The Possibility of Polarisation in the LHeC Ring-Ring Scenario

D.P. Barber\textsuperscript{a}, H.-U. Wienands\textsuperscript{b}, M. Fitterer\textsuperscript{c}, H. Burkhardt\textsuperscript{c}

\textsuperscript{a}Deutsches Elektronen-Synchrotron (DESY), Germany
University of Liverpool, UK
Cockcroft Institute, Daresbury, UK

\textsuperscript{b}SLAC National Accelerator Laboratory, Menlo Park, California 94025, USA

\textsuperscript{c}CERN, CH-1211, Geneva 23, Switzerland

1 September 2010
See Kurt Aulenbacher’s review of $e^\pm - p$ schemes: 27/09/2010
Linac/recirculator - ring schemes
Max Klein: at ICHEP Paris: Future Machines and Projects 24.7.2010
Linac/recirculator - ring schemes
Max Klein: at ICHEP Paris: Future Machines and Projects 24.7.2010

Based on SLC, ILC and LHC experience.

Workpackages for CDR
- Baseline Parameters [Designs, Real photon option, ERL]
- Sources [Positrons, Polarisation]
- RF Design
- Injection and Dump
- Beam-beam effects
- Lattice/Optics and Impedance
- Vacuum and Beam Pipe
- Integration and Layout
- Interaction Region
- Powering Issues
- Magnets
- Cryogenics

BINP Novosibirsk
BNL
CERN
Cockcroft
Cornell
DESY
EPFL Lausanne
KEK
Liverpool U
SLAC
TAC Turkey
The ring - ring option.
Max Klein: at ICHEP Paris: Future Machines and Projects 24.7.2010

Newly built magnets installed on top of the LHC, bypassing LHC experiments in the twenties.

10 GeV injector into bypass of P1
2 \cdot 10^{10} \phi \ (LEP: 4 \cdot 10^{11})
\sim 10 \text{ min filling time synchronous ep + pp}

\int L = 100 \text{ fb}^{-1} \cdot E_c = 60 \text{GeV}
The ring - ring option.
Max Klein: at ICHEP Paris: Future Machines and Projects 24.7.2010

Based on HERA, LEP, LHC experience.

Workpackages for CDR

Baseline Parameters and Installation Scenarios
Lattice Design [Optics, Magnets, Bypasses, IR for high L and 1°]
Rf Design [Installation in bypasses, Crabs]
Injector Complex [Sources, Injector]
Injection and Dump
Beam-beam effects
Impedance and Collective Effects
Vacuum and Beam Pipe
Integration and Machine Protection
Powering Issues
e Beam Polarization
Deuteron and Ion Beams

BINP Novosibirsk
BNL
CERN
Cockcroft
Cornell
DESY
EPFL Lausanne
KEK
Liverpool U
SLAC
TAC Turkey
Precision QCD and Electroweak Physics
Max Klein: at ICHEP Paris: Future Machines and Projects 24.7.2010

Structure functions $[F_2, F_{UL}, F_{L}, g^Z, g^Z, F_{2cc}, F_{2bb}, F_{2ss}]$ in p/d and A
Quark distributions from direct measurements and QCD fits
Strong coupling constant $\alpha_s$ to per mille accuracy
Gluon distribution in full x range to unprecedented precision
Standard Model Higgs
Single top and anti-top quark production at high rate (5pb)
Electroweak couplings (light and heavy quarks and mixing angle)
Heavy quark fragmentation functions
Charm and beauty below and way beyond threshold at per cent accuracy
Heavy quarks in real photon-proton collisions [LR option]
Jets and QCD in photoproduction and DIS
Gluon structure of the photon
....
## Ring-ring design parameters
Max Klein: at ICHEP Paris: Future Machines and Projects 24.7.2010

