierarchies)
3. Opening of phase space (semi-hard emissions - not related to a divergence of $|M|^2$).
   Think (e.g.) multiple jets of fixed $p_t$, with increasing rapidity span (span=max difference in rapidity of two hard jets=$\Delta y$).
   All calculations will agree that number of additional jets increases - but the amount of radiation will differ (wildly) - e.g. due to limitations on the number (NLO) or hardness (shower) of additional radiation imposed by theoretical assumptions.
Increasing Rapidity Span $\rightarrow$ Increasing Number of Jets

$\langle N_{\text{jet}} \rangle$ vs. $\Delta y_{ab}$

h+dijets (at least 40GeV).

$\Delta y_{ab}$: Rapidity difference between most forward and backward hard jet

Compare NLO (green), CKKW matched shower (red), and High Energy Jets (blue).

All models show a clear increase in the number of hard jets as the rapidity span $\Delta y_{ab}$ increases.

Please recall this plot when I discuss the results of the ATLAS study of $\langle N_{\text{jets}} \rangle$. 

Goal (inspired by the great Fadin & Lipatov)

Sufficiently **simple** model for hard radiative corrections that the all-order sum can be evaluated explicitly (completely exclusive)

but...

Sufficiently **accurate** that the description is relevant
It is **well known** that QCD matrix elements **factorise** in certain kinematical limits: **Collinear limit** → enters many resummation formalisms, parton showers.

Like all good limits, the collinear approximation is applied **outside its strict region of validity**.

Will discuss the **less well-studied factorisation** of scattering amplitudes in a different kinematic limit, better suited for describing perturbative corrections from **hard parton emission**

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The Possibility for Predictions of $n$-jet Rates

The Power of Reggeisation

High Energy Limit

$|\hat{t}|$ fixed, $\hat{s} \to \infty$

$$A_{2 \to 2+n}^R = \frac{\Gamma_{A'A}}{q_0^2} \left( \prod_{i=1}^{n} e^{\omega(q_i)(y_{i-1} - y_i)} \frac{V^j_i(q_i, q_{i+1})}{q_i^2 q_{i+1}^2} \right) e^{\omega(q_{n+1})(y_n - y_{n+1})} \frac{\Gamma_{B'B}}{q_{n+1}^2}$$

$q_i = k_a + \sum_{l=1}^{i-1} k_l$

Maintain (at LL) terms of the form

$$\left( \alpha_s \ln \frac{\hat{s}_{ij}}{|\hat{t}_i|} \right)$$

to all orders in $\alpha_s$.

At LL only gluon production; at NLL also quark–anti-quark pairs produced. Approximation of any-jet rate possible.

$\Gamma_{AB} = \frac{1}{q_{AB}^2}$

$k_b, y_b$

$k_4, y_4$

$k_3, y_3$

$k_2, y_2$

$k_1, y_1$

$k_a, y_0$

LL: Fadin, Kuraev, Lipatov; NLL: Fadin, Fiore, Kozlov, Reznichenko
Comparison of 3-jet scattering amplitudes

Universal behaviour of scattering amplitudes in the HE limit:

\[ \forall i \in \{2, \ldots, n - 1\} : y_{i-1} \gg y_i \gg y_{i+1} \]

\[ \forall i, j : |p_i\perp| \approx |p_j\perp| \]

\[ \left| \mathcal{M}^{\text{MRK}}_{gg \rightarrow g \cdots g} \right|^2 = \frac{4 \, s^2}{N_C^2 - 1} \frac{g^2 \, C_A}{|p_1\perp|^2} \left( \prod_{i=2}^{n-1} \frac{4 \, g^2 \, C_A}{|p_i\perp|^2} \right) \frac{g^2 \, C_A}{|p_n\perp|^2}. \]

\[ \left| \mathcal{M}^{\text{MRK}}_{qg \rightarrow qg \cdots g} \right|^2 = \frac{4 \, s^2}{N_C^2 - 1} \frac{g^2 \, C_F}{|p_1\perp|^2} \left( \prod_{i=2}^{n-1} \frac{4 \, g^2 \, C_A}{|p_i\perp|^2} \right) \frac{g^2 \, C_A}{|p_n\perp|^2}. \]

\[ \left| \mathcal{M}^{\text{MRK}}_{qQ \rightarrow qg \cdots Q} \right|^2 = \frac{4 \, s^2}{N_C^2 - 1} \frac{g^2 \, C_F}{|p_1\perp|^2} \left( \prod_{i=2}^{n-1} \frac{4 \, g^2 \, C_A}{|p_i\perp|^2} \right) \frac{g^2 \, C_F}{|p_n\perp|^2}. \]

Allow for analytic resummation (BFKL equation).

