Birmingham High Energy Physics Group - Particle Physics Seminar,
University of Birmingham, 08.06.2016

Lattice QCD

Andreas Jüttner

UNIVERSITY OF Southampton

Outline

1. Lattice QCD (why and what)

2. Precision flavour physics

3. $(g\text{-}2)_\mu$ on the lattice

4. Pushing the frontiers (QED+QCD, rare decays)

UK Lattice community

- Cambridge
- Edinburgh
- Glasgow
- Liverpool
- Oxford
- Plymouth
- Southampton
- Swansea

Various collaborations
(UKQCD, HotQCD, HPQCD, ...)

- QCD flavour phenomenology
- QCD spectra
- BSM models (non-perturbatively)
- finite-T, finite-μ
- developments in quantum field theory, algorithms computing and hardware

34th International Symposium on
Lattice Field Theory

University of Southampton
24–30 July 2016

LATTICE
2016

http://www.southampton.ac.uk/lattice2016/

Motivation

- Standard Model of elementary particle physics describes electromagnetic, weak and strong (QCD) interactions consistently in terms of a renormalisable quantum field theory

- but there is substantial phenomenological evidence that it can't be the whole story: dark matter, CP-violation, … indicate that there must be sth. else

- despite decades of experimental and theoretical efforts we have not found a smoking gun

Motivation

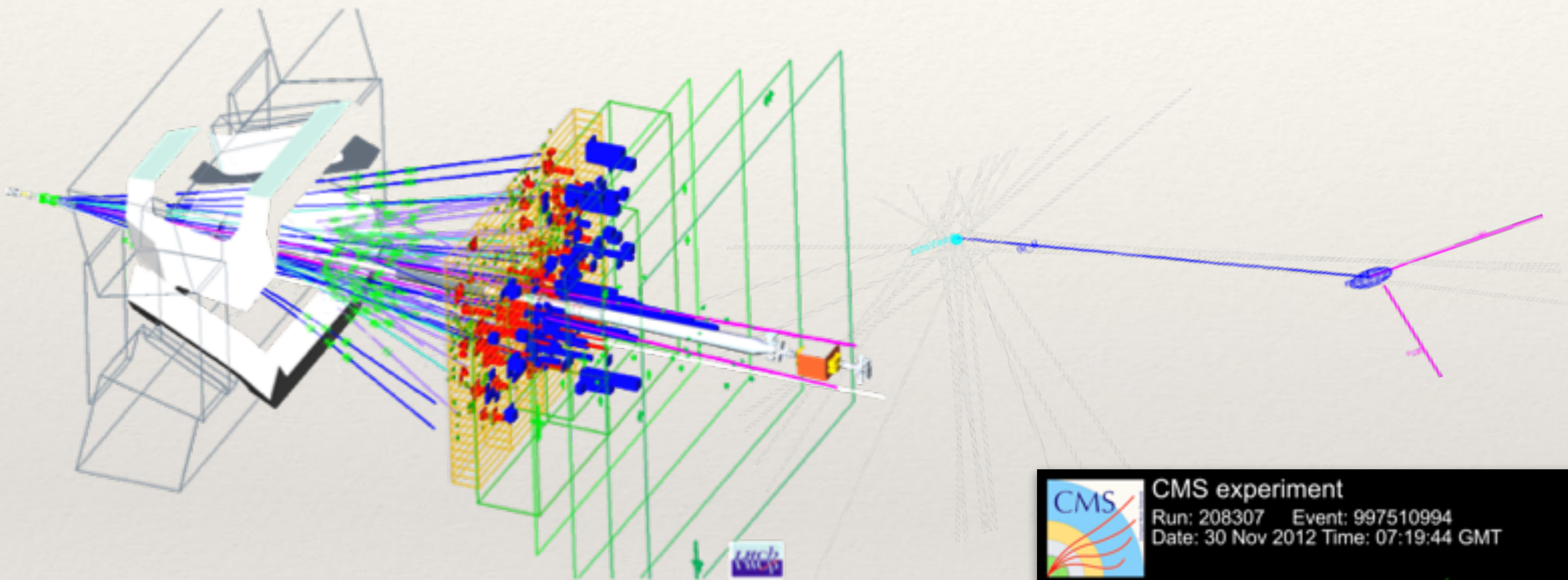
- searches for new physics: direct vs. indirect search:
  - ‘bump in the spectrum’
  - SM provides correlation between processes
    experiment + theory to over-constrain SM

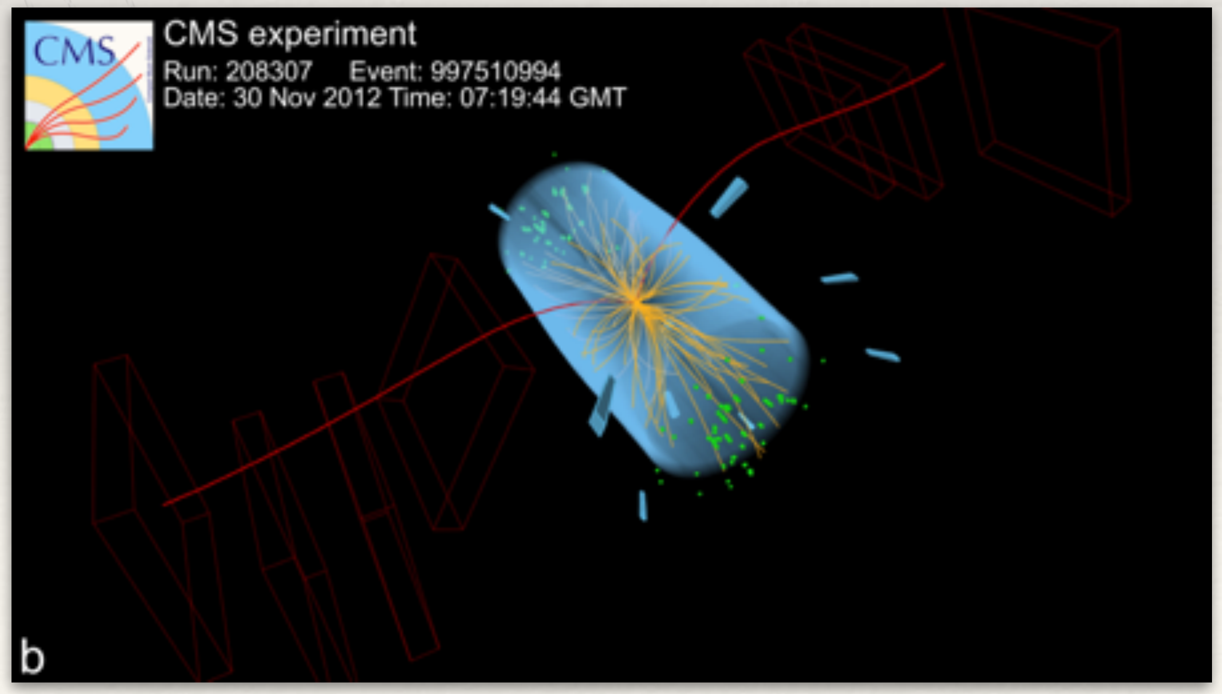
- hadronic (QCD) uncertainties dominating error budget

- lattice QCD can in principle provide the relevant input and is becoming increasingly precise in its predictions

$B_s \to \mu^+\mu^-$

First observed by LHCb, CMS





$B_s \rightarrow \mu^+\mu^-$

Standard Model prediction:

- Loop suppressed in the SM (FCNC) → sensitive to non-SM interaction?

$$\mathrm{Br}(B_s \to \ell^+\ell^-)^{\mathrm{SM}} = \tau_{B_s} \frac{G_F^2 \alpha^2}{16\pi^3} |V_{tb}V_{ts}^*|^2 m_{B_s} m_\ell^2 \beta_\ell(m_{B_s}^2) |C_{10}|^2 \boxed{f_{B_s}^2}$$

$$Br \propto (PT) \times \boxed{\langle 0|\bar{s}\gamma_\mu\gamma_5 b|\bar{B}_s\rangle^2} + \cdots$$

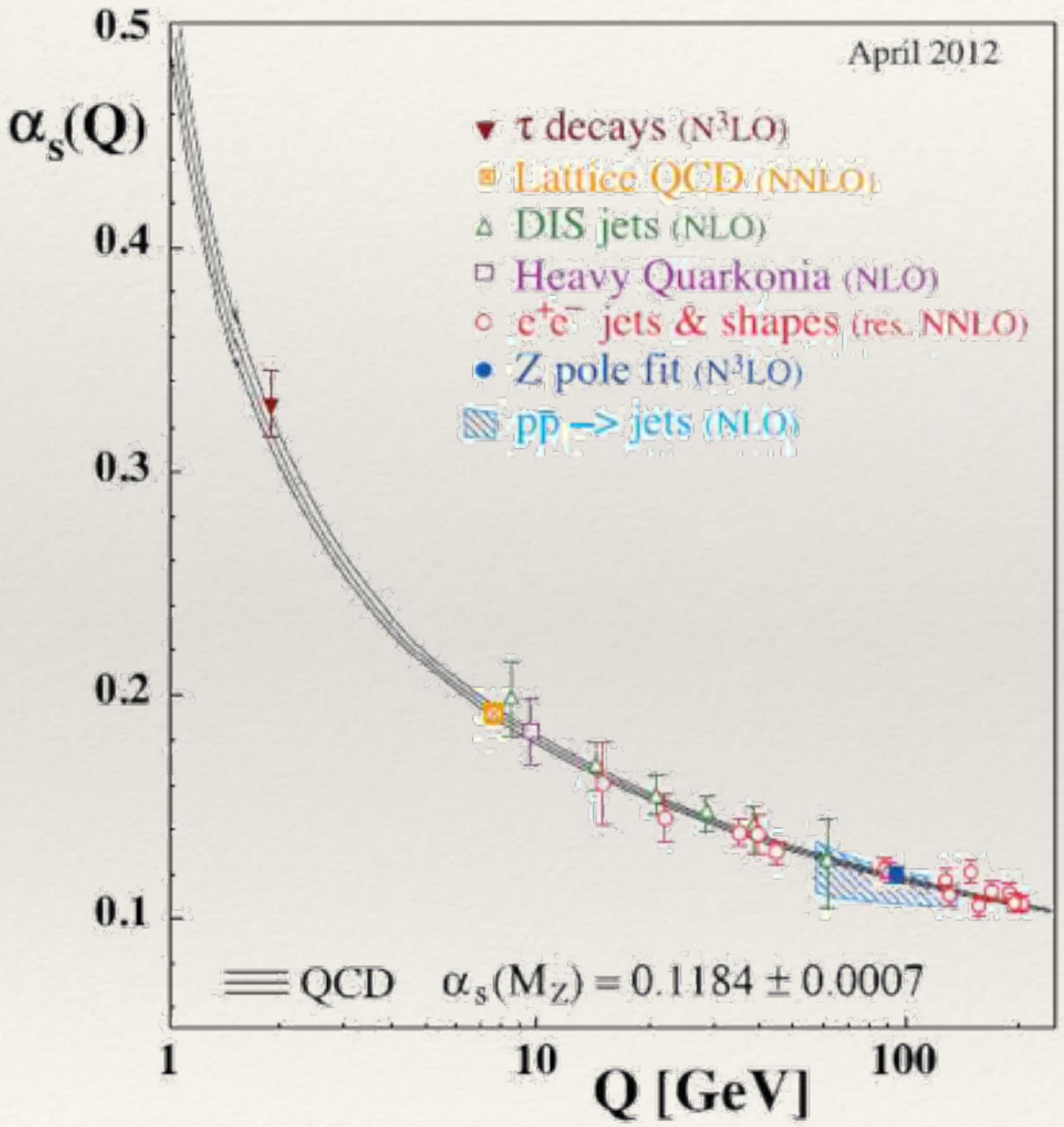
NNLO QCD
NLO EW

Hermann, Misiak, Steinhauser, JHEP 1312, 097 (2013)
Bobeth, Gorbahn. Stamou, PRD 89, 034023 (2014)

very precise and reliable prediction for the decay constant is needed

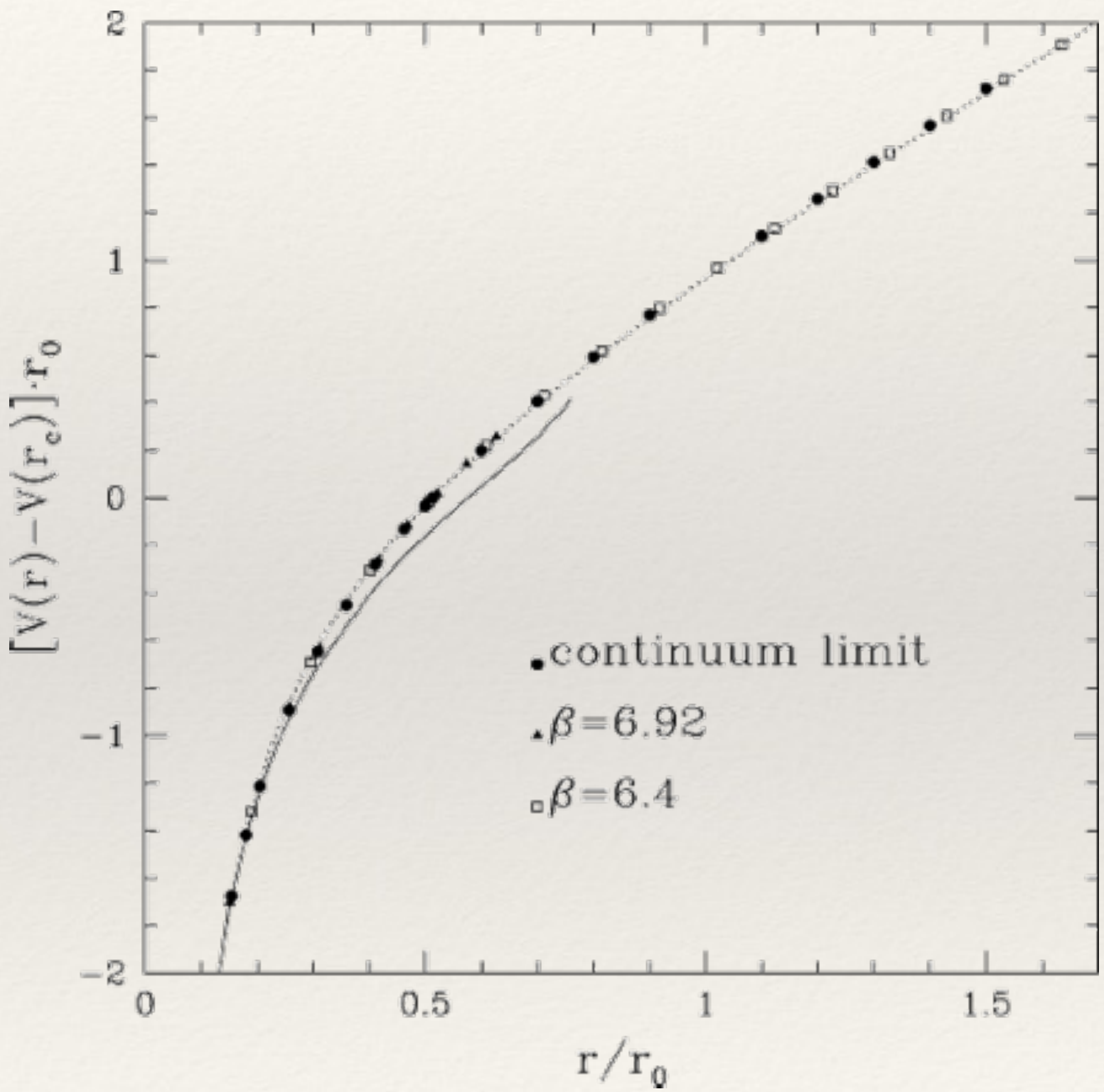
QCD

asymptotic freedom



PDG

confinement



Necco & Sommer NPB 622 (2002)

Lattice QCD

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \sum_f \bar{\psi}_f \left(i\gamma^\mu D_\mu - m_f\right) \psi_f$$

Free parameters:

- gauge coupling $g \rightarrow \alpha_s = g^2/4\pi$
- quark masses $m_f = u,d,s,c,b,t$

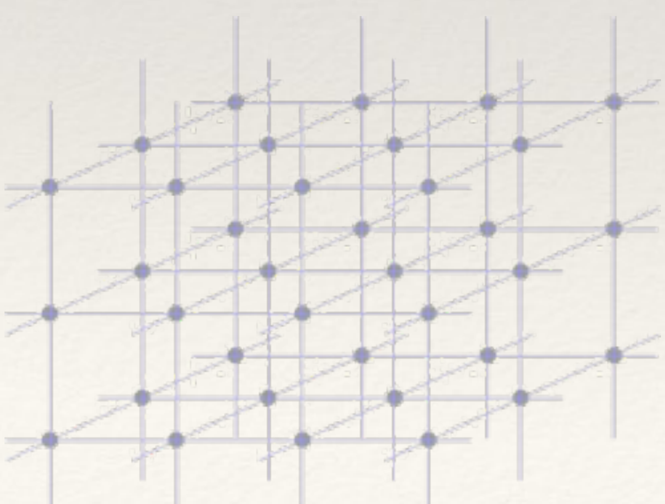
- Lagrangian of massless gluons and *almost massless quarks*
- what experiment sees are bound states, e.g. $m_\pi, m_P \gg m_{u,d}$
- underlying physics non-perturbative

Path integral quantisation:

$$\langle 0|O|0\rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}[U,\psi,\bar{\psi}] O e^{-iS_{\text{lat}}[U,\psi,\bar{\psi}]}$$

$$\langle 0|O|0\rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}[U,\psi,\bar{\psi}] O \boxed{e^{-S_{\text{lat}}[U,\psi,\bar{\psi}]}}$$

Euclidean space-time
Boltzmann factor



finite volume, space-time grid (IR and UV regulators)
$\propto L^{-1}$ $\propto a^{-1}$

→ well defined, finite dimensional Euclidean path integral
→ from first principles

Lattice QCD

- gauge-invariant regularisation (Wilson 1974)
- naively: replace derivatives by finite differences, integrals by sums
- finite volume lattice path integral still over large number of degrees of freedom > *O*(10^{10})
- Evaluate discretised path integral by means of Markov Chain Monte Carlo on state-of-the-art HPC installations
- UK computing time via STFC's DiRAC consortium