<table>
<thead>
<tr>
<th>electron beam</th>
<th>RR</th>
<th>LR</th>
<th>LR</th>
</tr>
</thead>
<tbody>
<tr>
<td>e- energy at IP[GeV]</td>
<td>60</td>
<td>60</td>
<td>140</td>
</tr>
<tr>
<td>luminosity (10^{32} \text{cm}^{-2}\text{s}^{-1})</td>
<td>17</td>
<td>10</td>
<td>0.44</td>
</tr>
<tr>
<td>polarization [%]</td>
<td>40</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>bunch population (10^3)</td>
<td>26</td>
<td>2.0</td>
<td>1.6</td>
</tr>
<tr>
<td>e- bunch length [mm]</td>
<td>10</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>bunch interval [ns]</td>
<td>25</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>transv.emit. (\gamma_{x,y}) [mm]</td>
<td>0.58, 0.29</td>
<td>0.05</td>
<td>0.1</td>
</tr>
<tr>
<td>rms IP beam size (\sigma_{xy}) [\mu m]</td>
<td>30, 16</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>e- IP beta funct. (\beta^*_{x,y}) [m]</td>
<td>0.18, 0.10</td>
<td>0.12</td>
<td>0.14</td>
</tr>
<tr>
<td>full crossing angle [mrad]</td>
<td>0.93</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>geometric reduction (H_{bg})</td>
<td>0.77</td>
<td>0.91</td>
<td>0.94</td>
</tr>
<tr>
<td>repetition rate [Hz]</td>
<td>N/A</td>
<td>N/A</td>
<td>10</td>
</tr>
<tr>
<td>beam pulse length [ms]</td>
<td>N/A</td>
<td>N/A</td>
<td>5</td>
</tr>
<tr>
<td>ER efficiency</td>
<td>N/A</td>
<td>94%</td>
<td>N/A</td>
</tr>
<tr>
<td>average current [mA]</td>
<td>131</td>
<td>6.6</td>
<td>5.4</td>
</tr>
<tr>
<td>tot. wall plug power [MW]</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>proton beam</th>
<th>RR</th>
<th>LR</th>
</tr>
</thead>
<tbody>
<tr>
<td>bunch pop. (10^{11})</td>
<td>1.7</td>
<td>1.7</td>
</tr>
<tr>
<td>tr.emit.  (\gamma_{x,y}) [\mu m]</td>
<td>3.75</td>
<td>3.75</td>
</tr>
<tr>
<td>spot size (\sigma_{xy}) [\mu m]</td>
<td>30, 16</td>
<td>7</td>
</tr>
<tr>
<td>(\beta^*_{x,y}) [m]</td>
<td>1.8, 0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>bunch spacing [ns]</td>
<td>25</td>
<td>25</td>
</tr>
</tbody>
</table>

“ultimate p beam”
present record \(N_p=1.3 \times 10^{11}\)
1.7 probably conservative

Design also for deuterons (new) and lead (exists)

RR = Ring − Ring
LR = Linac − Ring

Tentative: 8.7.2010
Max Klein: at ICHEP Paris: Future Machines and Projects 24.7.2010
For this talk, the ring-ring option:
a super HERA...

The ring-ring option would use conventional technology and would provide both polarised electrons and positrons.
LEP
Polarisation from the Sokolov-Ternov effect at 46 GeV and above – the highest energy so far!

Highest polarization achieved:

Vertical polarisation by the S-T effect, no rotators. “Deterministic” harmonic orbit correction.

46 GeV, $\tau_{st} \approx 5$ hours
HERA

The first and only $e^\pm$ ring to supply longitudinal polarisation at high energy
— via the Sokolov-Ternov effect — also at 3 IP’s simultaneously!
$\approx 30$ GeV, $\tau_{st} \approx 30$ mins. Depolarisation not too strong.
Perfectly balanced parameters
HERA electron/positron ring 2001 ---

\[ \mathbf{P}_{\text{meas}} \parallel \hat{n}_0 \]

"Transverse" polarimeter (TPOL)

"Longitudinal" polarimeter (LPOL)

Polarisation vertical in the arcs – to drive the Sokolov-Ternov effect
HERA MiniRotator: Buon + Steffen

56 m (“short”) → no quads.

27 – 39 GeV, both helicities, variable geometry

NO INTERNAL QUADRUPOLES!
3 pairs of rotators (so max. Sokolov-Ternov polarisation = 83 %), solenoids on, no beam-beam
Some theory and phenomenology
• Electrons (positrons) in storage rings can become spin POLARISED due to emission of synchrotron radiation: Sokolov–Ternov effect (1964).

• The polarisation is perpendicular to the machine plane in simple rings.

• The maximum value is then $P_{st} = 92.4\%$.

BUT!

• Sync. radn. also excites orbit motion. This leads to DEPOLARISATION!

• In any case, the value of the polarisation is the same at all azimuths — time scales.
The T-BMT equation.

\[
\frac{d\vec{S}}{ds} = \vec{\Omega}(\gamma, \vec{v}, \vec{B}, \vec{E}) \times \vec{S}
\]

Periodic solution \( \hat{n}_0 \) on closed orbit.