However, how well does this actually approximate the amplitude?
Comparison of 3-jet scattering amplitudes

Study just a slice in phase space, and compare full tree-level with $\alpha_s^3$-approximation from resummation:

40GeV jets in Mercedes star (transverse) configuration. Rapidities at $-\Delta y$, 0, $\Delta y$. 

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![Graph showing comparison of scattering amplitudes](image-url)
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\[
\frac{|M|^2}{256 s^2} \leq \text{MRK limit}
\]

- $A(qQ\rightarrow qQg)$
- $A(qg\rightarrow qgg)$
- $A(gg\rightarrow ggg)$

\[
\left(\frac{C_f}{C_A}\right)^2 A(gg\rightarrow ggg)
\]

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High Energy Jets (HEJ):

1) Inspiration from Fadin&Lipatov: dominance by \( t \)-channel
2) No kinematic approximations in invariants (denominator)
3) Accurate definition of currents (coupling through \( t \)-channel exchange)
4) Gauge invariance. Not just asymptotically.
Scattering of qQ-Helicity States

Start by describing quark scattering. Simple matrix element for \( q(a)Q(b) \rightarrow q(1)Q(2) \):

\[
M_{q^- Q^- \rightarrow q^- Q^-} = \langle 1|\mu|a\rangle g_{\mu\nu}^{\text{t}} \langle 2|\nu|b\rangle
\]

**t-channel factorised**: Contraction of (local) currents across \( t \)-channel pole

\[
|\overline{M}_t^{qQ \rightarrow qQ}|^2 = \frac{1}{4 \left( N_C^2 - 1 \right)} \left\| S_{qQ \rightarrow qQ} \right\|^2
\cdot \left( g^2 \ C_F \ \frac{1}{t_1} \right)
\cdot \left( g^2 \ C_F \ \frac{1}{t_2} \right).
\]

Extend to \( 2 \rightarrow n \ldots \)
Identification of the **dominant contributions** to the **perturbative series** in the limit of well-separated particles:

\[
\frac{1}{q^2} \exp (\hat{\alpha}(q) \Delta y)
\]

\[
q_i \rightarrow \mu \ V^\mu(q_{i-1}, q_i) \quad j^\nu = \bar{\psi} \gamma^\nu \psi
\]
Building Blocks for an Amplitude

$p_g \cdot V = 0$ can easily be checked (exact gauge invariance)

The approximation for $qQ \rightarrow qgQ$ is given by

$$
|\overline{\mathcal{M}}^t_{qQ \rightarrow qgQ}|^2 = \frac{1}{4 \left(N_C^2 - 1\right)} \left\| S_{qQ \rightarrow qQ} \right\|^2 \\
\cdot \left( g^2 C_F \frac{1}{t_1} \right) \cdot \left( g^2 C_F \frac{1}{t_2} \right) \\
\cdot \left( \frac{-g^2 C_A}{t_1 t_2} \right) V_{\mu}^{\mu}(q_1, q_2) V_{\mu}(q_1, q_2) .
$$
“What happens in $2 \rightarrow 2$-processes with gluons? Surely the $t$-channel factorisation is spoiled!"

Direct calculation ($q^- g^- \rightarrow q^- g^-$):

$$M = \frac{g^2}{\hat{t}} \times \frac{p_{2\perp}^*}{|p_{2\perp}|} \left( t_{ae}^2 t_{e1}^b \sqrt{\frac{p_b}{p_2}} - t_{ae}^b t_{e1}^2 \sqrt{\frac{p_2}{p_b}} \right) \langle b|\sigma|2 \rangle \times \langle 1|\sigma|a \rangle.$$

Complete $t$-channel factorisation!

J.M. Smillie and JRA
The $t$-channel current generated by a gluon in $qg$ scattering is that generated by a quark, but with a colour factor

$$\frac{1}{2} \left( C_A - \frac{1}{C_A} \right) \left( \frac{p_b^-}{p_2^-} + \frac{p_2^-}{p_b^-} \right) + \frac{1}{C_A}$$

instead of $C_F$. Tends to $C_A$ in MRK limit.