Euclidean correlation function

$$\langle 0|\mathcal{O}_{B_s}(t)\mathcal{O}_{B_s}(0)^\dagger|0\rangle = \frac{1}{Z}\int \mathcal{D}[\bar\psi,\psi,U]\mathcal{O}_{B_s}(t)\mathcal{O}_{B_s}(0)^\dagger e^{-S[\bar\psi,\psi,U]}$$

two-point function

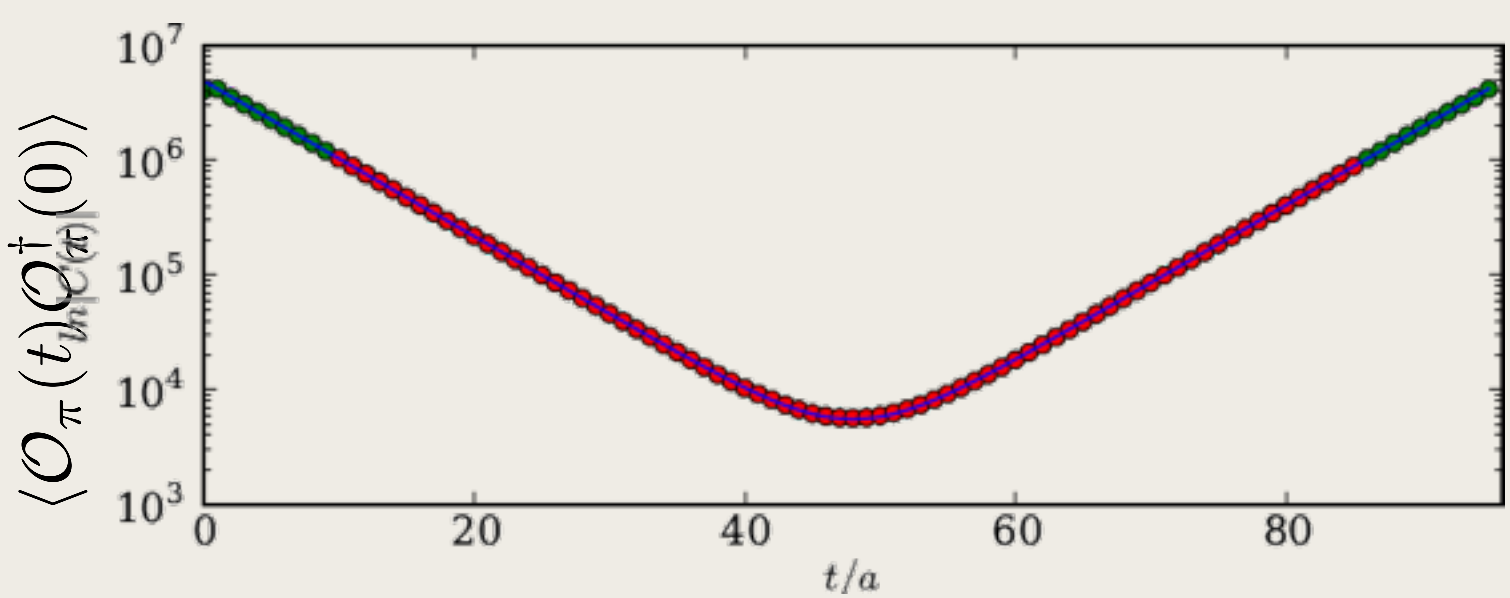
$$\begin{aligned}\langle 0|\mathcal{O}_{B_s}(t)\mathcal{O}_{B_s}(0)^\dagger|0\rangle &= \sum_{\vec{x},n}\langle 0|\mathcal{O}_{B_s}(x)|n\rangle\langle n|\mathcal{O}^\dagger_{B_s}(0)|0\rangle \\ &= \sum_n |\langle 0|\mathcal{O}_{B_s}(0)|n\rangle|^2\, e^{-E_n t_x} \\ &\overset{t\to\infty}{=} |\langle 0|\mathcal{O}_{B_s}(0)|B_s\rangle|^2\, e^{-m_{B_s} t_x}\end{aligned}$$

extract physical properties from fits to simulation data:

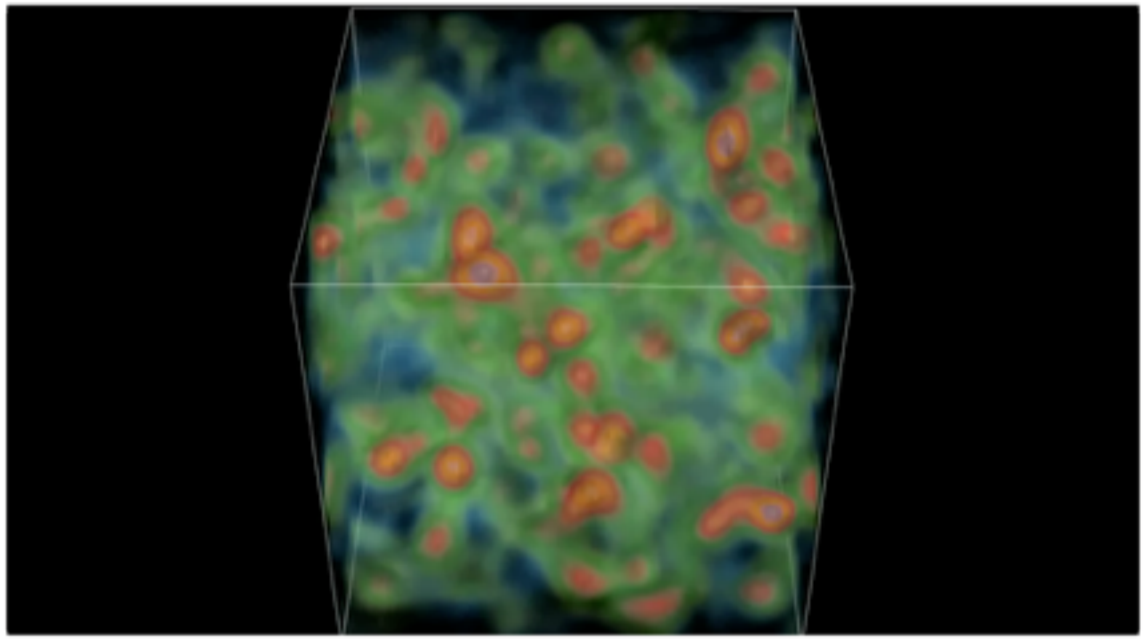
- normalisation → matrix element (e.g. decay constant)
- time-dependence → particle spectrum (e.g. meson mass)
- stat. errors from MC sampling over N field configurations

$$\langle \mathcal{O}\mathcal{O}^\dagger\rangle = \frac{1}{N}\sum_{n=1}^{N}[\mathcal{O}\mathcal{O}^\dagger]_n$$

(bootstrap, jackknife error analysis, autocorrelation analysis, …)



State of the art of lattice QCD simulations



action density of RBC/UKQCD physical point DWF ensemble

What we can do

- simulations of QCD with dynamical (sea) *u,d,s,c* quarks with masses as found in nature ➔ $N_f = 2,\ 2+1,\ 2+1+1$
- bottom only as valence quark
- cut-off $a^{-1} \leq 4\text{GeV}$
- volume $L \leq 6fm$

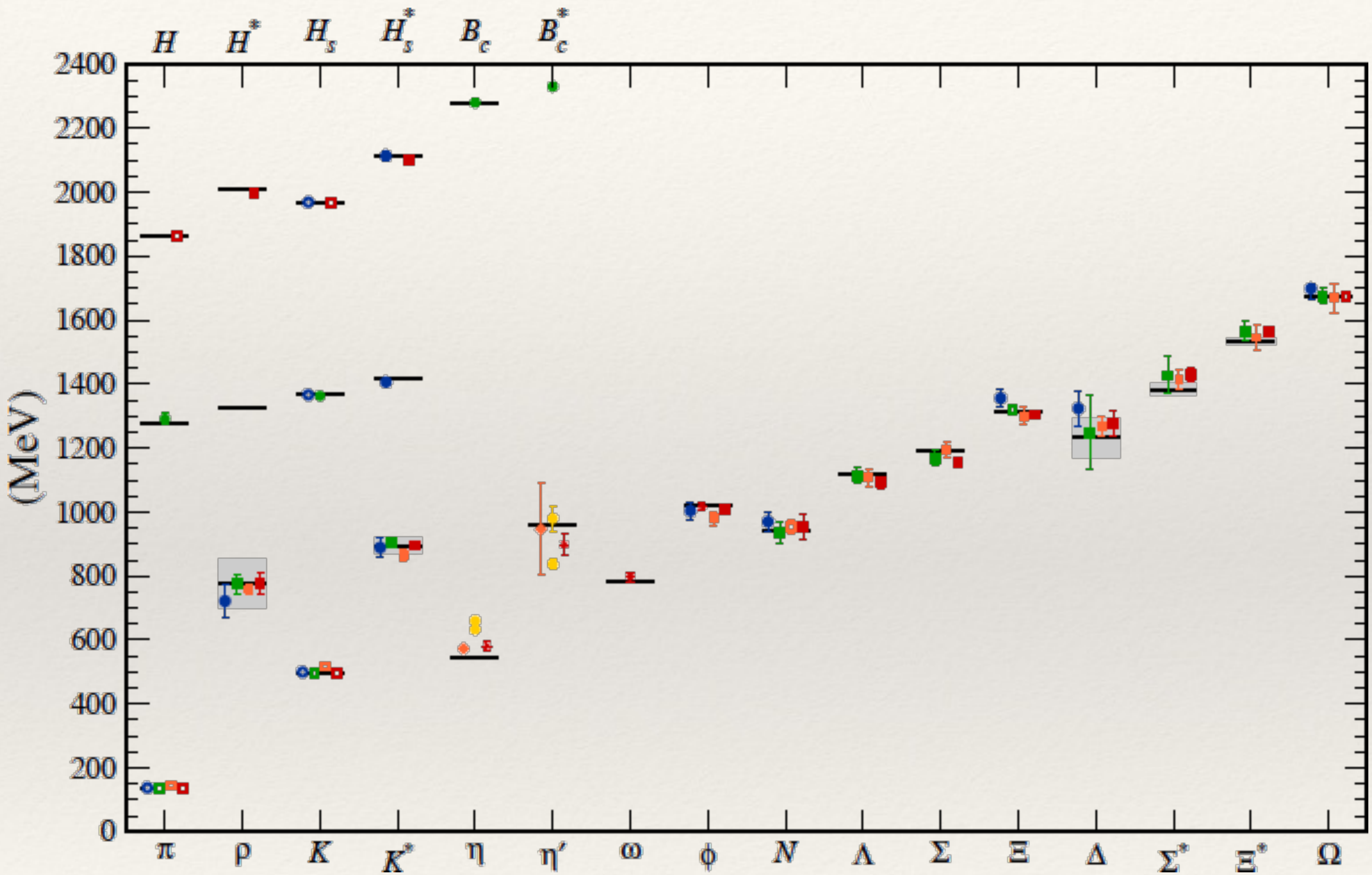
Parameter tuning

start from *educated guesses* and:

- tune light quark mass am_l such that $\frac{am_\pi}{am_P} = \frac{m_\pi^{PDG}}{m_P^{PDG}}$
- tune strange quark mass such that $\frac{am_\pi}{am_K} = \frac{m_\pi^{PDG}}{m_K^{PDG}}$
- determine physical lattice spacing $a = \frac{af_\pi}{f_\pi^{PDG}}$

IMPORTANT:
once the QCD-parameters are *tuned* no further parameters need to be fixed and we can make fully predictive simulations of QCD

benchmark - the hadron spectrum

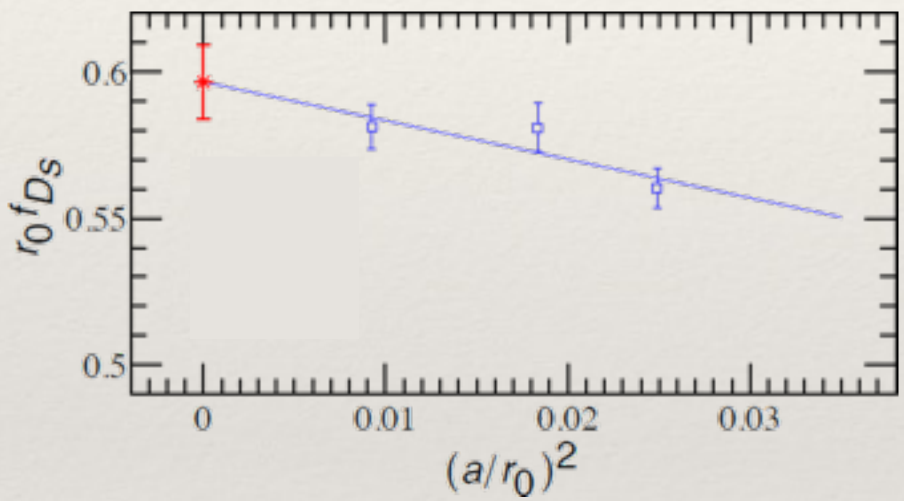


Kronfeld, Ann. Rev. of Nucl. Part. Sci 2012 62

lattice - systematics

In practice one needs to control a number of sources of systematic uncertainties, most notably:

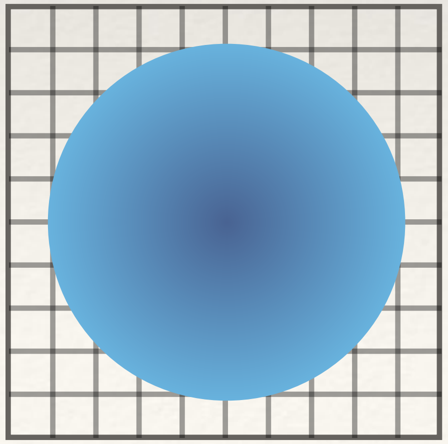
- **discr. errors** (lattice spacing a)



$$S_{\rm eff} = \int d^4x \, \{\mathcal{L}_0(x) + a\mathcal{L}_1(x) + a^2\mathcal{L}_2(x) + \ldots\}$$

Symanzik 1982,1983

- **finite volume errors** (box size L)



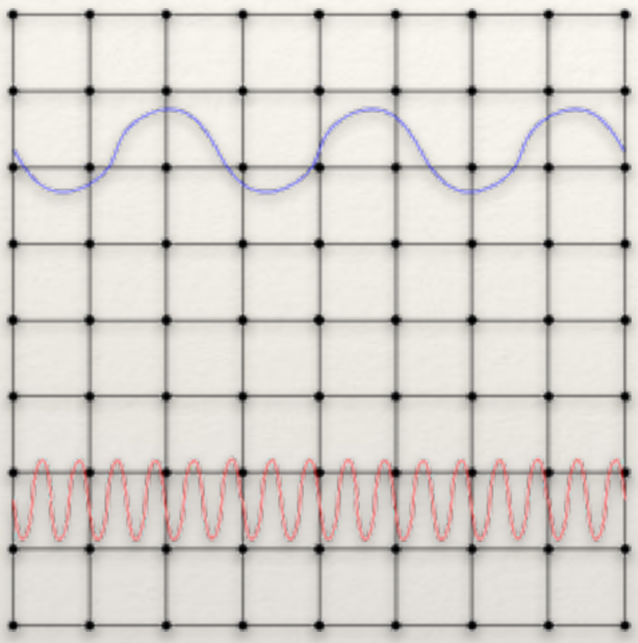
In QCD for simple ME $\propto e^{-m_\pi L} \propto O(1\%)$

more complicated for processes with several hadrons in initial or final state

Lüscher Commun.Math.Phys. 105 (1986) 153-188, Nucl. Phys. B354, 531 (1991)

quite crude, in practice more complicated

a state-of-the-art lattice



need to keep

$$\boxed{a^{-1} \ll \text{ relevant scales } \ll L^{-1}}$$

- for m_π=140MeV the constraint for controlled finite volume effects of $m_\pi L \gtrsim 4$ suggests L≈6fm
- for charm quarks to be well resolved $am_c < 1$ *e.g.* a^{-1} larger than ≈2.5GeV needed
- lattices with $L/a \gtrsim 80$ needed

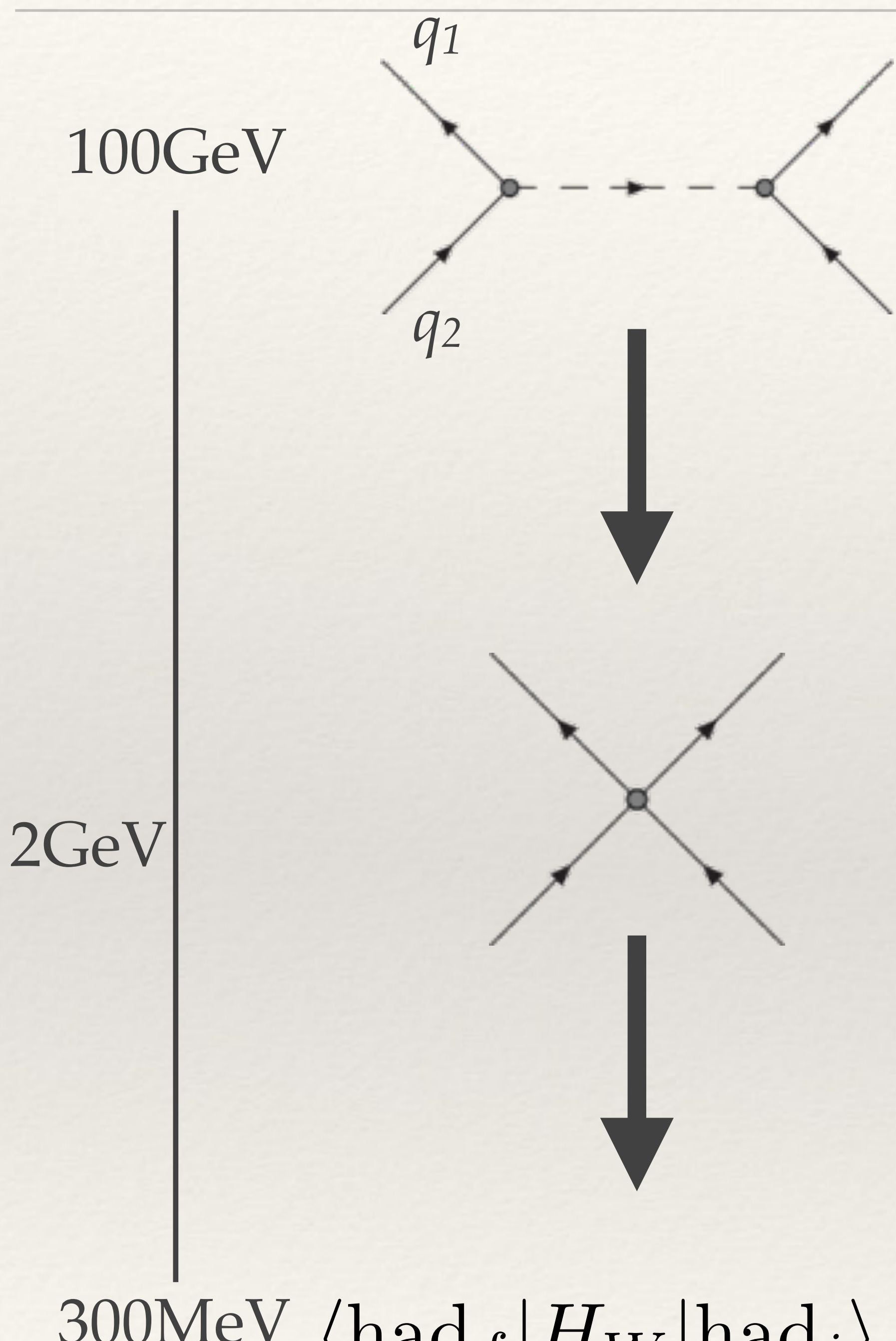
Fulfilling all the constraints is just starting to happen
(e.g. first $96^3 \times 192$ have been generated (MILC)) in the meantime most collaborations