The real unit eigenvector of:

\[
R_{3\times3}(s + C, s)\hat{n}_0 = \hat{n}_0
\]

\( \hat{n}_0 \) is 1-turn periodic: \( \hat{n}_0(s + C) = \hat{n}_0(s) \)

\( \hat{n}_0 \): direction of measured equilibrium radiative polarisation.

Closed orbit spin tune \( \nu_0 \): number of precessions per turn around \( \hat{n}_0 \) for a spin on the closed orbit. Extract from the eigenvalues of \( R_{3\times3}(s + C, s) \)
Spin motions

- Protons: largely deterministic — unless various noise (e.g. IBS).
- Electrons/positrons:
  If a photon causes a spin flip, what are the other $\approx 10^{10}$ photons doing? $\implies$

  Stochastic/damped orbital motion due to synchrotron radiation
  + inhomogeneous fields
  + spin–orbit coupling via T–BMT
  $\implies$ spin diffusion i.e. depolarisation!!!

Self polarisation: Balance of poln. and depoln. $\implies$

$$P_\infty \approx P_{BK} \frac{1}{1 + \frac{1}{(\frac{r_{dep}}{r_{BK}})^{-1}}} \quad (P_{ST} \rightarrow P_{BK})$$

In any case:

$$\tau_{dep}^{-1} \propto \gamma^{2N} \tau_{st}^{-1} \quad \text{(actually a polynomial in } \gamma^{2N} \text{)}$$

$\implies$ Trouble at high energy!
Spin–orbit resonances

\[ \nu_{\text{spin}} = k + k_I \nu_I + k_{II} \nu_{II} + k_{III} \nu_{III} \]

\( \nu_{\text{spin}} \): amplitude dependent spin tune \( \approx \) closed orbit spin tune = precessions /turn on CO

- Orbit “drives spins” \( \implies \) Resonant enhancement of spin diffusion
  AT FIXED ENERGY EVEN AWAY FROM RESONANCES!

- Resonance order: \( |k_I| + |k_{II}| + |k_{III}| \)

- First order: \( |k_I| + |k_{II}| + |k_{III}| = 1 \)  \( \text{e.g. SLIM like formalisms.} \)

- Strongest beyond first order:
  synchrotron sidebands of first order parent betatron or synchrotron resonances

\[ \nu_{\text{spin}} = k + k_i \nu_i + k_{III} \nu_{III}, \quad i = I, II \text{ or } III \]

- Proton-style resonances strengths are NOT helpful for estimating depolarising rates!
Sidebands of parent first order betatron resonances: a useful approximation

\[ \tau_{dep}^{-1} \propto \frac{A}{(v_0 \pm Q_y)^2} \quad \rightarrow \quad \tau_{dep}^{-1} \propto \sum_{m_s=-\infty}^{\infty} \frac{A B(\xi; m_s)}{(v_0 \pm Q_y \pm m_s Q_s)^2} \]

\( A \) is an energy dependent factor

\( B(\xi; m_s) \)'s: enhancement factors, contain modified Bessel functions

\( I_{|m_s|}(\xi) \) and \( I_{|m_s|+1}(\xi) \) depending on the modulation index

\[ \xi = \left( \frac{a \gamma \sigma_\delta}{Q_s} \right)^2 \]

in a simple flat ring.

\[ \Rightarrow \text{very strong effects at high energy — dominant source of trouble} \]

Analogous formula for sidebands of first order synchrotron resonances.
R. Assmann, SPIN2000, Osaka, Japan
For longitudinal polarisation the polarisation vector must be rotated into the longitudinal direction before an IP and back to the vertical afterwards \( \Rightarrow \) spin rotators.

- Vertical bends must be neutralised – otherwise \( \hat{n}_0 \) is not vertical \( \Rightarrow \) strong depolarisation

- Depolarisation can be strongly enhanced by misalignments, regions where the polarisation vector (\( \hat{n}_0 \)) is horizontal between spin rotators etc, etc.....
Linear spin matching

Skip the invariant spin field and the Derbenev-Kondratenko formula for today!

Heuristics instead!
N.B. this is not the trivial business of ensuring that a spin behaves as required in a string of dipoles!!

\[ \vec{S} \approx \hat{n}_0(s) + \alpha \hat{m}_0(s) + \beta \hat{l}_0(s) \]

\( \alpha, \beta \): 2 small spin tilt angles — have subtracted out the big rotations!

\[ \hat{M}_{8 \times 8} = \begin{pmatrix} M_{6 \times 6} & 0_{6 \times 2} \\
G_{2 \times 6} & D_{2 \times 2} \end{pmatrix} \]

acting on \( \vec{u} = (x, x', y, y', l, \delta) \) and \( \alpha, \beta \)

This is the SLIM formalism for estimating depolarisation analytically at first order (Chao 1981).