Similar results for e.g. $g^+g^- \rightarrow g^+g^-$. **Exact, complete $t$-channel factorisation.**

By using the formalism of **current-current scattering**, we get a better description of the $t$-channel pole than by using just the BFKL kinematic limit.
Have prescription for $2 \rightarrow n$ matrix element, including virtual corrections: Lipatov Ansatz $\frac{1}{t} \rightarrow \frac{1}{t} \exp(-\omega(t)\Delta y_{ij})$

Organisation of cancellation of IR (soft) divergences is easy

Can calculate the sum over the $n$-particle phase space explicitly ($n \sim 30$) to get the all-order corrections (just as if one had provided all the $N^{30} LO$ matrix elements and a regularisation procedure)

Merge $n$-jet tree-level MEs (by merging $m$-parton momenta to $n$ hard jet-momenta) where these can be evaluated in reasonable time

Extension of merging mechanism to $NLO$ ongoing

Resummation of HEJ recently merged with a parton shower (Ariadne)
Expression for the Cross Section

\[ \left| \mathcal{M}_{\text{reg}}^{\text{HEJ}}(\{p_i\}) \right|^2 = \frac{1}{4(N_C^2 - 1)} \left| S_{t_1 f_2 \rightarrow f_1 f_2} \right|^2 \cdot \left( g^2 K_{t_1} \frac{1}{t_1} \right) \cdot \left( g^2 K_{t_2} \frac{1}{t_{n-1}} \right) \]

\[ \cdot \prod_{i=1}^{n-2} \left( g^2 C_A \left( \frac{-1}{t_i t_{i+1}} V_{\mu}(q_i, q_{i+1}) V_{\mu}(q_i, q_{i+1}) - \frac{4}{p^2_j} \theta \left( p^2_i < \chi^2 \right) \right) \right) \]

\[ \cdot \prod_{j=1}^{n-1} \exp \left[ \omega^0(q_j, \lambda)(y_{j-1} - y_j) \right], \quad \omega^0(q_j, \lambda) = -\frac{\alpha_s N_C}{\pi} \log \frac{q_j^2}{\chi^2}. \]

\[ \sigma_{\text{resum,match}} = \sum_{f_1, f_2} \sum_{n=2}^{\infty} \prod_{i=1}^{n} \left( \int \frac{d^2 p_{i\perp}}{(2\pi)^3} \int \frac{dy_i}{2} \right) \frac{\left| \mathcal{M}_{\text{HEJ}}^{f_1 f_2 \rightarrow f_1 g \cdots g_{f_2}}(\{p_i\}) \right|^2}{s^2} \]

\[ \times \sum_{m} \mathcal{O}_{m_j}(\{p_i\}) \mathcal{O}_{m_{-\text{jet}}}(\{p_i\}) \]

\[ \times x_a f_{A, f_1}(x_a, Q_a) x_2 f_{B, f_2}(x_b, Q_b) (2\pi)^4 \delta^2 \left( \sum_{i=1}^{n} p_{i\perp} \right) \mathcal{O}_{2j}(\{p_i\}). \]

\[ w_{n-\text{jet}} = \frac{\left| \mathcal{M}_{f_1 f_2 \rightarrow f_1 g \cdots g_{f_2}}(\{p_{J_i}(\{p_i\})\}) \right|^2}{\left| \mathcal{M}_{t^2, f_1 f_2 \rightarrow f_1 g \cdots g_{f_2}}(\{p_{J_i}(\{p_i\})\}) \right|^2}. \]
Comparison to data

Two drivers for multi-jet production:
- large ratio of transverse scales (shower resummation)
- Colour exchange over a range in rapidity

The LHC has the energy to explore the second mechanism. Several interesting studies already with the first (2010) year of data!
This Atlas analysis did not cleanly separate the two “drivers” of jet production. (cut on $\bar{\vec{p}}_t$ induces large $\vec{p}_t$-hierarchy on forward/backward jet, besides the hierarchy between large $\bar{\vec{p}}_t$ and $Q_0$, the general jet scale)

HEJ slightly undershoots the jet activity when large ratios of transverse scales are imposed (shower region).