- weaken the finite volume effects by simulating unphysically heavy pions
- extrapolate from coarser lattices relying on assumptions for functional form of cutoff effects

Lattice pheno - what's possible

- **Standard:**
  - meson ME with single incoming and/or outgoing pseudo-scalar states
    $\pi, K, D_{(s)}, B_{(s)} \to \mathrm{QCD-vacuum},\ \pi \to \pi, K \to \pi, D \to K, B \to \pi, ..., B_K, (B_D), B_B$
  - QCD parameters: quark masses, strong coupling constant
  - meson/baryon spectroscopy of stable (in QCD) states
- **Challenging:**
  - two initial/final hadronic states, one channel $\pi\pi \to \pi\pi, K\pi \to K\pi, K \to \pi\pi, \ldots$
  - elm. effects in spectra
  - long-distance contributions in e.g. rare Kaon decays, K-mixing
- **Very challenging - new ideas needed/no clue:**
  - multi-channel final states (hadronic D, B) (e.g. Hansen, Sharpe PRD86, 016007 (2012))
  - transition MEs with unstable in/out states (Briceño et al. arXiv:1406.5965)
  - electromagnetic effects in hadronic MEs

Quark Flavour Physics



$$\langle \mathrm{had}_f | H_W | \mathrm{had}_i \rangle$$

$$\langle 0 | H_W | \mathrm{had}_i \rangle$$

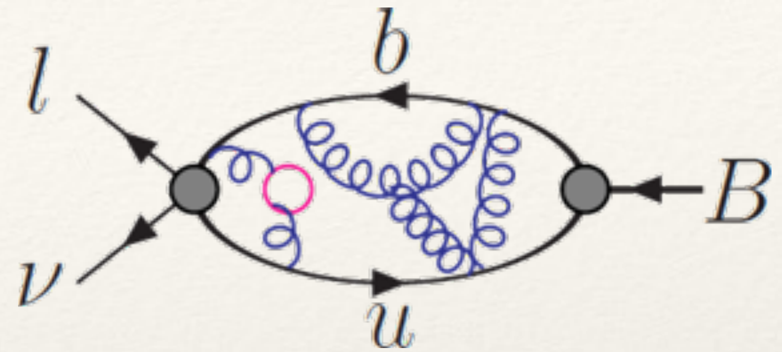
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

3x3 unitary matrix
4 unknown parameters

- quark mixing
- CP-violation (one complex phase)
- constraints on SM processes
- high energy reach
- inconsistencies → failure of the SM?

Quark Flavour Physics

e.g tree level leptonic B decay:



Assumed factorisation: $\Gamma_{\text{exp.}} \overset{???}{=} V_{\text{CKM}}(\text{WEAK})(\text{EM})(\text{STRONG})$

currently EFT

$$\underbrace{\Gamma(B \to l\nu_l)}_{\text{theory}} = \underbrace{|V_{ub}|^2}_{\text{output}} \underbrace{\frac{m_B}{8\pi} G_F^2 m_l^2 \left(1 - \frac{m_l^2}{m_B^2}\right)^2 f_B^2}_{\text{theory prediction}}$$

Experimental measurement + theory prediction allows for extraction of CKM MEs

Flavour Physics

Determine CKM elements $\longleftrightarrow$ (indirect) test of SM:

- over-determine elements of V_{CKM} and check consistency of CKM paradigm
- unitarity tests:
  - rows and columns are (in SM) complex unit vectors
  - rows (columns) are orthogonal to other rows (columns)

  violation of unitarity would indicate non-SM physics

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \stackrel{?}{=} 1$$

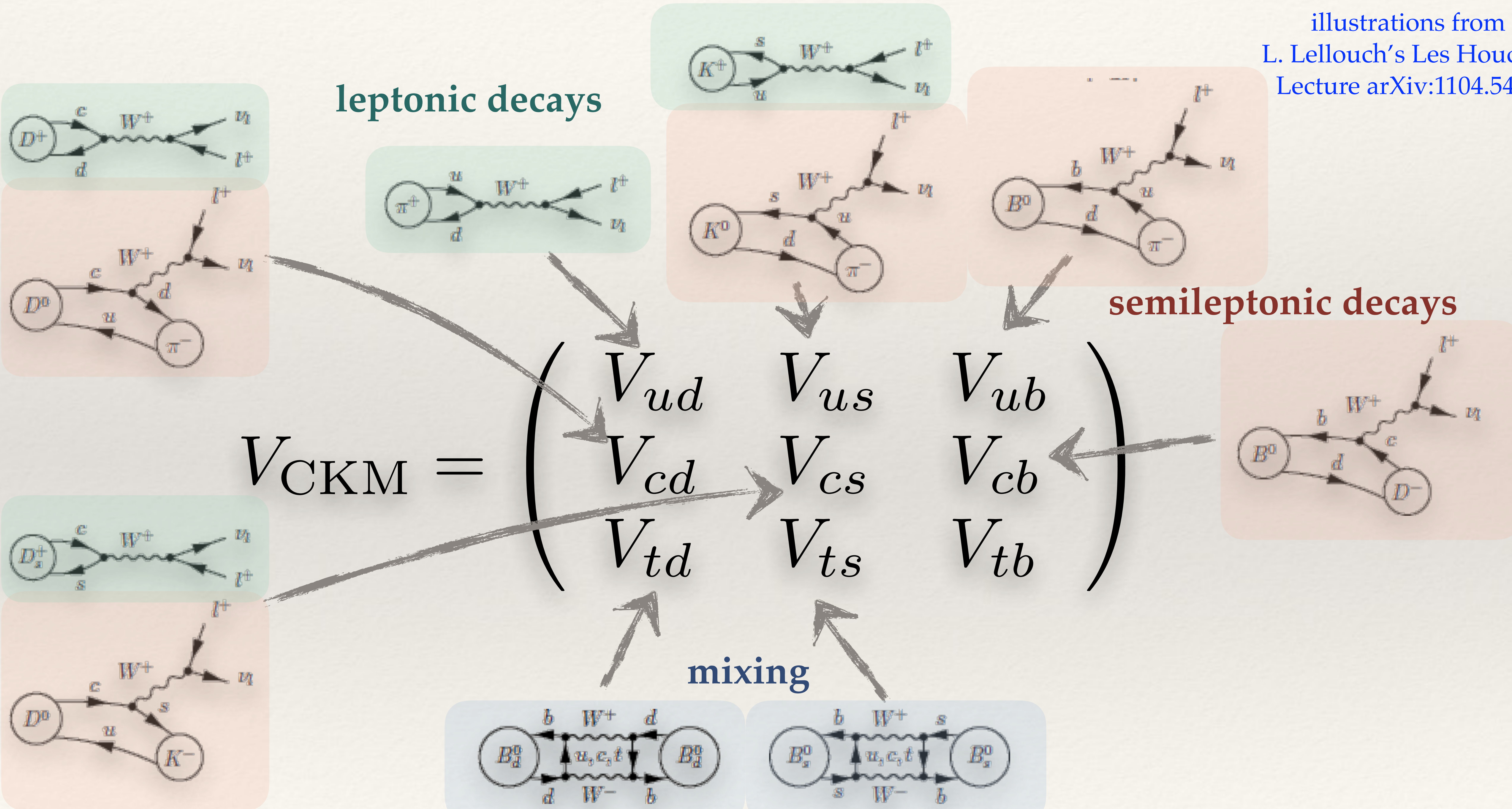
row-test

$$\sum_{U=u,c,t} V_{Ud} V_{Ub}^* \stackrel{?}{=} 0$$

triangle-test

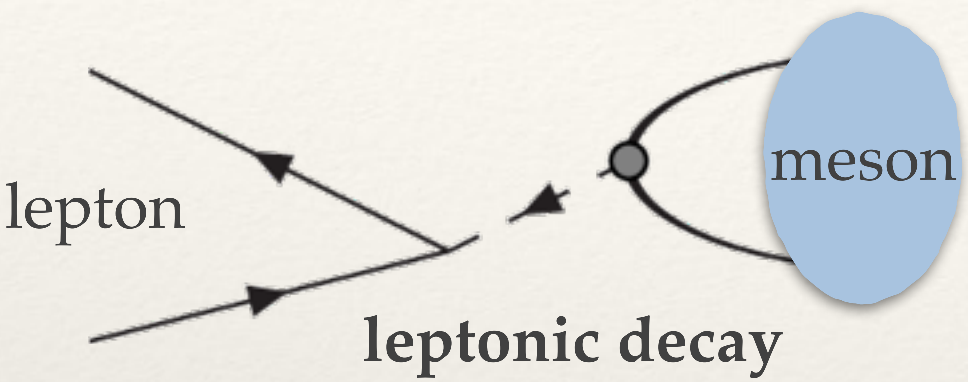
- Which channels still allow room for NP?
  How much NP would be compatible with measurements?
  What would be the properties of NP?

Lattice flavour physics and CKM



illustrations from
L. Lellouch's Les Houches
Lecture arXiv:1104.5484

“tree” kaon/pion decays

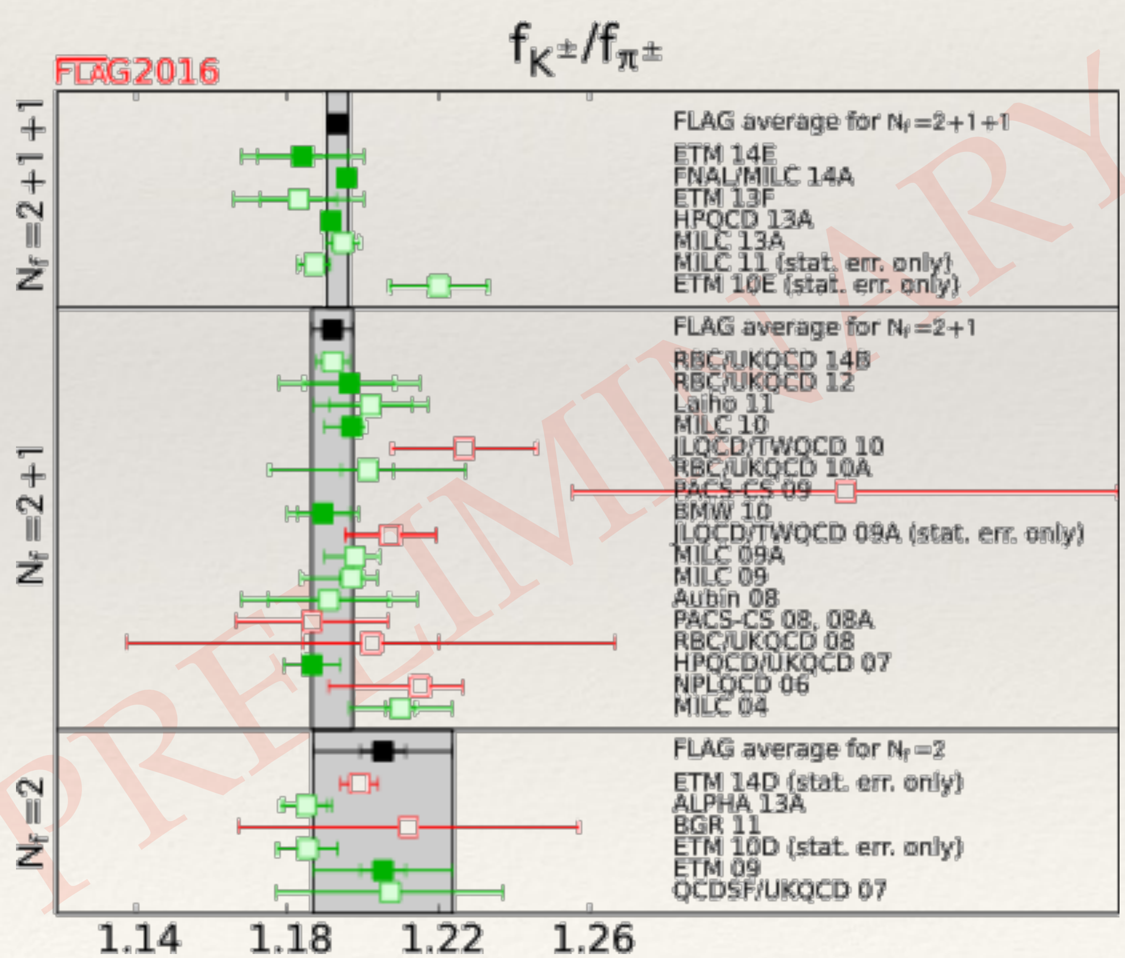


$$\Gamma(K \to \mu\bar{\nu}_\mu) = \frac{G_F^2}{8\pi} f_K^2 m_\mu^2 m_K \left(1 - \frac{m_\mu^2}{m_K^2}\right)^2 |V_{us}|^2$$

$$\langle 0|\bar{s}/\bar{d}\gamma_\mu\gamma_5 u|K/\pi(p)\rangle = i f_{K/\pi} p_\mu$$

$$\frac{\Gamma(K\to\mu\bar{\nu}_\mu)}{\Gamma(\pi\to\mu\bar{\nu}_\mu)} = \frac{|V_{us}|^2}{|V_{ud}|^2}\left(\frac{f_K}{f_\pi}\right)^2 \frac{m_K(1-m_\mu^2/m_K^2)^2}{m_\pi(1-m_\mu^2/m_\pi^2)^2} \times 0.9930(35)$$

Marciano, Phys.Rev.Lett. 2004



1‰!!!

Standard calculations and results - FLAG

Flavour Lattice Averaging Group

"What's currently the best lattice value for a particular quantity?"

FLAG-1 **(Eur. Phys. J. C71 (2011) 1695)**

FLAG-2 **(http://itpwiki.unibe.ch/flag/, Eur.Phys.J. C74 (2014) 2890)**

FLAG-3 - working on it

- quantities:

$$m_{u,d,s,c,b}$$

$$f_K/f_\pi,\, f_+^{K\pi}(0),\, B_K,\, SU(2) \text{ and } SU(3) \text{ LECs}$$

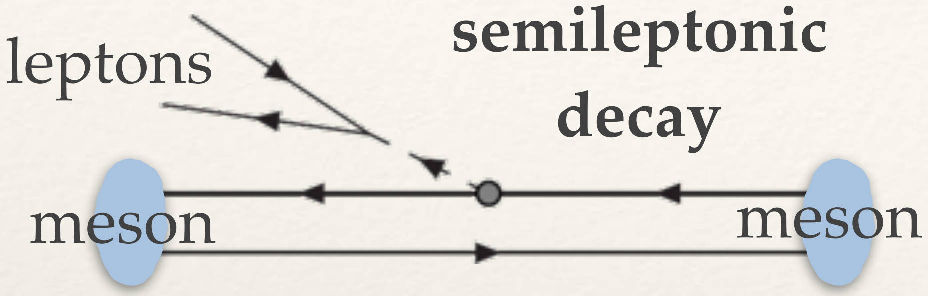
$$f_{D_{(s)}},\, f_{B_{(s)}},\, B_{B_{(s)}},\, B_{(s)}- \text{ and } D_{(s)}-\text{semileptonics}$$

$$\alpha_s$$

- summary of results
  - evaluation according to FLAG quality criteria (colour coding)
  - averages of best values where possible
  - detailed summary of properties of individual simulations

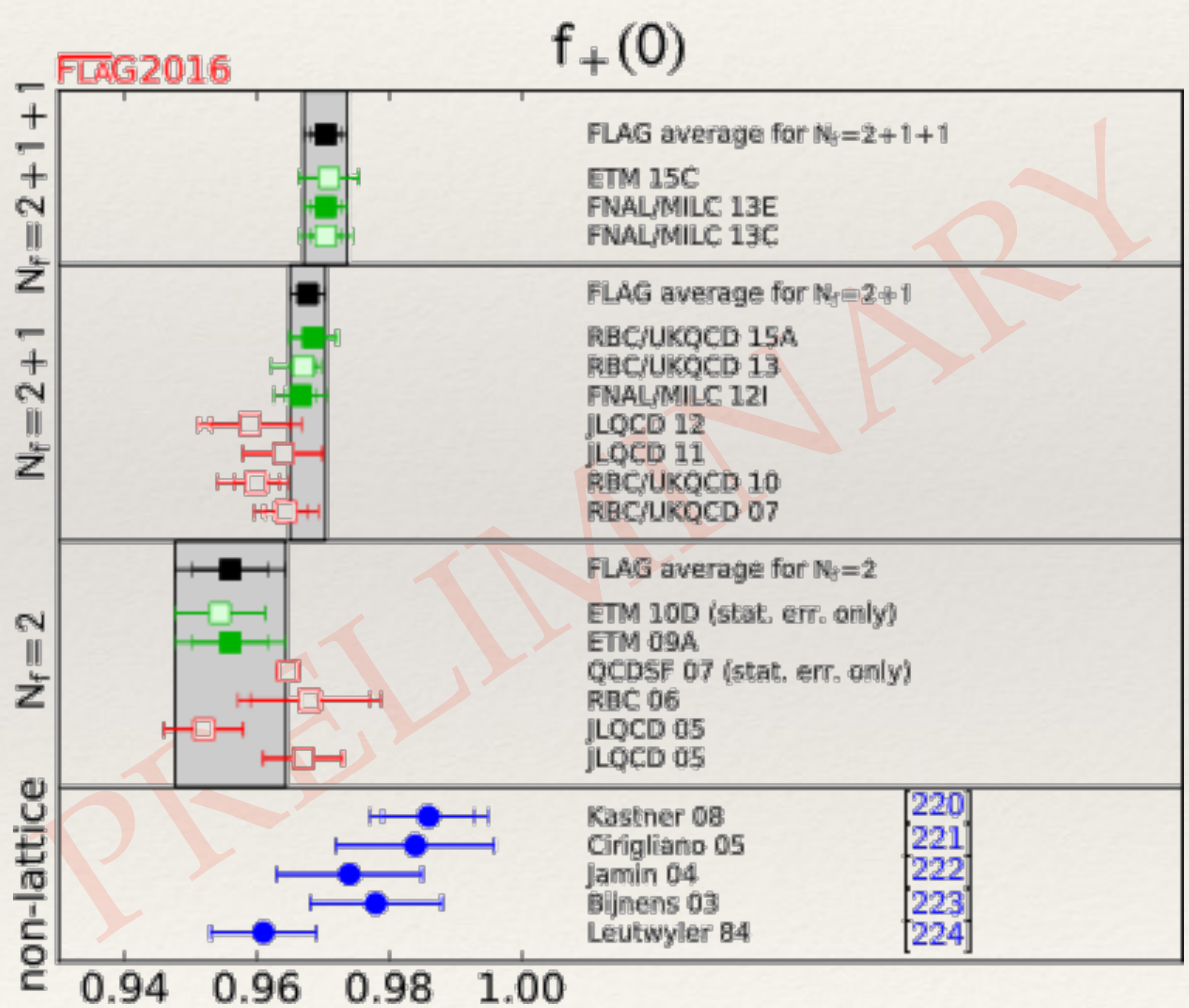
“tree” kaon/pion decays

semileptonic decay



$$\Gamma_{K\to\pi l\nu} = C_K^2 \frac{G_F^2 m_K^5}{192\pi^2} S_{\rm EW}(1+\Delta_{SU(2)}+\Delta_{\rm EM})^2\, I\, |f_+^{K\pi}(0)|^2 |V_{us}|^2$$

$$\langle \pi(p_\pi)|V_\mu(0)|K(p_K)\rangle = f_+^{K\pi}(q^2)(p_K+p_\pi)_\mu + f_-^{K\pi}(q^2)(p_K-p_\pi)_\mu$$



3‰!!!