To minimize depolarisation:
minimize appropriate bits of \( G_{2 \times 6} \) for appropriate stretches of ring
\[ \Rightarrow \] lots of independent quadrupole circuits.
The structure of SLICKTRACK

- **Main**
  - Read optic/layout and control files
  - Choose misalignments
  - Correct the C.O. “in line”
  - 6x6 formalism
  - 6x6 symplectic linearised optic wrt C.O.
  - 6x6 damped linearised optic wrt C.O.
  - Orbit excitation from symp. E.V.s

- **Main**
  - 6x6 damped non-linear M−C orbit tracking ‘big photon noise’ 3−D spin also beam−beam
    \( \rightarrow \tau_{\text{eq}} \rightarrow p_{\text{eq}} \)
  - 6x6 damped linearised M−C orbit tracking ‘big photon noise’ 3−D spin also beam−beam
    \( \rightarrow \tau_{\text{eq}} \rightarrow p_{\text{eq}} \)
  - 8x8 damped linearised M−C spin−orbit tracking with ‘big photon noise’
    8x8 covariance mat.
    \( \rightarrow \tau_{\text{eq}} \rightarrow p_{\text{eq}} \)
    as in analytical (D−K)
  - 6x6 damped linearised M−C orbit tracking with ‘big photon noise’
    \( \rightarrow \text{equil. 6x6 cov. mat. as in analytical} \)
    \( \rightarrow \tau_{\text{eq}} \rightarrow p_{\text{eq}} \)
  - Polarisation with linearised spin motion using 8x8 matrices + D−K
    \( \rightarrow \text{analytical} \)
    \( \rightarrow \tau_{\text{eq}} \rightarrow p_{\text{eq}} \)

= old (SLICK)  = New (done)  = Old + New (done)  = New (in progress)  = Planned

Also: acceleration and spin flip
Spin coordinates

\[ \dot{S} = \sqrt{1 - \alpha^2 - \beta^2} \hat{n}_0 + \alpha \hat{m} + \beta \hat{l} \]

Estimating depolarisation by M-C simulation \( \alpha^2 + \beta^2 << 1 \)

\[ \Delta P \approx -\frac{1}{2} \Delta \langle \alpha^2 + \beta^2 \rangle = -\frac{1}{2} \Delta (\sigma^2_\alpha + \sigma^2_\beta) \quad \implies \quad \frac{dP}{dt} \approx -\frac{1}{2} = -\frac{1}{2} \frac{d}{dt} (\sigma^2_\alpha + \sigma^2_\beta) \]

Spin–orbit covariance matrix

\[
\begin{pmatrix}
\sigma^2_x & \sigma_{xx'} & . & . & . & . \\
\sigma_{x'x} & \sigma^2_{x'} & . & . & . & . \\
. & . & \sigma^2_{x''} & . & . & . \\
. & . & . & \sigma^2_\delta & . & . \\
. & . & . & . & \sigma^2_\delta & . \\
\sigma_{\beta x'} & . & . & . & . & \sigma^2_\alpha \sigma_{\alpha \beta} \\
\end{pmatrix}
\]
Spin–orbit maps for sections

For linearised spin motion (SLIM/SLICK):
\[ \hat{M} = \begin{pmatrix} M_{6 \times 6} & 0_{6 \times 2} \\ G_{2 \times 6} & D_{2 \times 2} \end{pmatrix} \]

The \( G_{2 \times 6} \times (x, x', y, y', \Delta l, \delta) \) delivers changes to the 2 small angles \( \alpha \) and \( \beta \)

For full 3–D spin motion:
\[ \hat{M} = \begin{pmatrix} M_{6 \times 6} & 0_{6 \times 3} \\ G_{3 \times 6} & D_{3 \times 3} \end{pmatrix} \]

The \( G_{3 \times 6} \times (x, x', y, y', \Delta l, \delta) \) delivers rotations around \( \hat{n}_0, \hat{m}_0, \hat{l}_0 \)

The beam–beam (non–linear) kicks are applied at single points
Some advice to calculators:

Software, for linearised spin motion, that does not intrinsically include synchrotron motion and which does not include misalignments and orbit correction is useless above a couple of GeV.