Very good agreement in the most important regions of phase space

Obviously beyond NLO (more than one extra jet on average at $\Delta y \geq 3$)
CMS: Simultaneous prod. of central and forward jet

Jets: anti-kt, R=.5, $p_t > 35$GeV

- central: $|\eta| < 2.8$
- forward: $3.2 < |\eta| < 4.7$

(not particularly large rapidity spans, typically 1 unit).
Measure the $p_t$-spectrum of the central and the forward jet. Any difference is obviously due to additional radiation.
Comparison to Theory, I

CMS, \( pp \rightarrow \text{jet}_{\text{wrd}} + \text{jet}_{\text{cent}} + X, \sqrt{s}=7 \text{ TeV}, L_{\text{int}} = 3.14 \text{ pb}^{-1} \)

\[
\frac{d^2\sigma}{dp_T d\eta} \quad [\text{pb/(GeV/c)}]
\]

- Data
- PYTHIA 6 (D6T)
- PYTHIA 6 (Z2)
- PYTHIA 8 (Tune 1)
- POWHEG (+PYTHIA 6)
- CASCADE

\(|\eta| < 2.8\)

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Comparison to Theory, II

CMS, pp → jet_{fwd} + jet_{cent} + X, \sqrt{s}=7 \text{ TeV}, \mathcal{L}_{\text{int}} = 3.14 \text{ pb}^{-1}

\begin{align*}
\frac{d^2 \sigma}{d \eta d \mathbf{p}_T} &\text{ [pb/(GeV/c)]} \\
|\eta| &< 2.8 \\
\text{central jet } p_T (\text{GeV/c}) &
\end{align*}

\begin{align*}
\frac{d^2 \sigma}{d \eta d \mathbf{p}_T} &\text{ [pb/(GeV/c)]} \\
3.2 < |\eta| < 4.7 \\
\text{forward jet } p_T (\text{GeV/c}) &
\end{align*}

- Data
- HERWIG 6 (+JIMMY)
- HERWIG++
- POWHEG (+HERWIG)
- HEJ

Jeppe R. Andersen (IPPP)  Hard Jets and Higgs Bosons  Birmingham, April 30 2014  28 / 47
Predictions for ratio of Inclusive Jet Rates vs. $H_{T2}$


Inclusive dijet production
LHC @ 7 TeV

(d\sigma_{3}^{incl}/dH_{T2}) / (d\sigma_{2}^{incl}/dH_{T2})

**Similarities:** NLO+Shower, HEJ (all-order hard resummation)

**Difference:** NLO
Clear differences: NLO, POWHEG, HEJ
Simple set of cuts, combined with a exclusive dijet-analyses can discriminate clearly between the mechanisms of perturbative corrections implemented in NLO, POWHEG (NLO+Shower) and High Energy Jets.
Azimuthal angle between two hardest jets

Theory/Data

$\frac{d\sigma}{d\Delta\phi_{12}}$ [pb]
\[ \Delta R = \sqrt{\Delta y^2 + \delta \phi^2} \] between hardest jets
Good description everywhere, in particular also at large invariant mass between jets. Important region for HJJ-analyses (see later).
D0 measurement of the probability of at least one additional jet when requiring just a $W$ in association with two jets. Probability measured vs. rapidity separation of

1. the two most rapidity separated jets
2. the two hardest (in pt) jets
3. the two hardest (in pt) jets, counting additional jets only in the rapidity interval between the two hardest jets
CP Properties of Higgs-Boson Couplings from Hjj through Gluon Fusion
Stabilising the Extraction against Higher Order Corrections
Why study Higgs Boson production in Association with Dijets?

The distribution in the **azimuthal angle** between the **two** jets in $Hjj$ allows for a **clean extraction** of CP properties.

The Problem

... in a region of phase space where the **perturbative corrections** are large.

How do we deal with events with **three or more** jets?

The Solution

By constructing an azimuthal observable, which takes into account the information from all the **jets** of the event!
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Considerations for Weak Boson Fusion

\( W, Z \quad H \quad W, Z \)
Higgs Couplings through Azimuthal Correlations

...and gluon fusion (Higgs coupling to gluons through top loop)
Higgs Couplings through Azimuthal Correlations

\[ \mathcal{M} \propto \frac{j_1^\mu C_H^{\mu\nu} j_2^\nu}{t_1 t_2}, \quad j_1^\mu = \bar{\psi}_1 \gamma^\mu \psi_a \]

\[ C_H^{\mu\nu} = a_2 \left( q_1 q_2 g^{\mu\nu} - q_1^\nu q_2^\mu \right) + a_3 \varepsilon^{\mu\nu\rho\sigma} q_1^\rho q_2^\sigma. \]
Higgs Couplings through Azimuthal Correlations

\[ \mathcal{M} \propto \frac{j_1^\mu C_{\mu\nu}^H j_2^\nu}{t_1 t_2}, \quad j_1^\mu = \overline{\psi}_1 \gamma^\mu \psi_a \]

\[ C_{\mu\nu}^H = a_2 \left( q_2 g_{\mu\nu} - q_2^\mu q_2^\nu \right) + a_3 \varepsilon^{\mu\nu\rho\sigma} q_1^\rho q_2^\sigma. \]