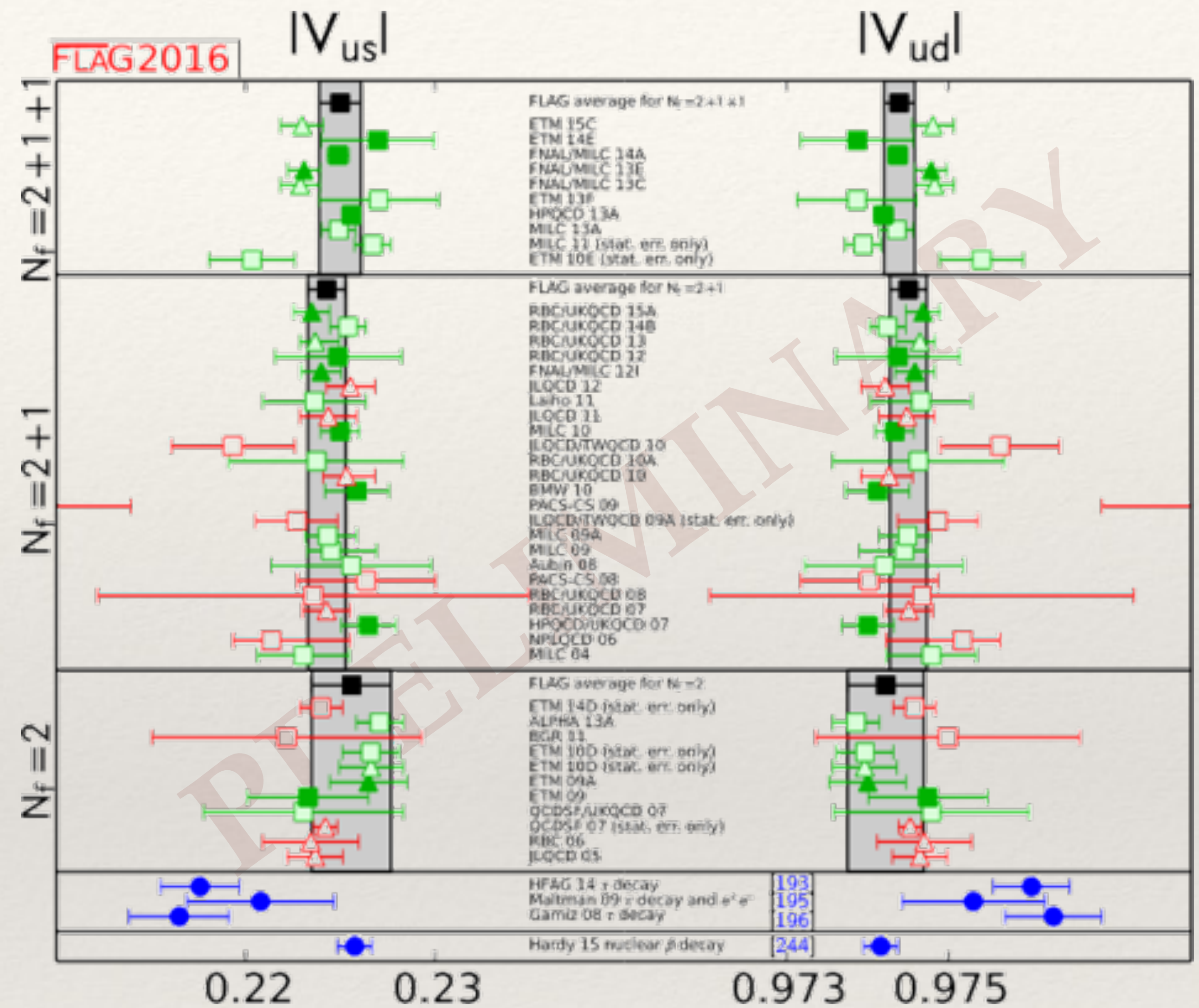
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \approx |V_{ud}|^2 + |V_{us}|^2 \stackrel{?}{=} 1$$

Experimental results:

$$|V_{us}| f_+(0) = 0.2163(5)$$

$$\frac{f_K}{f_\pi} \frac{|V_{us}|}{|V_{ud}|} = 0.2758(5)$$

FLAVIA Kaon WG EPJ C 69, 399-424 (2010)
KTeV, Istra, KLOE, NA48



Numerical results from FLAG2, illustrations (preliminary) from FLAG3

First row unitarity:

- $f_+^{K\pi}(0)$ and $|V_{ud}|$ from experiment
- f_K/f_π and $|V_{ud}|$ from experiment
- $f_+^{K\pi}(0)$ and f_K/f_π from lattice

	$f_+(0)$, $\mid V_{ud} \mid$	f_K/f_K, $\mid V_{ud} \mid$	combined
N_f=2+1	0.9993(5)	1.0000(6)	0.987(10)
N_f=2	1.0004(10)	0.9989(16)	1.029(35)

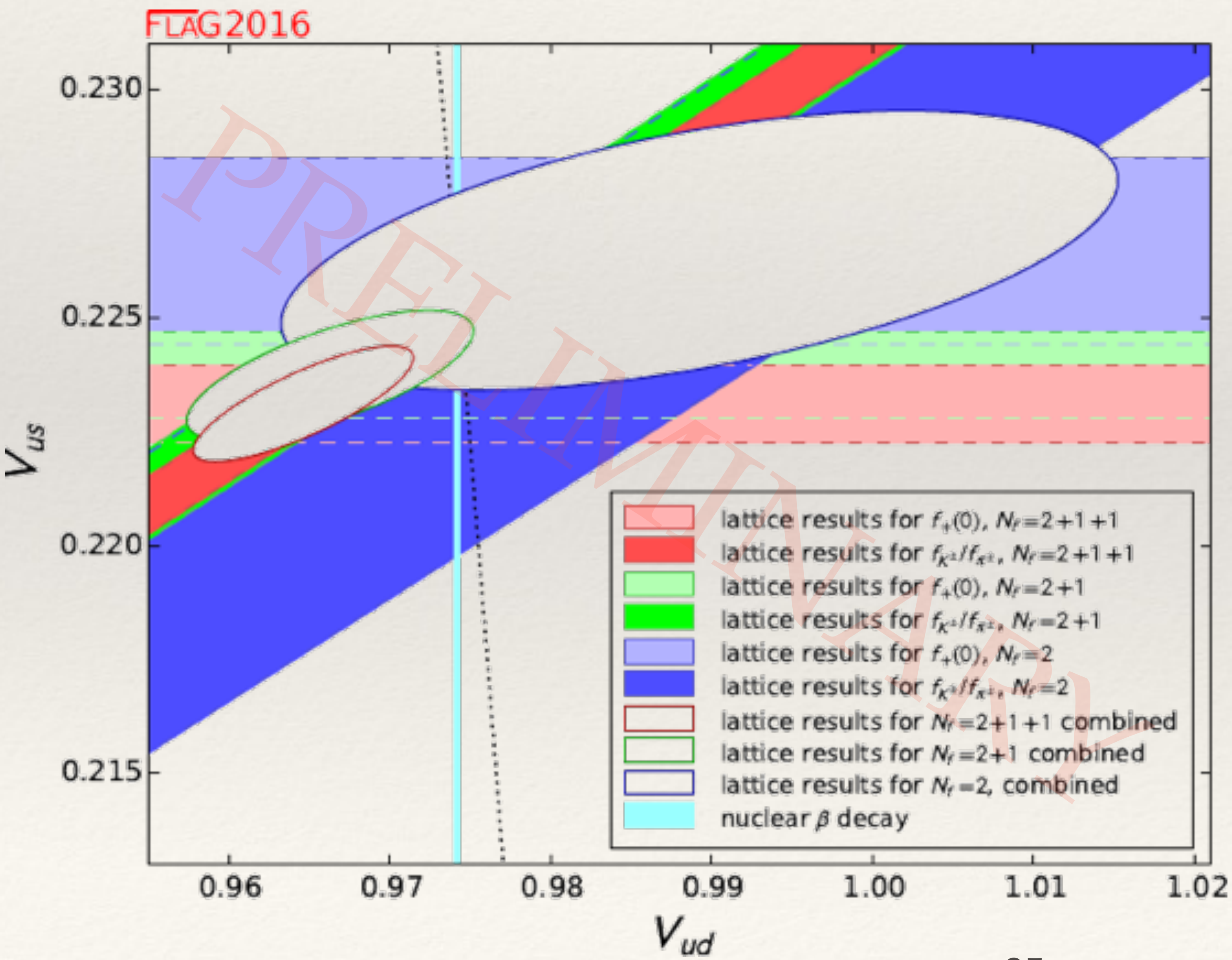
Eur.Phys.J. C74 (2014) 2890
arXiv:1310.8555

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \approx |V_{ud}|^2 + |V_{us}|^2 \stackrel{?}{=} 1$$

FLAG V_{us} Working Group (Boyle, Kaneko, Simula)

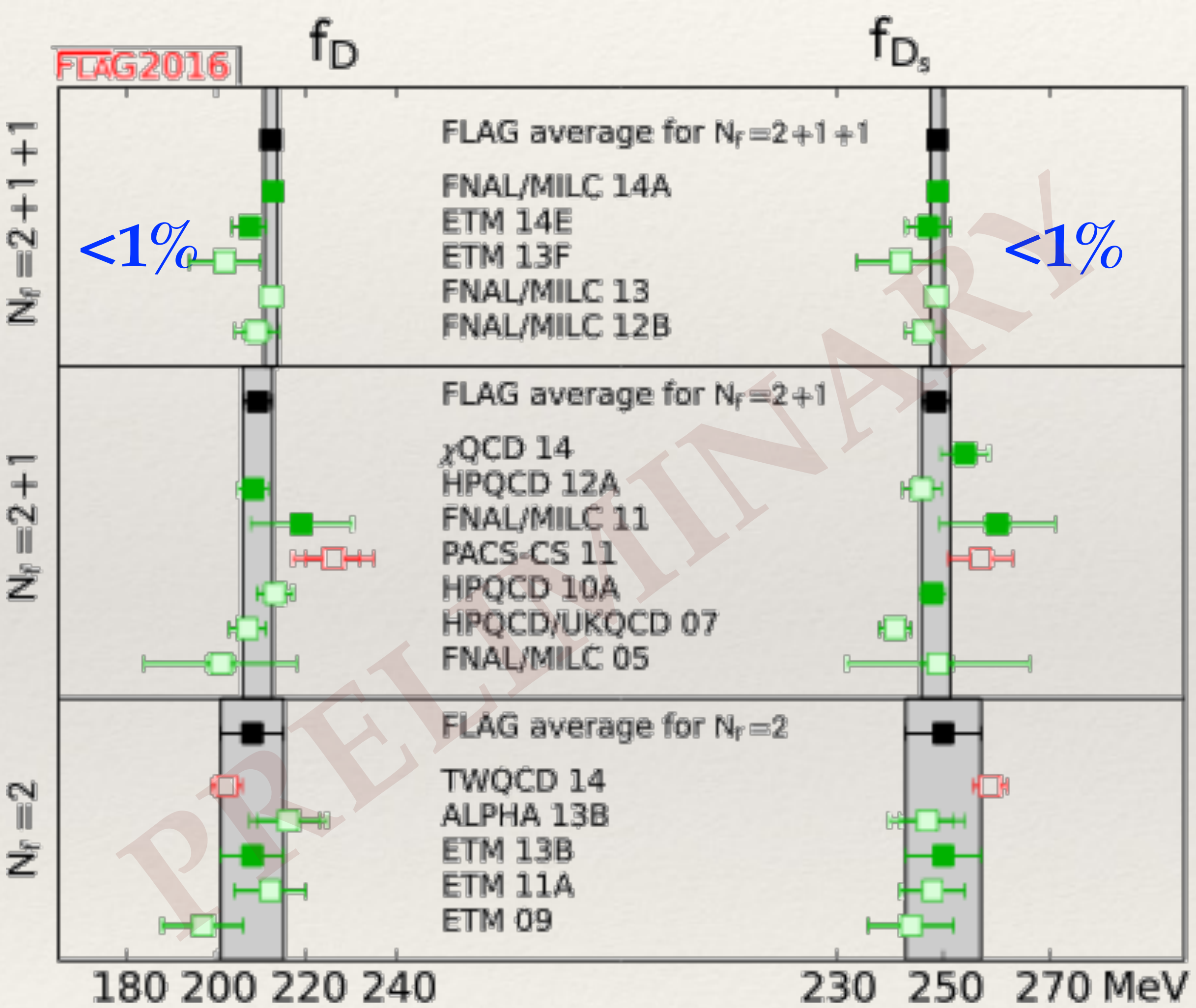
$$|V_{us}| f_+^{K^0\pi^-}(0) = 0.2163(5) \qquad \frac{f_{K^+}}{f_{\pi^+}} \frac{|V_{us}|}{|V_{ud}|} = 0.2758(5)$$

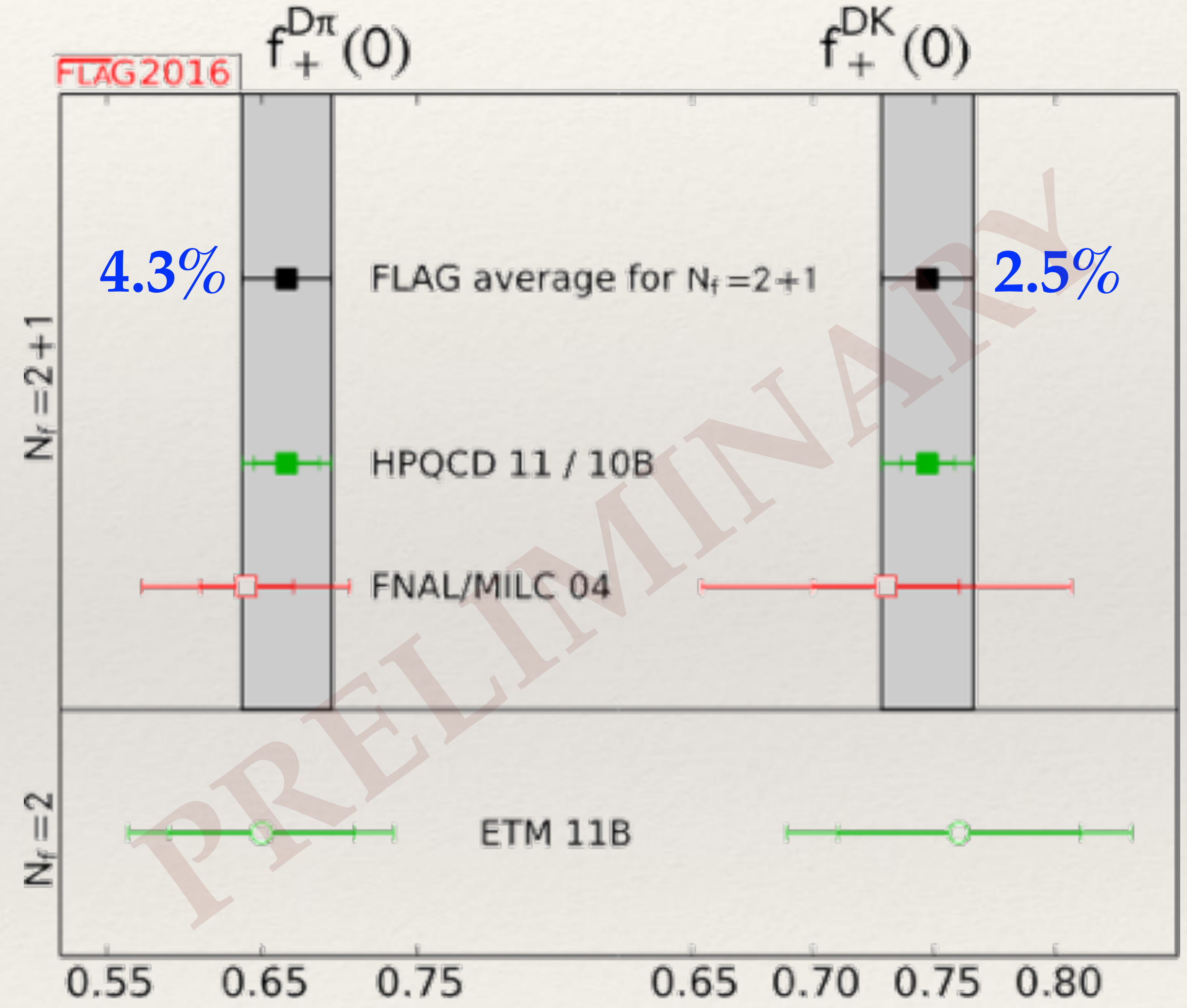
FLAVIANet Kaon WG
EPJ C 69, 399-424 (2010)
arXiv:1005.2323



high precision test of SM unitarity - no worrisome tension at sub-percent-level precision