Software that cannot, in addition, account for full 3-D spin motion, and which therefore cannot consistently account for synchrotron sidebands, is useless above about 10 GeV.
A first look!!
Flat ring with vertical polarisation near 60 GeV

- $Q_x = 123.83$
- $Q_y = 85.62$
- $\sigma_{vco} = 75$ microns
- R.m.s. tilt of $\hat{n}_0 \approx 4$ mrad near the peak polarisation. No harmonic closed-orbit spin matching so far.
- Radiative energy loss: 430 MeV per turn
- $a \gamma_0 \frac{\sigma_{v0}}{\gamma_0} \approx 0.13$. 
$$Q_s = 0.06, \xi \approx 5$$  An example of just one misalignment
\( Q_s = 0.1, \xi \approx 1.9. \quad \text{An example of just one misalignment} \)
Energy dependence of maximum polarisation — an example of just one misalignment
Summary on the flat ring

- Initial calculations suggest that vertical polarisation would not be impossible with modern very good alignment.
- The dependence on $Q_s$ is qualitatively as expected.
- The attainable equilibrium polarisation is highest at low energy as expected.
Longitudinal polarisation

- Need rotators $\Rightarrow$ need serious spin matching.
- Rotators must be compatible with the constraints of the environment.
- Do Siberian Snakes help to suppress the effect of synchrotron sidebands by suppressing the oscillations of $\alpha\gamma$?
- Naive use of snakes kills the Sokolov-Ternov polarisation!
- So need asymmetric distribution of radiation.
A suggestion by Ya. Derbenev and H. Grote

n means $\hat{n}_0$ here !!!
LHeC rotators
very strong depolarisation – of course!

But we can switch spin-orbit coupling off/on to see what does what: the $G$ matrix

So make the interaction region and rotators spin transparent in software.

Diagnostics! – since 1982
The D–G set-up near 60 GeV

- $Q_x = 124.36$
- $Q_y = 88.80$
- $\sigma_{vco} = 75$ microns
- R.m.s. tilt of $\hat{n}_0 \approx 8$ mrad near the peak polarisation. No harmonic closed-orbit spin matching so far.
- Radiative energy loss: 586 MeV per turn
- $\alpha \gamma_0 \frac{\sigma_{\gamma_0}}{\gamma_0} \approx 0.13$.
- An ideal thin lens snake which is transparent for orbital motion.
- $\nu_0$ is almost independent of machine energy: around 0.41 (not 0.5 – because of the rotators).
Perfect alignment and with the IR $G$ matrix off

$Q_s = 0.1$

Equilibrium polarizations with misalignments

- Sokolov-Ternov Polarization
- Total Polarization (linear approx.)
- Total Polarization (M-C)
**An example of just one misalignment and with the IR $G$ matrix off**

$Q_s = 0.06$

*Equilibrium polarizations with misalignments*

- Sokolov-Ternov Polarization
- Total Polarization (linear approx.)
- Total Polarization (M-C)
An example of just one misalignment and with the IR $G$ matrix off

$Q_s = 0.10$

Equilibrium polarizations with misalignments

- Sokolov-Ternov Polarization
- Total Polarization (linear approx.)
- Total Polarization (M-C)
Summary on the model ring with rotators and a snake

- The maximum S-T polarisation is limited by the need for the asymmetric radiation distribution.
- In these calculations $\hat{n}_0$ is tilted from the vertical twice as much as for the flat ring $\implies$ lower polarisation compared to the maximum S-T polarisation.
- With this rotator, $\hat{n}_0$ is tilted in the arcs away from design energy.
- Initial indications that the snake supresses the synchrotron sidebands. Much more investigation needed.
- --- the first time in the field that this topic has been investigated.
- Essential to provide optical spin matching of the IR and arcs — obviously!.
- The dogleg rotator fits the need to bring the electron beam down to the proton beam.
- A practical snake design is needed.
- Optical spin matching is a big but necessary challenge.
- Harmonic closed orbit spin matching should be tested.
- In any case it would be essential to align the ring extremely well – but modern rings do have good alignment.
This has been a very first look but:

with modern alignment and the use of the Derbenev-Grote scheme,

optical spin matching
will be well worth pursuing as the next step.
SPINERELLA AND THE UGLY SISTERS
ENERGIA AND ΖΕΥΜΙΝΟΣΑ

SPIN IS IN

B. MONTAGUE
1980

By Brian Montague during the lead-up to LEP and HERA polarisation.