Take e.g. the term \( \varepsilon^{\mu\nu\rho\sigma} q_1^\rho q_2^\sigma \): for \( |p_{1,z}| \gg |p_{1,x,y}| \) and for small energy loss (i.e. \( \overline{\psi}_1 \gamma^\mu \psi_a \to 2p_a, \overline{\psi}_2 \gamma^\mu \psi_b \to 2p_b, p_{a,e} \sim p_{1,e} \)):

\[ \left[ j_1^0 j_2^3 - j_1^3 j_2^0 \right] (q_1 \perp \times q_2 \perp). \]

In this limit, the azimuthal dependence of the propagators is also suppressed: \( |\mathcal{M}|^2: \sin^2(\phi) \) (CP-odd), \( \cos^2(\phi) \) (CP-even).
Azimuthal distribution

$1/\sigma \, d\sigma / d\phi_{ja|jb}$

$\phi_{ja|jb}$

$y_{sep} = 0.5$

$y_{sep} = 0.75$

$y_{sep} = 1.0$

$\frac{1}{\sigma} \, d\sigma / d\phi_{ja|jb}$

$\phi_{ja|jb}$

$CP$-even, $p_{j\perp} > 40$ GeV, $y_{ja} < y_h < y_{jb}$,

$|y_{ja,jb}| < 4.5, \min (|y_h - y_{ja}|, |y_h - y_{jb}|) > y_{sep}$. 

JRA, K. Arnold, D. Zeppenfeld

\[ \Delta y = \left| y_{ja} - y_{jb} \right|, \quad y^* = y_h - \frac{y_{ja} + y_{jb}}{2}. \]

Rapidity separation between the jets and the Higgs Boson **enhance** the azimuthal correlation.
Increasing Rapidity Span → Increasing Number of Jets

All models show a clear increase in the number of hard jets as the rapidity span increases.

How to extract the $CP$-structure of the Higgs boson coupling from events with three or more jets?

Develop Insight Into the Perturbative Corrections

High Energy Limit

\[ |p_{\perp,i}| \text{ fixed, } \hat{s}_{ij} \to \infty \]

\[ |\mathcal{M}_{gg \to g...ghg...g}|^2 \to \frac{4 \hat{s}^2}{N_C^2 - 1} \left( \prod_{i=1}^{j} \frac{C_A g_s^2}{p_{i\perp}^2} \right) \frac{|C^H(q_{a\perp}, q_{b\perp})|^2}{q_{a\perp}^2 q_{b\perp}^2} \left( \prod_{i=j+1}^{n} \frac{C_A g_s^2}{p_{i\perp}^2} \right) \]

\[ C^H(q_{a\perp}, q_{b\perp}) = -i \frac{\alpha_s}{3\pi v} q_{a\perp} \cdot q_{b\perp}, \quad y_0 < \cdots < y_j < y_H < y_{j+1} < y_n \]

The High Energy Limit tells us to investigate the azimuthal angle between the sum of the jet vectors either side in rapidity of the Higgs Boson!
And It Even Works!

Three subsamples of tree-level three-jet events: two jets on same side of the Higgs boson parallel (S1), perpendicular (S2) or anti-parallel (S3). Azimuthal correlation almost unchanged from hjj.
Les Houches Comparison of HJJ Predictions

Good agreement of inclusive $Hjj$-cross section and differential distributions (e.g. rapidity of the Higgs boson).

Variations within the uncertainty quoted for each calculation.
Differences arising at large invariant mass between the hard jets. (as expected)

Vector-Boson-Fusion cuts select region of large $m_{jj}$.

(Please recall that HEJ gives a good description of WJJ at large invariant mass).
The difference in the distribution of $m_{jj}$ (and $\Delta y_{12}$) induce a difference in the cross section after VBF-cuts.

The difference in behaviour between shower-approaches and HEJ appear at large rapidities and large $m_{jj}$ - where HEJ resums virtual corrections that are not treated systematically in any of the other approaches.
Conclusions

The LHC probes hard (=jets) perturbative corrections beyond pure NLO
... already at 7TeV!

*High Energy Jets* provides a new approach to the perturbative description of LHC physics
... and compares favourably to data in several analyses
... several ongoing improvements in the formal accuracy of the perturbative approximations

*http://cern.ch/hej*