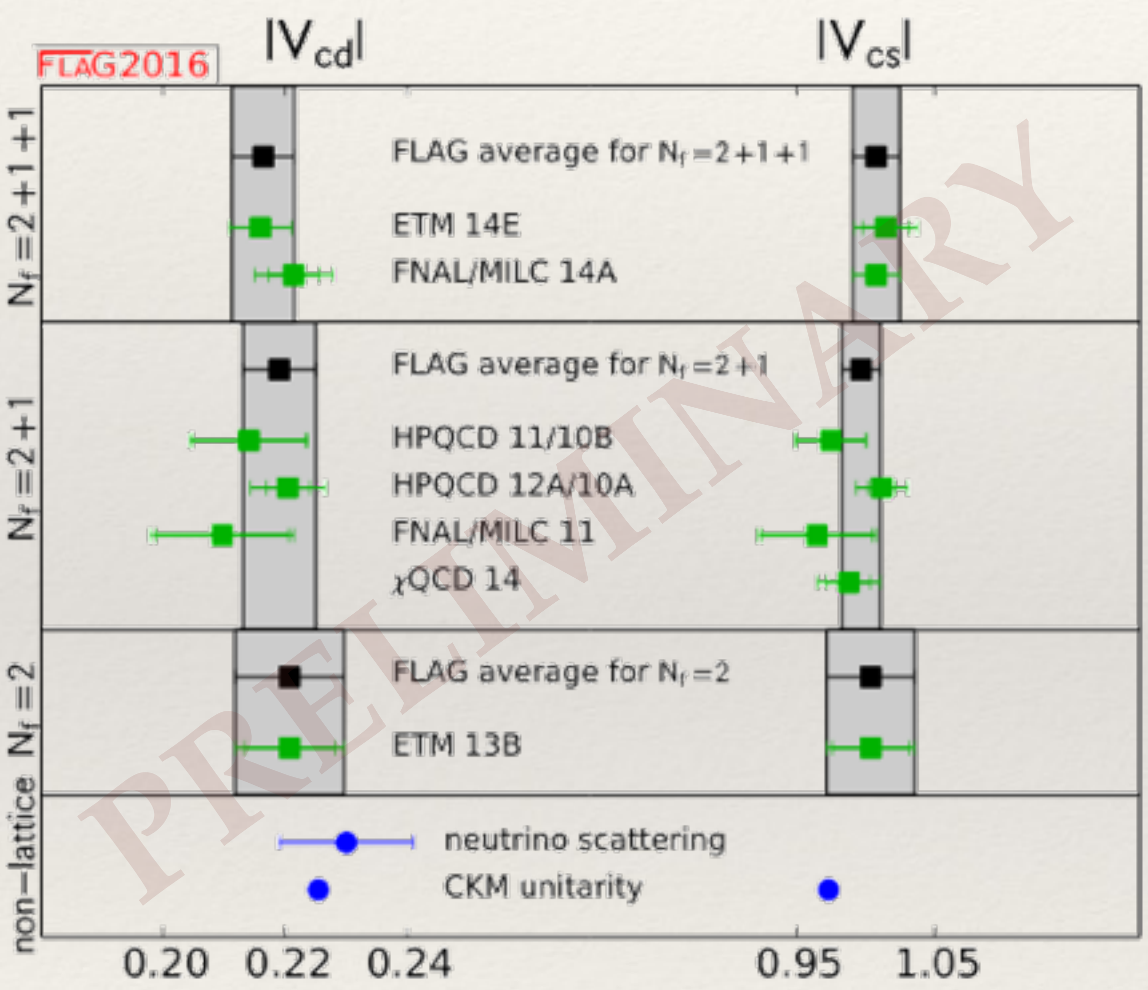
Leptonic $D_{(s)}$ meson decays



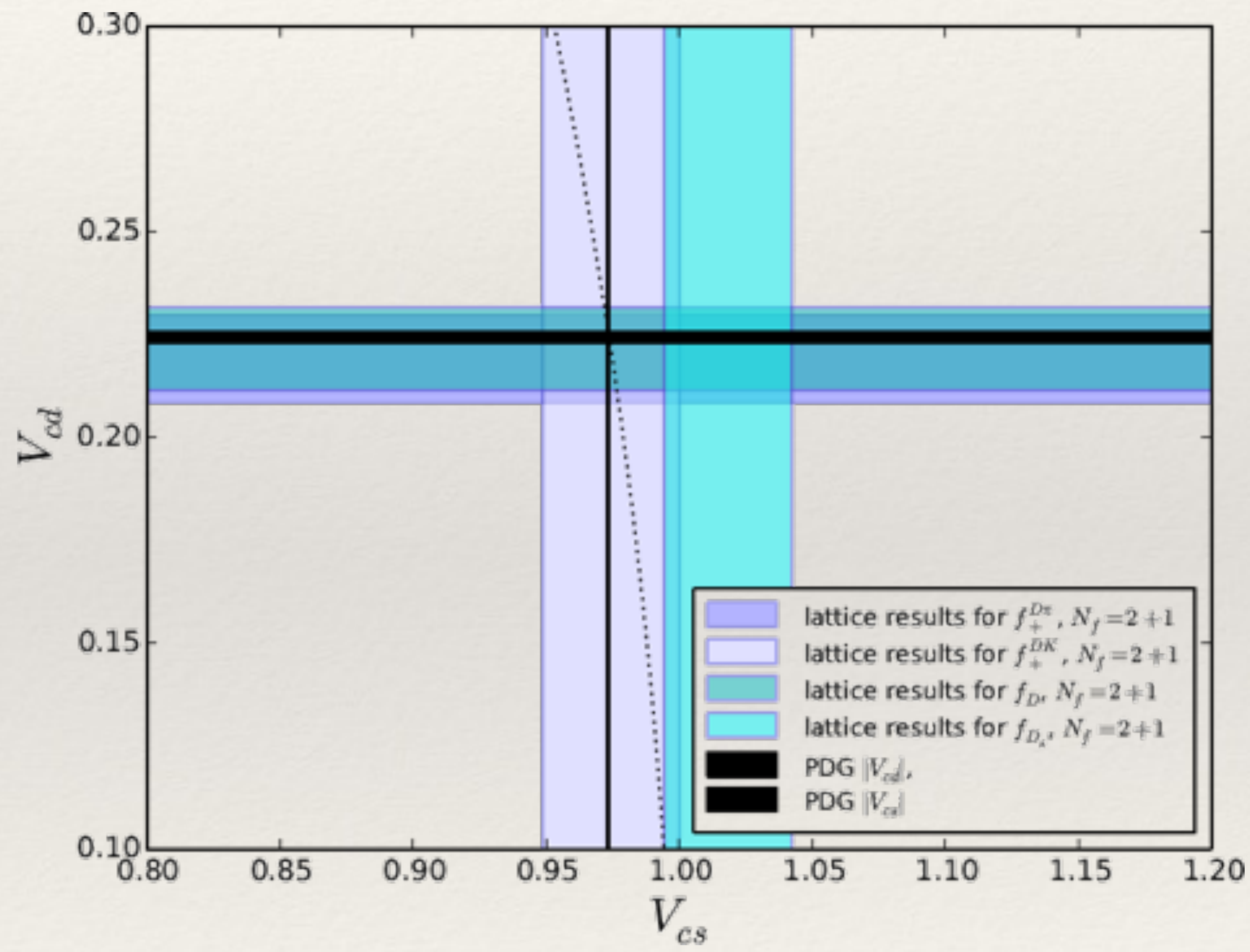


Results for $|V_{cd}|$ and $|V_{cs}|$

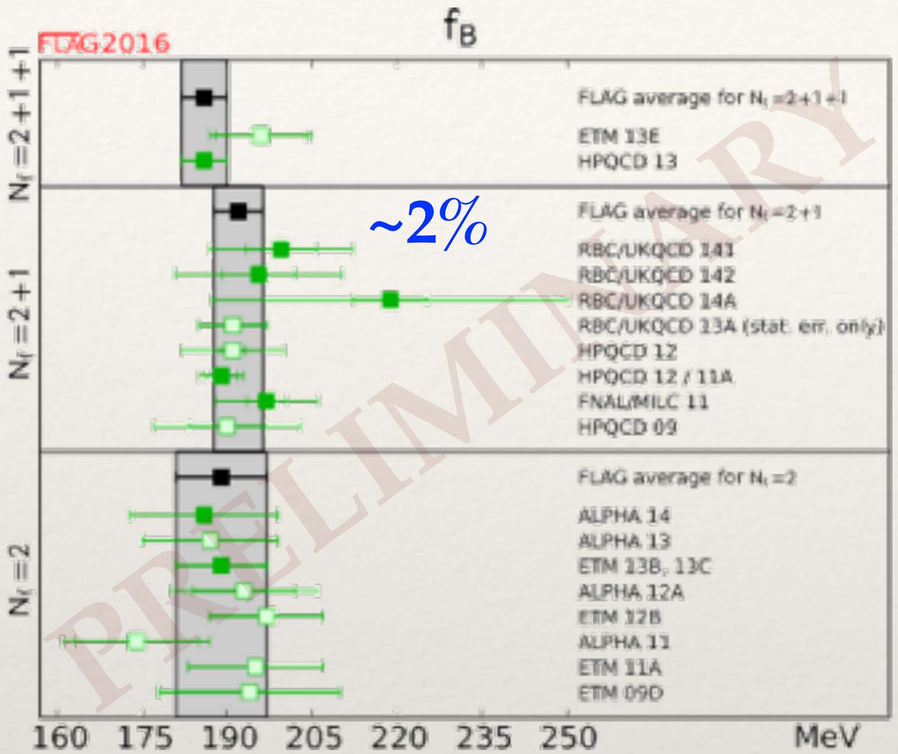
$f_D|V_{cd}| = 45.91(1.05)$ MeV , $f_{D_s}|V_{cs}| = 250.9(4.0)$ MeV PDG

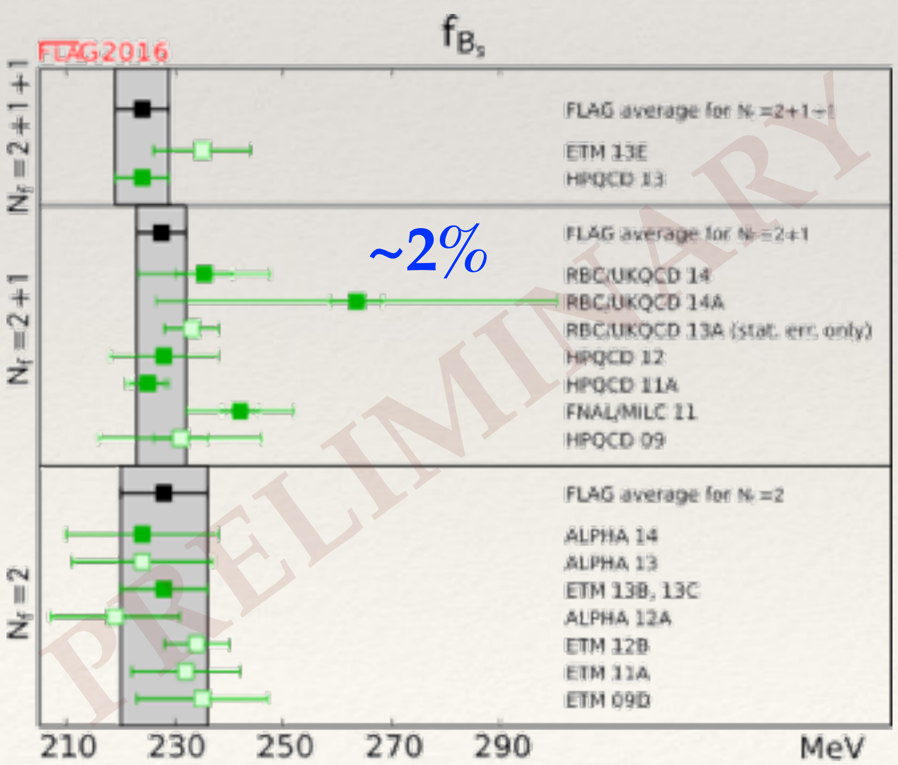


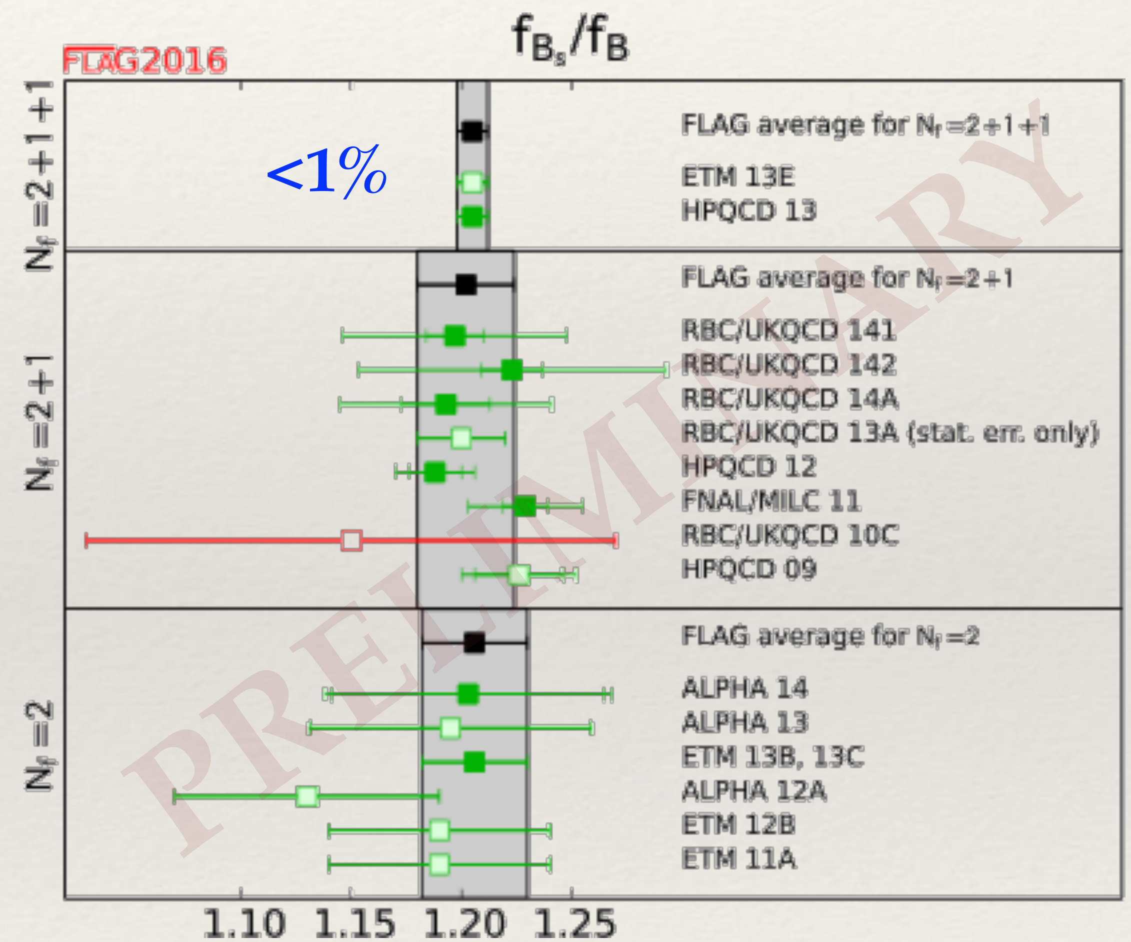
N_f=2+1 unitarity analysis



Leptonic beauty decays



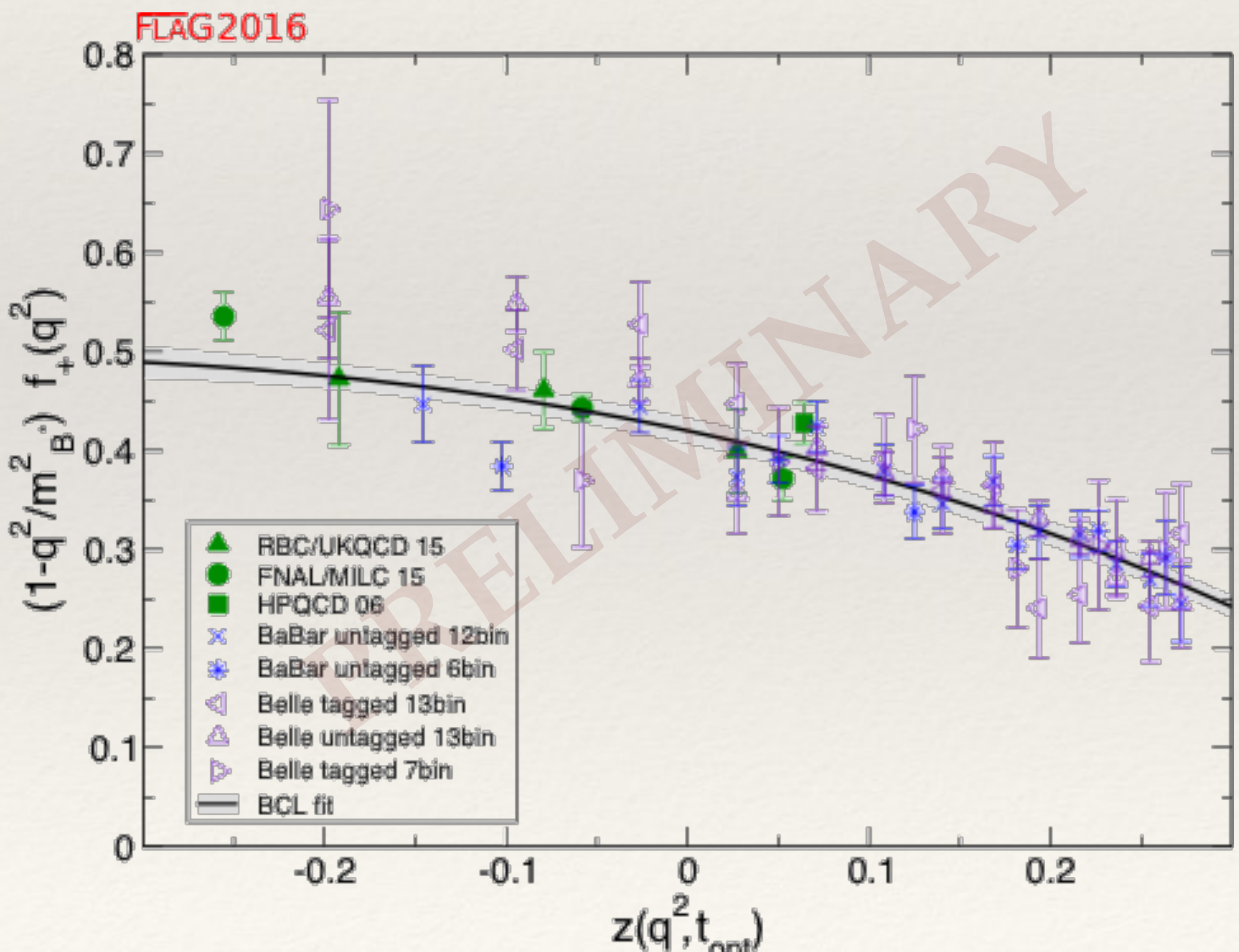




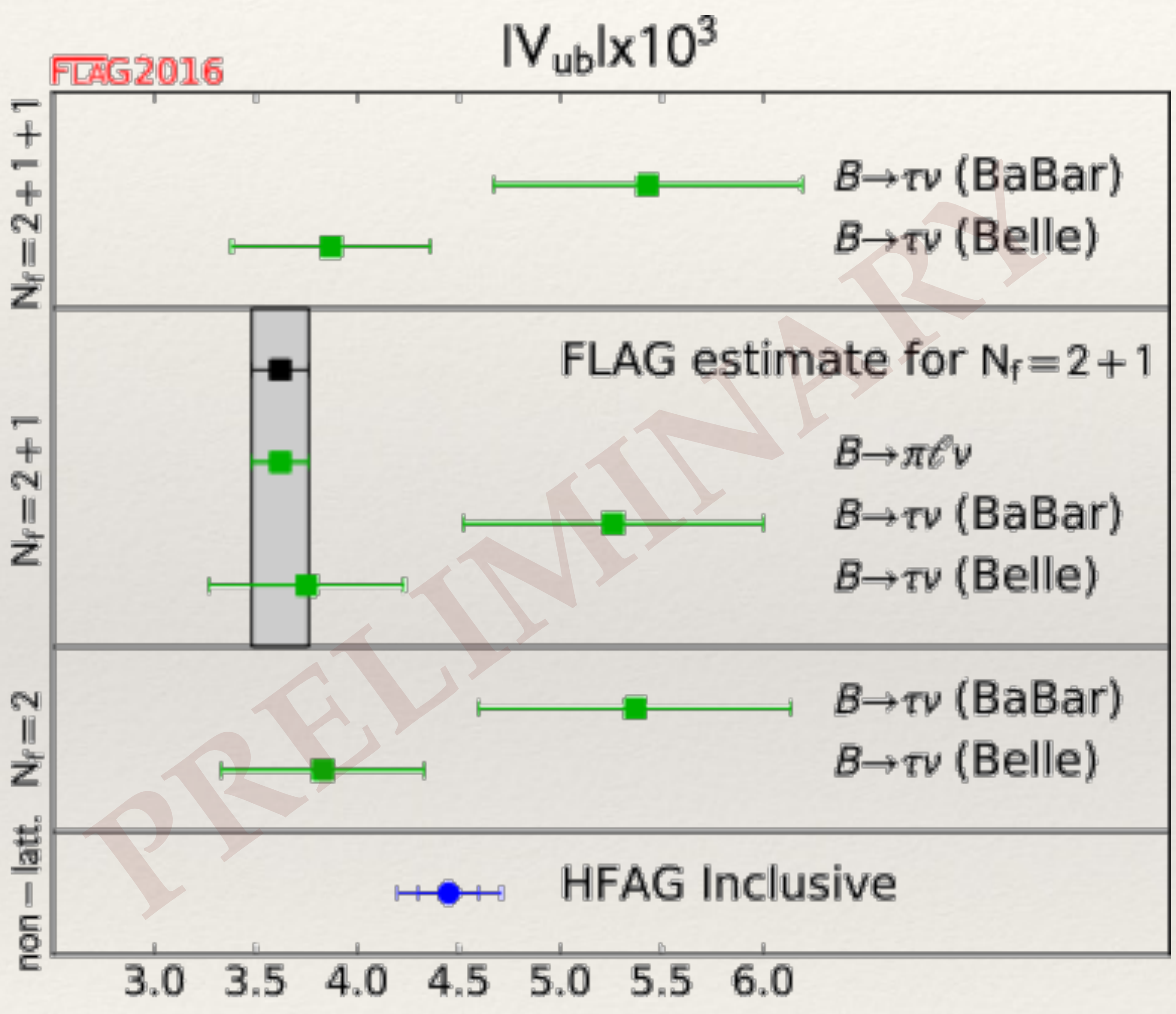
Semileptonic beauty decays

Kinematical reach limited in lattice QCD → extract value of V_{ub} from simultaneous analysis of exp. and lattice data

$$q^2 = \left(E_B - E_{\text{light}}\right)^2 - \left(\vec{p}_B - \vec{p}_{\text{light}}\right)^2 \qquad \vec{p} = \frac{2\pi}{L}\vec{n}$$



Results for $|V_{ub}|$

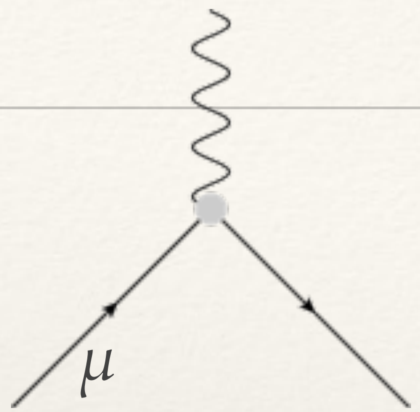


- tension between betw. incl. and excl. semileptonic decays
- slight tension in exp data Belle/BaBar
- looking forward to Belle II

Flavour summary

- (non-rare) Lattice Flavour Physics is a mature research field
- many independent groups competing
- sub-percent precision for some quantities
- FLAG summarises particularly mature quantities for use in SM and BSM phenomenology (FLAG-3 very very soon)

$(g\text{-}2)_\mu$

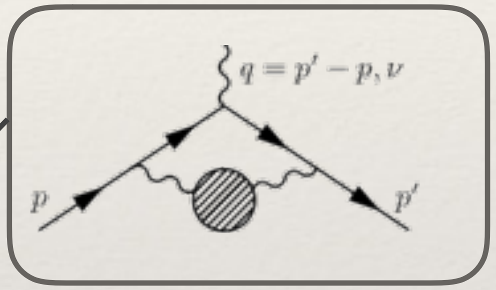


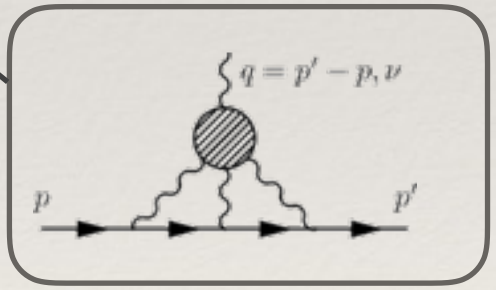
$$\langle e(\vec{p}\,')|j_\nu|e(\vec{p})\rangle = -e\,\bar{u}(\vec{p}\,')\left[F_1(q^2)\gamma_\nu + i\frac{F_2(q^2)}{4m}[\gamma_\nu,\gamma_\rho]q_\rho\right]u(\vec{p})$$

$$F_2(0) = \frac{g-2}{2} \equiv a_\mu$$

PDG 2013

contribution	val$\times 10^{11}$	$\delta\times 10^{11}$
QED	116584718.95	0.08
EW	153.6	1.0
HVP LO	6923	42
HVP NLO	-98.4	0.6
HVP LBL	105	26
SM	116591803	49
EXP	116592091	63

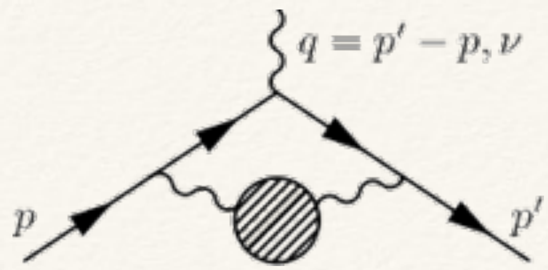




- discrepancy > 3σ
- new experiments (Fermilab, J-PARC)
- HVP LO from $e^+e^- \to$hadrons
- HVP LBL model based

→ warrants any attempt at first principles computation

LO HVP



$$\boxed{a_\mu^{LO\,HVP} = 4\alpha^2 \int_0^\infty dQ^2 f(Q^2)\left(\Pi(Q^2) - \Pi(0)\right)}$$

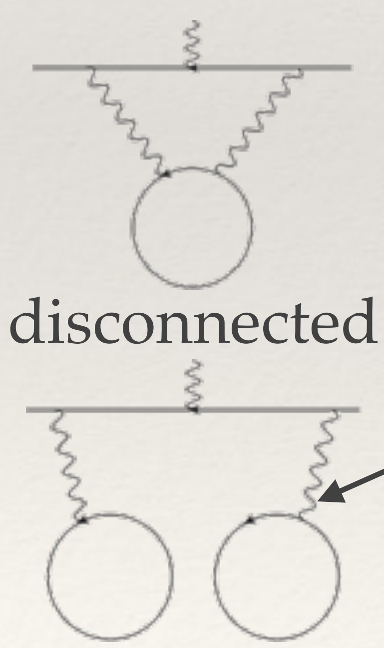
Euclidean momenta!

T. Blum PRL 91 (2003) 052001

$$\Pi_{\mu\nu}(Q) = \int d^4x e^{-iQx} \underbrace{\boxed{\langle j_\mu(x) j_\nu(0)\rangle}}_{\text{vector-vector 2pt-function}} \qquad \Pi(Q^2) = \frac{\Pi_{\mu\nu}(Q^2)}{\delta_{\mu\nu}Q^2 - Q_\mu Q_\nu}$$

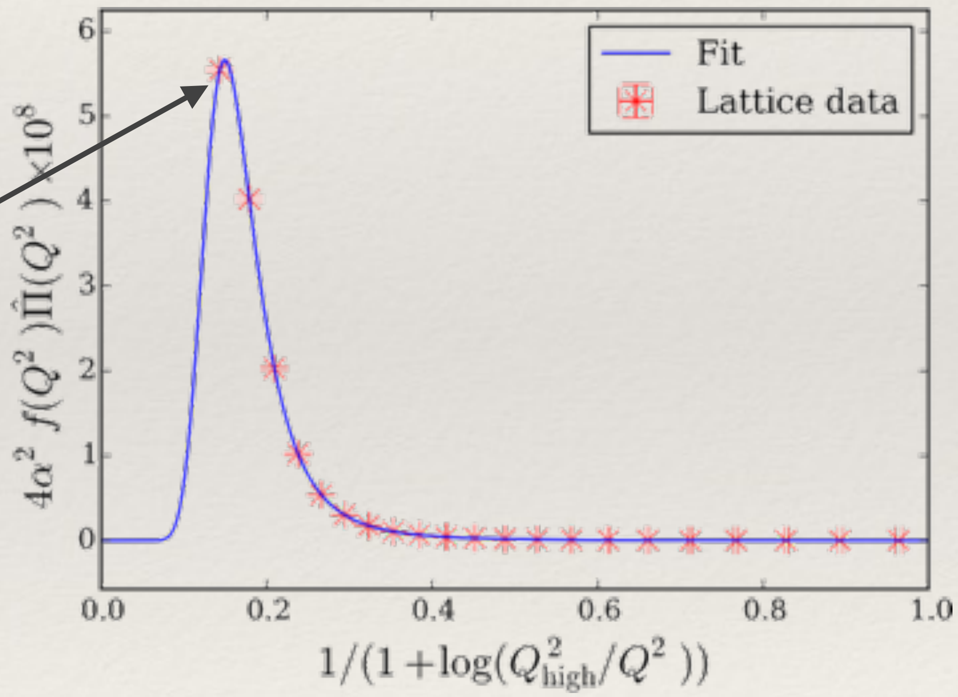
connected

disconnected



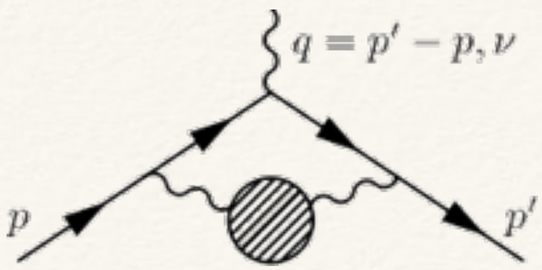
non-trivial:

- bad signal/noise ratio → stat. error
- integrand peaked at small Q^2 which is inaccessible on current lattices due to $p_i \sim 2\pi/L$
- Π not defined at $Q^2 = 0$
- quark-disconnected contributions
- isospin breaking



But last years have seen tremendous progress !!!

LO HVP



arXiv:1601.03071 · JHEP 1604 (2016) 063 arXiv:1602.01767 · arXiv:1512.09054

a_μ x 10^{10}	HPQCD	RBC/UKQCD
light	598(11)	work in progress
strange	53.4(6)	52.4(2.1)
charm	14.4(4)	work in progress
disconnected		−9.6(3.3)(2.3)
all	666(6)(12)	—
SM OK exp all	720(7)	720(7)

- strange, charm and bottom sufficiently precisely known
- getting the disconnected in full LQCD was a big achievement (previously considered show stopper)

- first results (HPQCD) indicate tension confirmed

Need to concentrate on:

- stat. error on light contribution
- strong and elm. isospin breaking effects (later)

There is significant work on light-by-light going on as well!

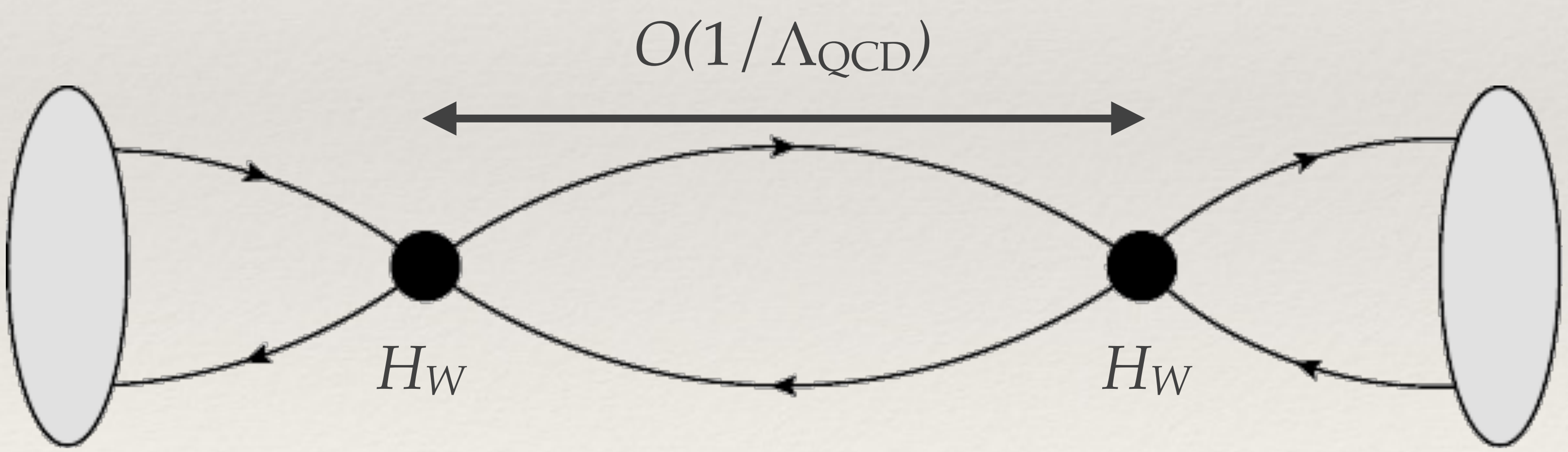
Blum et al., Phys.Rev. D93 (2016) no.1, 014503 arXiv:1510.07100
Green et al., Phys.Rev.Lett. 115 (2015) no.22, 222003 arXiv:1507.01577

Beyond precision lattice QCD

Go beyond factorisation

$$\Gamma_{\mathrm{exp.}} \stackrel{???}{=} V_{\mathrm{CKM}}(\mathrm{WEAK})\underbrace{(\mathrm{EM})(\mathrm{STRONG})}_{\text{treat jointly in lattice QCD+QED}}$$

Go beyond short distance physics



Including QED in meson decay MEs

- Precision on MEs such that EM and strong isospin effects important remember: so far mostly only QCD ($m_l=m_u=m_d$, $\alpha_{EM}=0$)
- we should go beyond EFT treatment (e.g. replace ChPT estimates)
- need to understand how this can be done conceptually
- already many results for spectroscopic quantities but not for matrix elements
- finite size effects with photons pose a substantial problem

Isospin corrections are important

- e.g. $K \to \pi l \nu$: $\Gamma_{K\to\pi l\nu} = C_K^2 \frac{G_F^2 m_K^5}{192\pi^2} S_{\mathrm{EW}} (1 + \underbrace{\Delta_{SU(2)} + \Delta_{\mathrm{EM}}}_{3\%})^2 \, I \, |f_+^{K\pi}(0)|^2 |V_{us}|^2$

Kastner & Neufeld
Eur. Phys. J. C 57 (2008) 541

- precision now such that corrections need to be improved:

			Approx contrib to % err			
Mode	$V_{us} f_+(0)$	% err	BR	τ	Δ	I
$K_{L\,e3}$	0.2163(6)	0.26	0.09	0.20	0.11	0.05
$K_{L\,\mu3}$	0.2166(6)	0.28	0.15	0.18	0.11	0.06
$K_{S\,e3}$	0.2155(13)	0.61	0.60	0.02	0.11	0.05
$K_{e3}^{\pm}$	0.2172(8)	0.36	0.27	0.06	0.23	0.05
$K_{\mu3}^{\pm}$	0.2170(11)	0.51	0.45	0.06	0.23	0.06

Moulson@CKM 2014
arXiv:1411.5252

QCD+QED

Action:

$$S[U, A, \bar{\psi}, \psi] = S_g[U; g] + S_\gamma[A] + \sum_f \bar{\psi}_f D[U, A; e, q_f, m_f]\psi_f$$

$$S_\gamma^{naive} = -\frac{a^4}{4} \sum_{\mu,\nu,x} (\partial_\mu A_{\nu,x} - \partial_\nu A_{\mu,x})^2$$

- MC simulation of discretised theory
- QCD has a mass gap → finite volume effects $\propto e^{-m_\pi L}$ (for simple MEs)
- photon is massless and interacts over long range
  → power-like finite volume effects $\propto 1/L, 1/L^2, \ldots$ from exchange of photon around torus
- sufficiently large volumes currently not feasible, so use effective field theory to subtract finite volume effects

QCD+QED

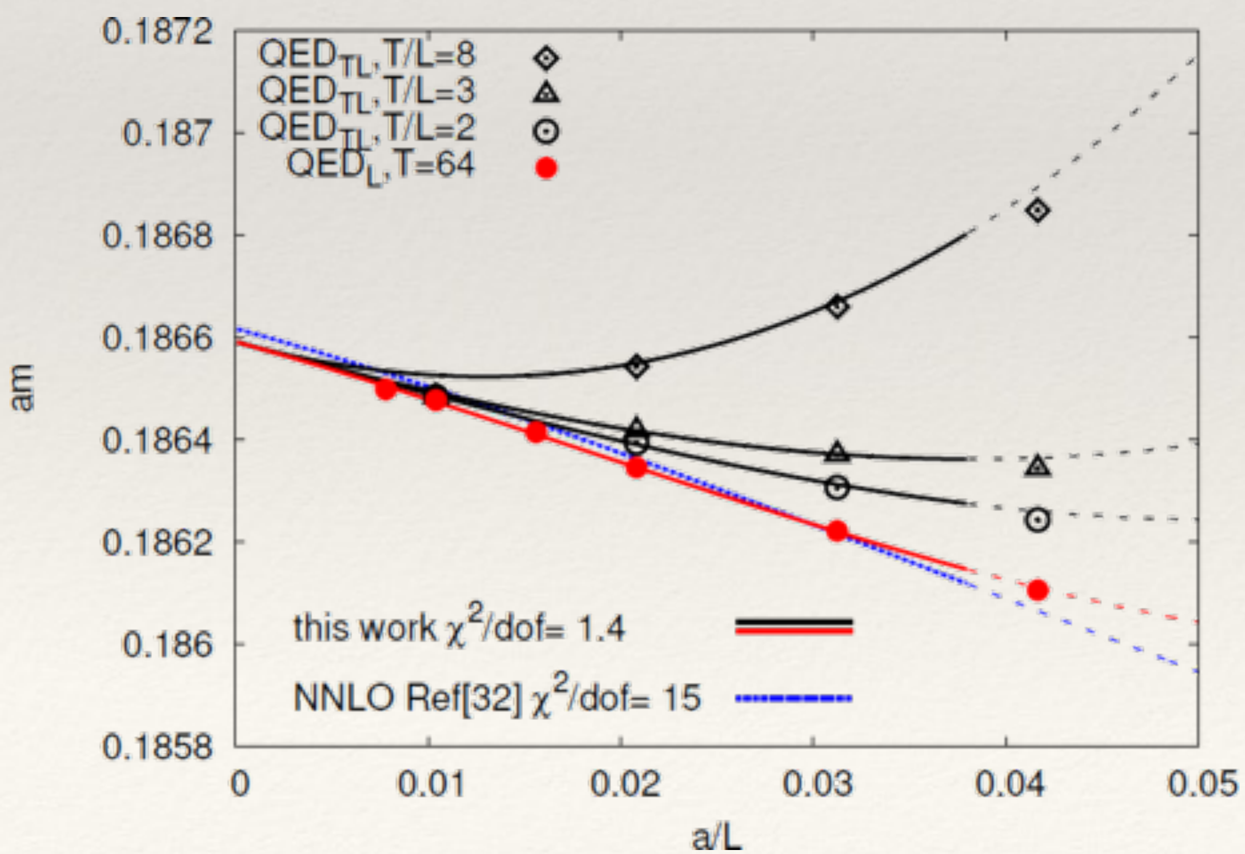
BMW Collaboration
Science 347 (2015) 1452-1455
arXiv:1406.4088

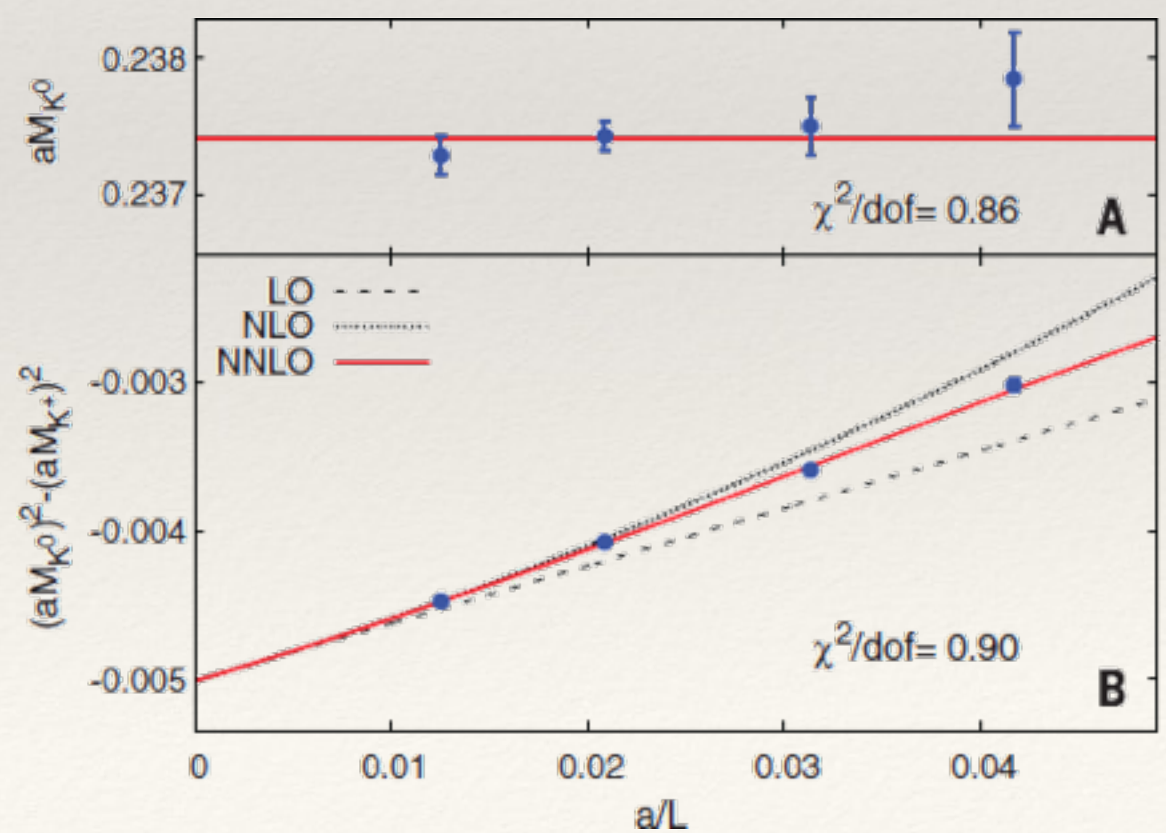
Example: FV correction to mass of a spin-1/2 particle in QED

analytically compute the difference of the *finite volume* and *infinite volume* self energies Σ:

$$m^2(T,L) \overset{T,L\to\infty}{\propto} m^2 \left\{1 - q^2\alpha\left[\frac{\kappa}{2mL}\left(1+\frac{2}{mL}\right) - \frac{3\pi}{(mL)^3}\right]\right\}$$

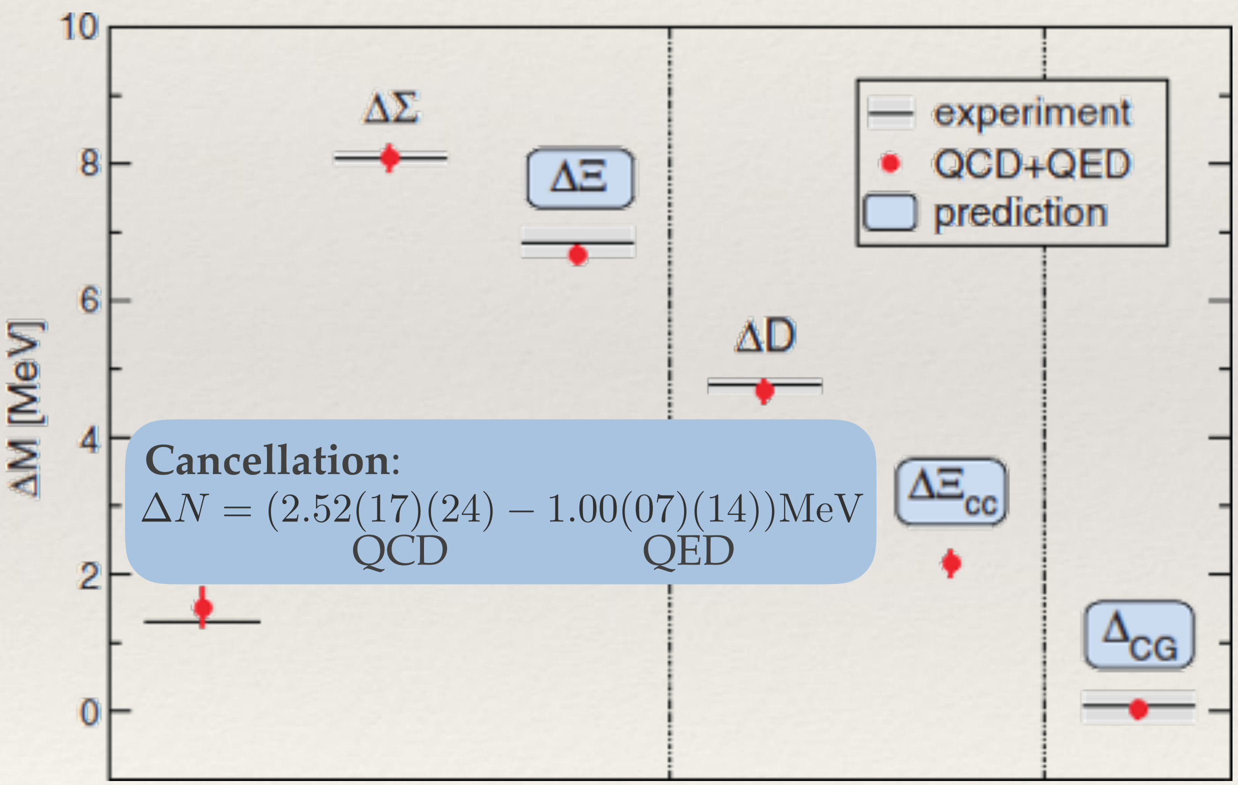
leading behaviour universal in κ (structure- and spin-independent)





QCD+QED: baryon mass splitting

- relative neutron-proton mass difference found in nature 0.14%
- the value has significant implication for nature
  - smaller value → inverse β-decay of H
  - much larger value → faster β-decay for neutrons in BBN



Cancellation:
$$\Delta N = (\underbrace{2.52(17)(24)}_{\text{QCD}} - \underbrace{1.00(07)(14)}_{\text{QED}})\text{MeV}$$

BMW carried out simulations of $N_f = 1{+}1{+}1{+}1$ *QCD+QED* simulations and determined the light baryon isospin splitting

BMW Collaboration
Science 347 (2015) 1452-1455
arXiv:1406.4088

Including QED in meson decay MEs

- leptonic decay at $O(\alpha^0)$:

$$\Gamma(\pi^+ \to l^+\nu_l) = \frac{G_F^2|V_{ud}|^2 f_\pi^2}{8\pi} m_\pi m_l^2 \left(1 - \frac{m_l^2}{m_\pi^2}\right)^2$$

- including elm. effects @ $O(\alpha)$:

$$\begin{aligned}\Gamma(\pi^+ \to l^+\nu_l(\gamma)) &= \Gamma(\pi^+ \to l^+\nu_l) + \Gamma(\pi^+ \to l^+\nu_l\gamma) \\ &\equiv \quad \Gamma_0 \quad + \quad \Gamma_1\end{aligned}$$

IR div. cancel between terms on r.h.s.
between virtual and real photons
(Bloch Nordsieck)

Including QED in meson decay MEs

Carrasco et al. PRD 91 074506 (2015) arXiv:1502.00257

- cut on small photon momentum < ΔE → γ *sees* point-like π
  $\Delta E \approx 20$MeV experimentally accessible and π point like

point approximation

$$\Gamma(\Delta E) = \lim_{V\to\infty} \boxed{(\Gamma_0 - \Gamma_0^{\mathrm{pt}})} + \lim_{V\to\infty} \boxed{(\Gamma_0^{\mathrm{pt}} + \Gamma_1^{\mathrm{pt}}(\Delta E))}$$

$\Gamma(\pi^+ \to l^+ \nu_l)$
lattice and analytical
finite V

$\Gamma(\pi^+ \to l^+ \nu_l \gamma(\Delta E))$
analytically in $V \to \infty$

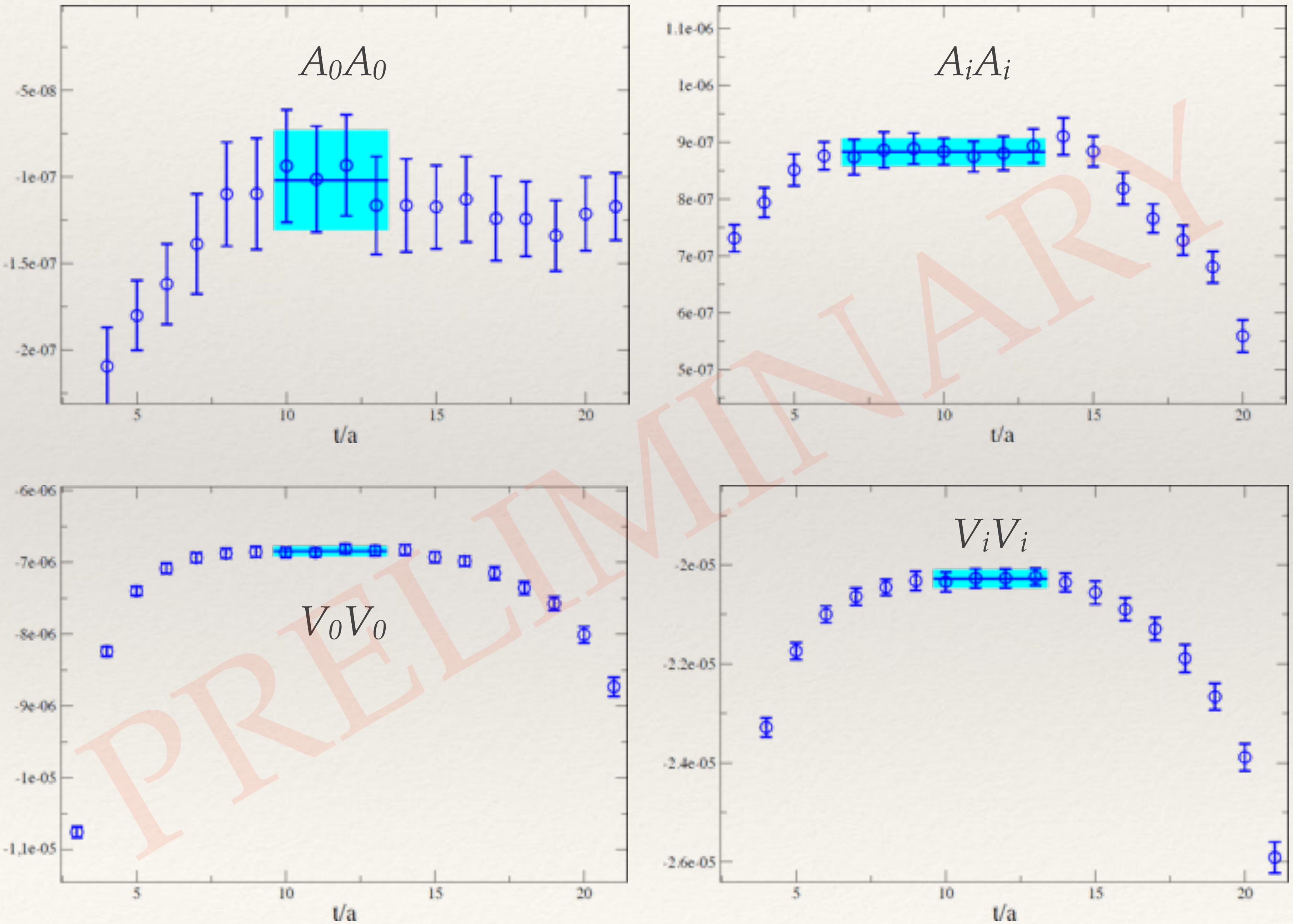
both terms separately IR finite, gauge invariant on its own

- analytical calculation for pt. approximation is done:

$$\mathcal{L}_{\pi\text{-}\ell\text{-}\nu_\ell} \;=\; i\, G_F f_\pi V_{ud}^* \,\{(\partial_\mu - ieA_\mu)\pi\} \left\{ \bar{\psi}_{\nu_\ell} \frac{1+\gamma_5}{2} \gamma^\mu \psi_\ell \right\} + \mathrm{H.C.}\,.$$

Including QED in meson decay MEs

$24^{3};\ m_\pi \approx 500\text{MeV}$



- first time ever conceptually clean attempt of calculation of leptonic decay at $O(\alpha)$
- disconnected pieces need to be included
- Γ_0 works, now needs to be combined wt. analytical results for $\Gamma_0^{\rm pt}$, $\Gamma_1^{\rm pt}(\Delta E)$
- ~20% stat. error would be sufficient for use in phenomenology

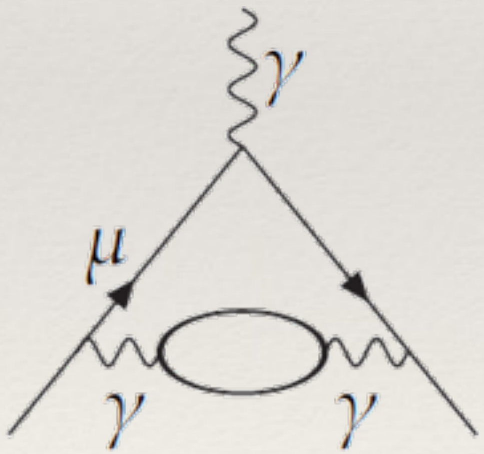
QCD+QED applications

- start with light flavour matrix elements f_π, f_K, $f_+(0)$, …

$$\Gamma_{K\to\pi l\nu} = C_K^2 \frac{G_F^2 m_K^5}{192\pi^2} S_{\mathrm{EW}} (1 + \Delta_{SU(2)} + \Delta_{\mathrm{EM}})^2 \, I \, |f_+^{K\pi}(0)|^2 |V_{us}|^2$$

$$\langle \pi(p_\pi) | V_\mu(0) | K(p_K) \rangle = f_+^{K\pi}(q^2)(p_K + p_\pi)_\mu + f_-^{K\pi}(q^2)(p_K - p_\pi)_\mu$$

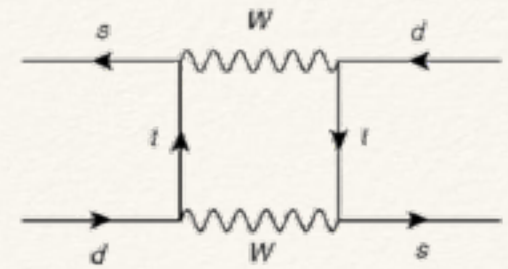
- lattice predictions of leading hadronic contribution to muon g-2

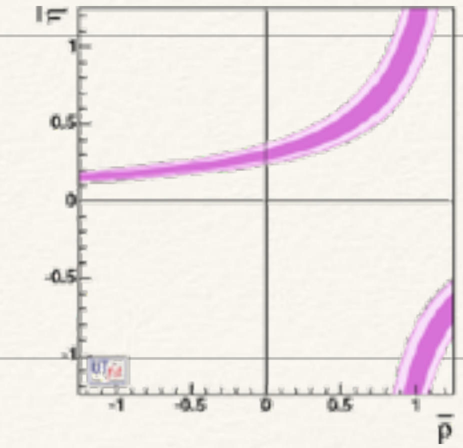


  - lattice (isospin symmetric, α_{EM}=0 is getting competitive with experimental determination ($e^+e^-\to$*hadrons*)
  - next step would be inclusion of isospin breaking effects

- inclusion of QED effects will be one of the big challenges over the next years

Long distance effects in kaon physics - mixing



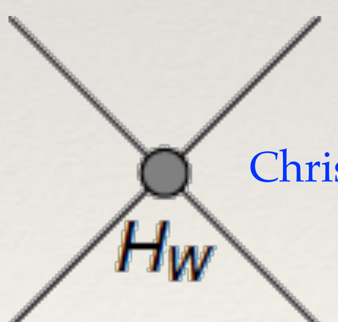


$$\epsilon_K = \frac{A(K_L \to (\pi\pi)_{I=0})}{A(K_S \to (\pi\pi)_{I=0})} = e^{i\Phi_\epsilon} \sin\phi_\epsilon \left(\frac{\mathrm{Im}\langle \bar{K}^0 | H_W^{\Delta S=2} | K^0 \rangle}{\Delta M_K} + \text{L.D. effects} \right)$$

L.D. effects: Buras, Guadagnoli PRD 78 (2008); Buras, Guadagnoli, Isidori, PLB 688 (2010)

Long Distance effects amount to O(5%), so certainly worth considering on the lattice

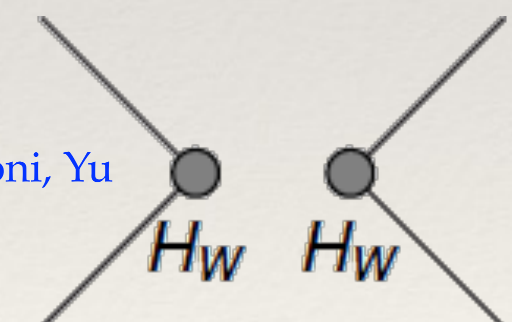
1st order Weak



single 4-quark OP, length scale 10^{-18}m

Christ, Izubuchi, Sachrajda, Soni, Yu arXiv:1212.5931

2nd order Weak

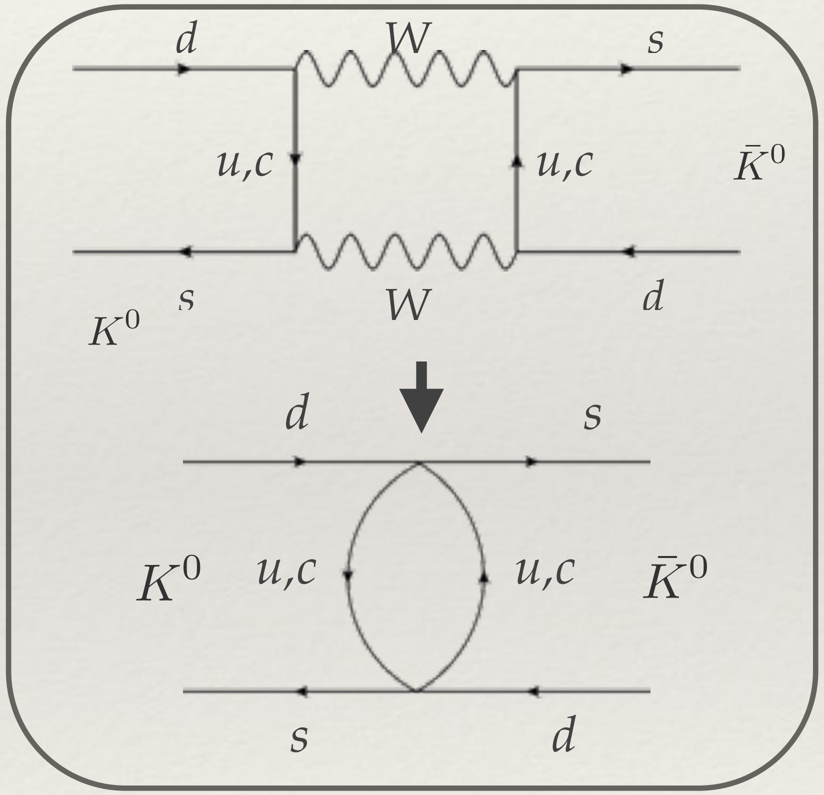


two 4-quark OP length scale $1/\Lambda_{QCD}$

Beyond short distance: e.g. ΔM_K

$$\boxed{\Delta M_K = m_{K_S} - m_{K_L} = \boxed{2\mathrm{Re}M_{\bar{0}0}}} \qquad M_{\bar{0}0} = \mathcal{P}\sum_{\lambda} \frac{\langle \overline{K^0}|H_W|\lambda\rangle\langle\lambda|H_W|K^0\rangle}{m_K - E_\lambda}$$

- experimentally ΔM_K=3.483(6)×10^{-12}MeV (PDG)
- suppressed by 14 orders of magnitude with respect to QCD → poses strong BSM constraints (e.g. $(1/\Lambda)^2\ \bar{s}d\bar{s}d$ BSM contribution) knowing ΔM_K at 10%-level → **Λ≥10^{4}TeV**
- SD about 70% of experimental value - rest LD?
- PT large contributions at $\mu \sim m_c$ where PT turns out to converge badly (NLO->NNLO constitutes 36% correction) Brod, Gorbahn PRL 108 121801 (2012) arXiv:1108.2036



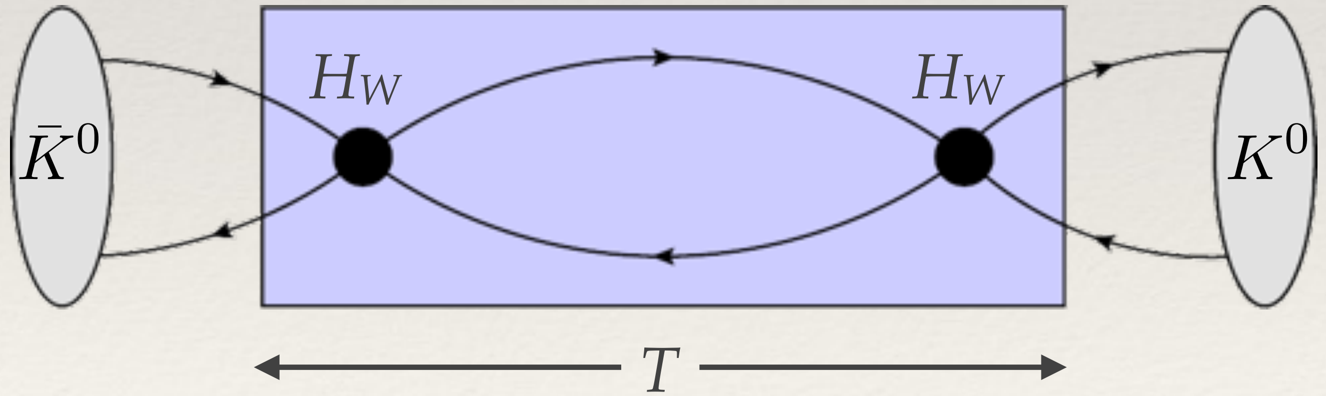
long distance effects – ΔM_K

N. Christ et al. PRD 88 (2013) 014508 **arXiv:1212.5931**
Bai et al. PRL 113 (2014) 112003 **arXiv:1406.0916**

$$M_{\bar{0}0} \quad = \quad \mathcal{P}\sum_\lambda \frac{\langle \overline{K^0}|H_W|\lambda\rangle\langle\lambda|H_W|K^0\rangle}{m_K - E_\lambda}$$

$$\mathcal{A} = \langle 0|T\left\{K^0(t_f)\frac{1}{2}\int_{t_A}^{t_B} dt_2 \int_{t_A}^{t_B} dt_1 H_W(t_2)H_W(t_1){K^0}^\dagger(t_i)\right\}|0\rangle$$

Integrate operators (here H_W) over time interval where initial and final kaon dominate



long distance effects – ΔM_K

N. Christ et al. PRD 88 (2013) 014508 **arXiv:1212.5931**
Bai et al. PRL 113 (2014) 112003 **arXiv:1406.0916**

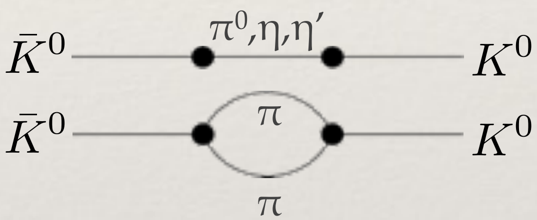
$$\mathcal{A} = N_K^2 e^{-M_K(t_f - t_i)} \sum_n \frac{\langle \bar{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{M_K - E_n} \left(-T - \frac{1}{M_K - E_n} + \frac{e^{(M_K - E_n)T}}{M_K - E_n} \right)$$

amplitude Δm_K^{FV}

irrelevant constant

exponential term needs to be subtracted

- multiple hadrons in intermediate states causing difficulties and need to be subtracted

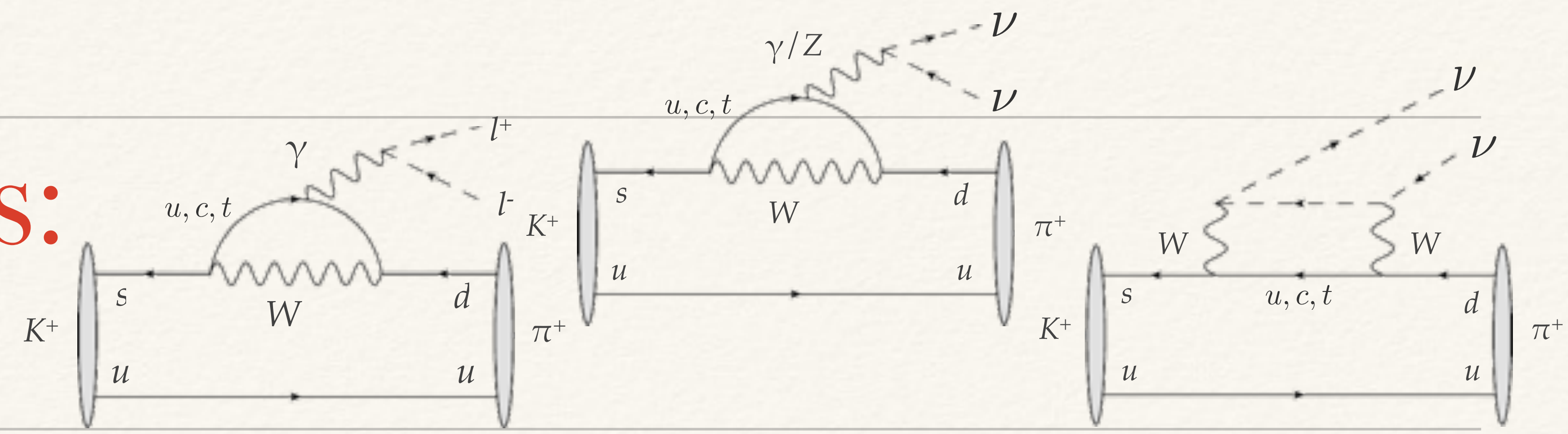


- finite volume corrections from two-particle intermediate state can be sizeable
  N. Christ et al. PRD91 (2015) 114510 **arXiv:1504.01170** also: Briceno, Hansen **arXiv:1502.04314**
  extension of Lellouch-Lüscher correction to 2nd order weak MEs
  $\Delta^{\mathrm{FV}}(\Delta M_K) = -\cot\left(\phi(M_K) + \delta_0(M_K)\right) \frac{d(\phi(E) + \delta_0(E))}{dE}\Big|_{E=M_K} |\langle \bar{K}^0 | H_W | \pi\pi, M_K \rangle^{\mathrm{V}'}|^2$

- what happens when the two H_W approach each other (GIM in action)?

- RBC/UKQCD is working very hard on this (and ε_K) and there are first promising (but exploratory) results

long distance effects: Rare kaon decays



Two new experiments dedicated to rare kaon decays NA62 (CERN) and KOTO (J-PARC) are running

- FCNC (W-W or γ/Z-exchange diagrams)
- deep probe into flavour mixing and SM/BSM due to suppression in the SM
- can determine V_{td}, V_{ts} and test SM

$K_L \to \pi^0 \nu \bar{\nu}$

- KOTO (J-PARC)
- direct CP violation
- exp. BR $\leq 2.6 \times 10^{-8}$
  theory BR $3.0(3) \times 10^{-11}$
- GIM → top dominated and charm suppressed, purely SD

$K^+ \to \pi^+ \nu \bar{\nu}$

- NA62 (CERN)
- CP conserving
- exp. BR $1.73(^{+1.15}_{-1.05}) \times 10^{-10}$
  theory BR $0.911(72) \times 10^{-10}$
- small LD contribution, candidate for lattice

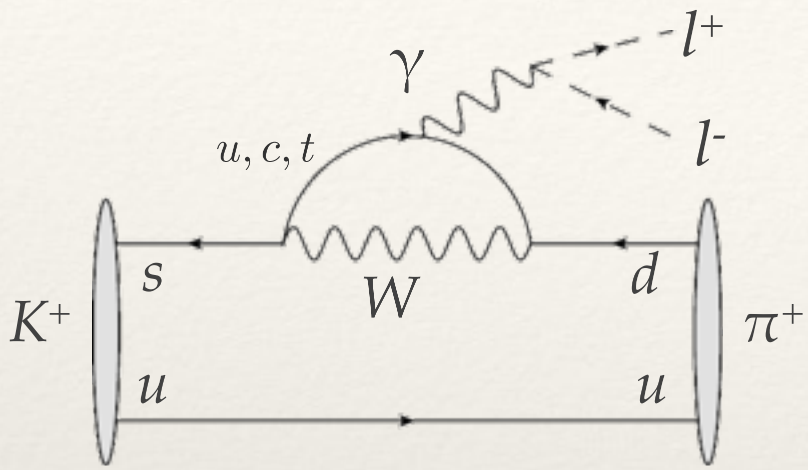
compute in lattice QCD

$K^+ \to \pi^+ l^+ l^-$ $K_S \to \pi^0 l^+ l^-$

- 1-photon exchange LD dominated
- indirect contribution to CP-violating rare K_L decay
- SM prediction mainly ChPT
- lattice can predict ME and LECs
- experimenters will be able to look at these channels as well

Rare kaon decays $K^+ \to \pi^+ l^+ l^-$

N. Christ et al. arXiv:1507.03094
arXiv:1602.01374



LD contribution given through $K \to \gamma^*$ contribution which is computed as

$$\mathcal{A}_\mu = (q^2) \int d^4x \langle \pi(p) | T\left[J_\mu(0) H_W(x) \right] | K(k) \rangle$$

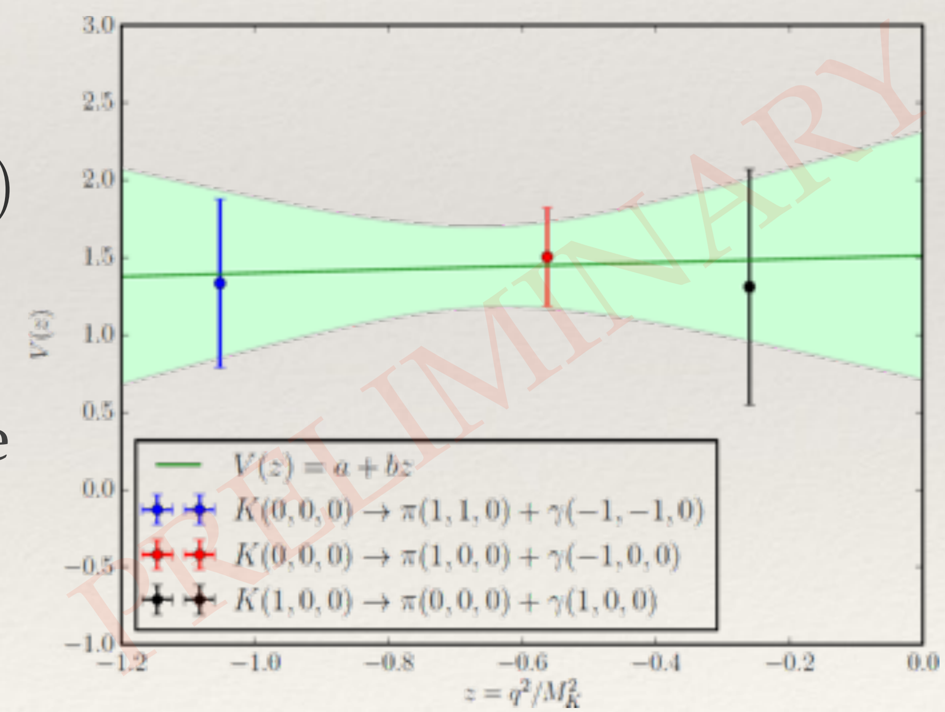
dominant operators: $Q_1^q = (\bar{s}_i \gamma_\mu^L d_i)\,(\bar{q}_j \gamma_\mu^L q_j)\,, \qquad Q_2^q = (\bar{s}_i \gamma_\mu^L q_i)\,(\bar{q}_j \gamma_\mu^L d_j)$

Decay amplitude in terms of elm. transition form factor

$$A_i = -\frac{G_F \alpha}{4\pi} V_i(z)(k+p)^\mu \bar{u}_l(p_-)\gamma_\mu \nu_l(p_+) \qquad (i = +, S)$$

$$V_i(z) = a_i + b_i z + V_i^{\pi\pi}(z)$$

- the a_S and a_+ can be extracted from experiment or lattice
- a_S parameterises also the CP-violating contribution to the K_L decay



Summary/Conclusions

- considerable set of SM parameters, spectra and matrix elements now reliably and precisely predicted in full lattice QCD
- Flavour Lattice Averaging Group (FLAG)
- precision such that isospin breaking effects in matrix elements and spectra needs to be taken into account
- long distance effects:
  - neutral kaon system
  - rare kaon decays (new experimental facilities!!!)

  with loads of new questions and theoretical problems and potential impact on SM and BSM phenomenology

Summary/Conclusions

- this talk is by far not inclusive:
  - $K \to \pi\pi$, g-2, …
  - finite-T,μ
  - BSM
  - …

34th International Symposium on Lattice Field Theory

LATTICE 2016

University of Southampton
24–30 July 2016

registration and abstract submission open!

Looking forward to see you in Southampton!

http://www.southampton.ac.uk/lattice2016/